Newsletter

Vol. 6 Spring 2019

NCTS
National Center for Theoretical Sciences

Content

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Director's Message

As one of the major and perhaps the most important mission of NCTS is the nutrition of younger generation in theoretical sciences, I would like to share with you some interesting recent and upcoming events.

The 2019 NCTS Spring Day was held on Mar. 21. It is the day that all NCTS postdoc fellows share their research interests and seek for possible collaborations. It is also the day that NCTS Young Theoretical Scientists were awarded. Prof. Ye-Kai Wang (NCKU) and Zhi-You Chen (NCUE) are the recipients awarded this year.

There are some extra efforts made to share to beauty and joy of mathematics to general public. We celebrated the international π-Day at NCHU with more than 100 high school students on-site and another 1500 students connected online. The lecture and show of Peter Frankl, scheduled on Mar 30 is another moment of enjoyment.

For undergraduate students and graduate students, there are events that you don't want to miss. The 2019 USRP, Undergraduate Summer Research Program, are about to be announced. In this program, undergraduate students are encouraged to experience the hand-on projects and make friends from many different places. The joint Summer School with MSRI on Toric Varieties is the highlight of this summer for those who are interested in algebraic geometry and related topics.

There are much more that you can expect and experience at NCTS here. Come join the fun of working on mathematics at NCTS!

NCTS Director
Prof. Jungkai Chen

Highlights of Events

12/7	NCTS Interdisciplinary Lectures - Applying Physics to Mathematics
12/27 – 12/29	ICCM
1/2 – 1/3	Workshop on Global Differential Geometry
1/23 – 1/28	The 7th Trilateral Meeting (Australia - Italy - Taiwan) on Nonlinear PDEs and Applications
2/15	Young Dynamics Day
3/8	2019 Optimization Workshop
3/10 – 3/14	Workshop on Harmonic Analysis
3/14	The International Day of Mathematics - π day
3/15 – 3/16	NCTS Workshop on Applied Mathematics
3/21	NCTS 2019 Spring Day

Upcoming Events

5/20 – 5/22	2019 Spring Probability Workshop
5/20 – 5/24	Arithmetic of Function Fields and Diophantine Geometry
5/23 – 5/25	2019 NCTS Workshop on Dynamical Systems
5/27 – 5/31	International Workshop on Geometric and Harmonic Analysis
6/17 – 6/21	NCTS Workshop on Analytic Trends in Complex Geometry and Related Fields
7/1 – 8/9	USRP

ICCM Sze Lim Lectures

Tokieda Tadashi

Professor of Department of Mathematics at Stanford University

Research Interest:
Mathematical Physics

Interests:
Inventing, collecting, and studying toys

The ICCM Sze Lim Lecture supported by ICCM Sze Lim Lecture Fund, was held on December 7th, 2018, with the aim of promoting Mathematics development to public. It was jointly hosted and organized by the Department of Mathematics at NTNU and NCTS.

We were pleased to invite Prof. Tadashi Tokieda, faculty of Department of Mathematics at Stanford University, who devoted in studying toys that uniquely reveal and explore real-world surprises of applied mathematics.

Two lectures were given. One academic lecture, the topic "Applying Physics to Mathematics" was delivered at NCTS in the morning, and there were 129 participants. Not only students, professors, and researchers from universities and research institution, but also students from high school came to participate in this event.

Another was a general lecture with the topic "Two Magic Tricks" at NTNU auditorium in the evening, and there were about 200 participants, the attendees were from secondary schools, universities, and research institution.

These two lectures attracted a wide range of audiences. Seeing that the subject "Mathematics" was no longer so difficult that public cannot understand; it can be fun for everyone.

Highlights of Courses

2/18 – 6/14	NCTS-Taiwan Math School: Boltzmann Equations
2/18 – 6/14	NCTS-Taiwan Math School: Algebraic Surfaces
2/18 – 6/14	NCTS-Taiwan Math School: Introduction to Machine Learning and Deep Learning with Python
2/18 – 6/14	NCTS Course on PDE: Nonlinear Waves and Related Physical Problems
3/8 – 3/29	NCTS Course on Mathematical Finance
3/14 – 3/28	NCTS Mini-Course on Dynamical Systems
5/10 – 5/20	NCTS Courses on Dynamical Systems
6/20 – 7/4	NCTS Mini-course: Art and Practice of Regression Trees and Forests
7/8 – 7/19	NCTS Summer Course: Mathematical Modeling and Analysis of Infectious Diseases
7/15 – 7/23	NCTS Summer Course: Spectrally Negative Levy Processes and Applications
7/29 – 8/9	MSRI-NCTS Joint Summer School on ToricVarieties

Winter Courses

1/11	NCTS Winter Course on Flowering Biology
1/22 – 1/29	NCTS Winter Course on Abelian Varieties: Analytic Theory
2/11 – 2/14	NCTS-Taiwan Math School: Introduction to Parallel Programming for Multicore/ ManycoreClusters

Upcoming Courses

6//20 - 7/4	Art and Practice of Regression Trees and Forests by Prof. Wei-Yin Loh (University of Wisconsin-Madison)
7/1 - 7/5	Dynamical Systems: Combinatorics in Holomorphic Dynamics by Prof. Russell Lodge (Indiana State University)
7/8 - 7/19	Mathematical Modeling and Analysis of Infectious Diseases by Prof. Hiroshi Nishiura (University of Hokkaido), Joseph Wu (Hong Kong University), Don Klinkenberg (RIVM), Naveen K. Vaidya (San Diego State University) and several doctors from Centers for Disease Control
7/15 - 7/23	Spectrally Negative Levy Processes and Applications by Prof. Xiaowen Zhou (Concordia University)
8/19 - 8/30	Cloud and Machine Learning by Dr. I-Hsin Chung (IBM)
Date TBC	SC4SC by Prof. Kengo Nakajima (The University of Tokyo)
Date TBC	Deep Learning on HPC by Prof. Rio Yokota (Tokyo Tech)

NCTS Undergraduate Summer Research Program will start on July 1st - August 9th, 2019.

There will be 6 programs in USRP this year. The programs are as follows:

1. Compressive Sensing and Phase Retrieval by Prof. Pengwen Chen (NCHU)
2. 3D High-Performance Computation of PNP Equations in Ion Channels by Prof. Tzyy-Leng Horng (FCU) and Prof. Tai-Chia Lin (NTU)
3. Hidden Markov Models and Its Applications by Prof. Gi-Ren Liu (NCKU) and Prof. Yuan-Chung Sheu (NCTU)
4. Generating and Detecting Deepfakes by Prof. Tzer-Jen Wei (NDHU)
5. Mathematical Analysis and Computation of Synchronization and its Applications by Prof. Ming-Cheng Shiue (NCTU) and Prof. Chun-Hsiung Hsia (NTU)
6. Structures of Unitary Groups and Applications by Prof. Chia-Fu Yu (AS)

And NCTS will also hold a Joint Summer School on Toric Varieties with MSRI, which starts from July 29th to August 9th. We'll have Prof. David Cox (University of Massachusetts, Amherst) and Prof. Henry Schenck (Iowa State University) as speakers. See our website to learn more about this joint school.

2018 ICCM

December 27-29, 2018 @NCTS

The second annual meeting of the International Consortium of Chinese Mathematicians (ICCM) was held at the campus of National Taiwan University from Dec. 27 to Dec. 29, 2018. This meeting is jointly hosted by National Center for Theoretical

Sciences (NCTS), the Institute of Mathematics of Academia Sinica and Shing-Tung Yau Center at National Chiao Tung University. There were about 300 participants attending this annual meeting of ICCM 2018. The activities of ICCM annual meetings include academic talks, prize presentations, reports and discussions of ICCM matters.

The highlight of this event was the announcement of ICCM best paper award sponsored by TCL. Twenty-six medals were awarded at the ICCM annual meeting. Each awardee received the award of USD $5000. The article "Fixed energy universality for generalized Wigner matrices" by NCTS Distinguished Scholar, Horng-Tzer Yau (Harvard) and his collaborators was particularly mentioned in the award ceremony. Prof. Horng-Tzer Yau visits NCTS on a regular basis and has very close cooperation with NCTS. Another awarded article "A Minkowski type inequality for hypersurfaces in the Anti-deSitter-Schwarzschild manifold" was published by Mu-Tao Wang (Columbia), Pei-Ken Hung and Simon Brendle. Prof. Mu-Tao Wang was actively involved in the Topical Program in Differential Geometry and Geometric Analysis.

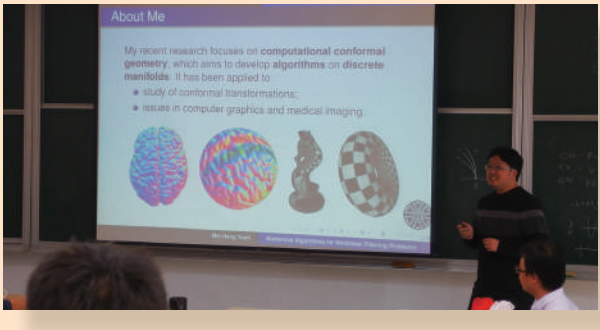
The other highlights during those 3 days were the public talk "Contribution of geometry to modern science and technology" given by Prof. Shing-Tung Yau (Harvard) and the Distinguished Lecture by Prof. George Lusztig (MIT) on "Positive structures in Lie theory".

The next ICCM meeting will be held in Tsinghua University, Beijing on June 9-June 14, 2019.

2019 NCTS Young Dynamics Day

February 15, 2019 @NUK

The "Young Dynamics Day" conference is a one-day workshop which has been hosted each February since 2012 by NCTS. This conference is initiated by Professor Kuo-Chang Chen (National Tsing-Hua University), and the purpose of this workshop is to provide a forum for young generations within the dynamical systems community in Taiwan. Speakers are encouraged to present their research interests or projects with ambition, contents need not be finished works of their own. In this regard, the workshop is a meeting for people to share their "mathematical dreams", instead of limited to their research accomplishments. This year we invited 7 40-minute speakers (the venue of this year was the National University of Kaohsiung) and the topics were around the dynamics of the Boltzmann equation, mathematical problem of traffic, pattern formation theory, the mathematical model of the infection of diseases and the learning problem in artificial intelligence. There are over 45 participants and Tamkang University is the tentative venue of the 2020 Young Dynamics Day conference.



2019 The International Day of Mathematics

3.14 Happy π Day !

March 14, 2019 @NCHU

“Have you ever heard that you can do experiments on math?” The first celebration activity of International Day of Mathematics in Taiwan was held by The Mathematical Society (TMS), National Center for Theoretical Science Math Division (NCTS), and National Chung Hsing University (NCHU). There was a “Math Experiment Calls for Thousands of People” proposal, having over 1,500 people (including teachers, students or people who are interested in math) participate in this experiment that made it the largest mathematical experiment ever in Taiwan’s history. The purpose of Buffon’s Needle Problem that posed by the famous French mathematician Georges-Louis Leclerc Comte de Buffon in 18 centuries, is to explore “If a needle is thrown on a parallel-lined paper, what is the probability of intersecting of needle and the parallel-lines?” By collecting thousands results of the experiment, we can roughly estimate the value of “π” and the number had posted on Facebook page of “Numeracy Lab”. This experiment made math no more doll and abstract but more fun and concrete.

This crow-pulling experiment was designed to respond to the project of International Mathematical Union (IMU) called The International Day of Mathematics, which has been applied to UNESCO this year and will be officially starting on March 14th 2020. During “The International Day of Mathematics”, we can promote mathematical activities and provide opportunities to the public, having people to take a closer but different look at math. This day allows the public to recognize math-related issues and understand the importance of math in life.

Many math teachers were invited to “Pop-up Math Booth” to display their math collections and introduce them to people with simple theories. One of our speakersas well as the well-known math science writer, Iwei Lai, shared that besides paper and pencil calculations, drawing lots in Taiwanese temples and constellation analysis statistics are actually related to math theories. We also invited Mr. Steven Chuang, a card consultant of the famous magician Qian Liu, to let the audience draw cards randomly that match the number of 3.14159256. Taiyuan Cheng, an Education Science Researcher from Taichung City Government Education Bureau, said that the government is excited to witness the success of this meaningful event, “With those easy concepts, math is more acces-




sible." NCTS Director Jungkai Chen, one of the organizers, encouraged students on-site to learn math well,"The better you learn, the more advantage you'll have in the future."

"I also expect that this activity will attract more people to pay more attention to math and more teachers can be inspired then try other more life-connected materials in teaching."

The organizers said that the accuracy of the result of the estimated value of "π" is not the goal we pursued in this activity. The main purposes are attracting children's attention and arousing their curiosity in math. Deep in many people's minds, math is monotonous, complex and intricate. What we aim for is to overthrow these unfair stereotypes that textbooks had implant in math. Through various accesses, people are aware of the facts that life is full of math and there are many different aspects of math not shown on the textbooks we used in school. We hope that the next generation can get to know more about math in a life-oriented way and lay a solid foundation of theoretical science for our future life.

NCTS Workshop on Harmonic Analysis

March 10-14, 2019 @NCTS

The workshop brought together mathematicians from Korea and Japan whose research focuses on Harmonic analysis and its applications. For example, Prof. Akihiko Miyachi talked about the boundedness of multilinear Fourier multipliers operators. Prof. Sanghyuk Lee talked about the uniform estimates for resolvent of Laplacian operators in which the connection to restriction conjecture was also discussed. Prof. Naohito Tomita continued to talk about his joint work with Prof. Miyachi on the same topic. There were also many exciting talks given by Prof. Hitoshi Tanaka, Joonil Kim, Hiroki Saito, Jason Cunanan, Bae Jun Park. During the workshop the interactions between the participants were very active in particular the feedbacks from graduate students were very positive. In all, it was a successful activity for people working on Harmonic analysis in Asia. We will continue to hold Harmonic analysis events in the near future.

2019 NCTS Spring Day

March 21, 2019 @NCTS

The 2019 NCTS Spring Day was held on Mar. 21. During this day, each postdoc fellows gave a 30 minutes talk on their work or their interests. NCTS research assistants were also invited to join this event to know more about the seniors' work. There were 11 talks among the research fields of Number Theory, Algebraic Geometry, Differential Geometry, Differential Equations, Scientific Computing and Interdisciplinary Studies.

We also had Award Ceremony of Young Theoretical Scientists and their short presentations. Prof. Ye-Kai Wang (NCKU) and Zhi-You Chen (NCUE) are the recipients awarded this year. Congratulations to these 2 outstanding scholars!

Young Theoretical Scientist Award

Dr. Zhi-You Chen completed his Ph.D. in Mathematics at National Central University. He is now an associate professor in National Changhua University of Education. Dr. Chen has been working mainly on nonlinear elliptic partial differential equations and systems. His research topics include the studies on some elliptic systems, which arise from the Maxwell-Chern-Simons model. Solutions of such systems can be classified as "topological" or "non-topological" depending on their behavior near the spatial infinity. His main contribution is about the existence, multiplicity, and structure of topological and non-topological solutions of the Maxwell-Chern-Simons system and related systems.

First, he proved the non-degeneracy of the corresponding linearized systerns at single-vortex topological solutions of Maxwell-Chern-Simons O(3) sigma model, and additionally, proved the uniqueness of single-vortex topological solutions. He also proved the existence of all radial solutions and classified the structure. Second, he dealt with single-vortex non-topological solutions of semi-local Chern-Simons model and studied the sharp range of the integrability of nonlinear terms over the whole space. Moreover, he has succeeded in proving the uniqueness of topological solutions of Chern-Simons CP(1) model when the Chern-Simons constant is sufficiently large. The main idea is to combine the non-degeneracy of linearized systems at singlevortex topological solutions, which provides a powerful method to prove the uniqueness of multi-vortex topological solutiorx for various related systems. More recently, he proved the uniqueness of topological solutions in the case of multiple vortices.

These remarkable results can be regarded as a breakthrough in this research area, and attract wide attentions from researchers in mathematics and physics. The major part of these works has been done by Dr. Chen himself. Therefore, there is no doubt that he is one of the best young analysts in Taiwan.

Dr. Ye-Kai Wang got his Ph.D. in Mathematics at Columbia University, New York and is now an assistant professor in National Cheng Kung University. He works in the fields of differential geometry, partial differential equations and general relativity.

Dr. Wang studied the problem of isometrically embedding a closed surface in hyperbolic spaces. Together with his collaborator, he proved that any metric on a 2-sphere with Gauss curvature no less than -1 admits a $C^{1,1}$ isometric embedding in the standard hyperbolic space. Solution to such problems were first given by Nirenberg and Pogorelov if the metric has Gauss curvature strictly greater than the sectional curvature of the ambient space forms. Besides extending Nirenberg's PDE method to accommodate the degenerate case of the isometric embedding equation, Dr. Wang made ingenious use of 2-dimension Ricci flow in his proof. This demonstrates well that he has an exceptional ability in combining PDE skills and geometric insights in conducting research on geometric analysis.

Besides, Dr. Wang is one of the few experts who initiated research on rigidity of time-flat surfaces in a spacetime. Time-flat surfaces are analogues of surfaces sitting in a hyperplane in the Minkowski spacetime. It was introduced by Bray and Jauregui who found a monotone property of Hawking mass along foliations of time-flat surfaces. In a joint work with his collaborators, Dr. Wang obtained both local and global rigidity theorems for co-dimension two submanifolds in Minkowski spacetimes and Schwarzschild spacetimes. So far these are the only rigidity results known in the literature concerning time-flat surfaces.

Dr. Wang is also among researchers who initiated study of Miknowski formulae in spacetimes. The classic Minkowski formulae relate the integral of σ_k-curvature of a hypersurface in the Euclidean space to a suitably weighted integral of the σ_{k+1}-curvature. Such formulae play a basic role in various rigidity theorems for surfaces in Euclidean spaces. Together with his collaborators, Dr. Wang was able to establish Minkowski formulae in Schwarzschild spacetimes and derived Alexandrov type theorems. Dr. Wang's achievement in this field shows that he is indeed an expert in both Riemannian and Lorentzian geometry.

We hope that this award will help Dr. Chen and Dr.Wang make further contribution and keep producing important impact in their research fields.

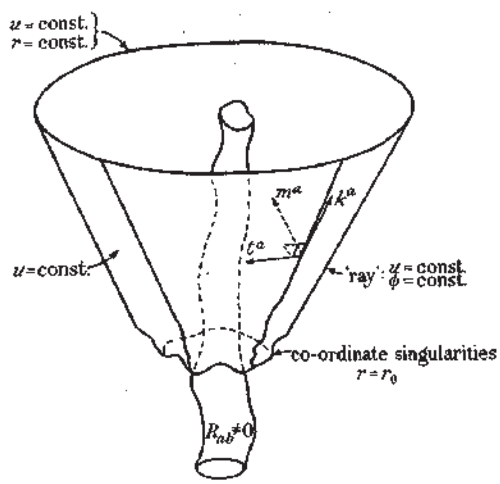
NCTS Research Spotlight

Prof. Ye-Kai Wang is an assistant professor at the Department of Mathematics of National Cheng Kung University. His research interest lies in mathematical general relativity.

The object of mathematical general relativity is the study of geometric and analytic properties of the Einstein equations

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab}$$

on 4-dimensional Lorentzian manifolds. The Einstein equations form a nonlinear second-order partial differential system with 10 unknown metric coefficients and 10 equations. In 1962, Bondi-Metzner-van der Burg [1] presented a method to solve the vacuum Einstein equations $T_{ab}=0$, which was later generalized by Sachs. The idea is to follow the outgoing null geodesics along which the gravitational waves travel.



The metric takes the Bondi-Sachs form

$$-\frac{V}{r}e^{2\beta}du^2 - 2e^{2\beta}dudr + r^2h_{AB}\left(dx^A - U^A du\right)\left(dx^B - U^B du\right).$$

Thanks to the second Bianchi identity and a determinant condition on hAB, one can distinguish 6 main equations ($R_{rr} = R_{rA} = R_{rB} = 0$) and the other 4 are consequence of the main equations. Moreover, the main equations separate into 4 hypersurface equations ($R_{rr} = R_{rA} = h^{AB}R_{AB} = 0$) and 2 evolution equations. The metric coefficients enjoy a remarkable hierarchy. One first prescribes h_{AB} freely on the initial null hypersurface and then integrates β, U^A, V in order using the hypersurface equation. The evolution equations then tell us how to evolve h_{AB}.

The Bondi-Sachs approach provided the first convincing evidence that an isolated system emits gravitational waves and there is a nonlinear mass loss effect. Moreover, it shows that the symmetry group at null infinity (Bondi-Metzner-Sachs group) is larger than the Poincare group.

Recently my research focuses on the conserved quantities at null infinity described by Bondi-Sachs formalism. In their seminal 2008 paper, Mu-Tao Wang and Shing-Tung Yau defined a quasi-local mass that addresses the first problem of R. Penrose's famous list of unsolved problems in classical general relativity. Together with Po-Ning Chen, they proved that the limit of quasi-local mass recovers the Bondi-Sachs energy-momentum 4-vector at null infinity. With Jordan Keller and Yau[4], we evaluate the total angular momentum and center-of-mass at null infinity by taking the limit of Chen-Wang-Yau quasi-local angular momentum and center-of-mass [3] as definition. As a final remark, there is currently no consensus in the physicists community on what the correct definition of total angular momentum at null infinity should be.

For readers interested in mathematical general relativity, we recommend the survey [2]. For more background knowledge of this article, please see [5].

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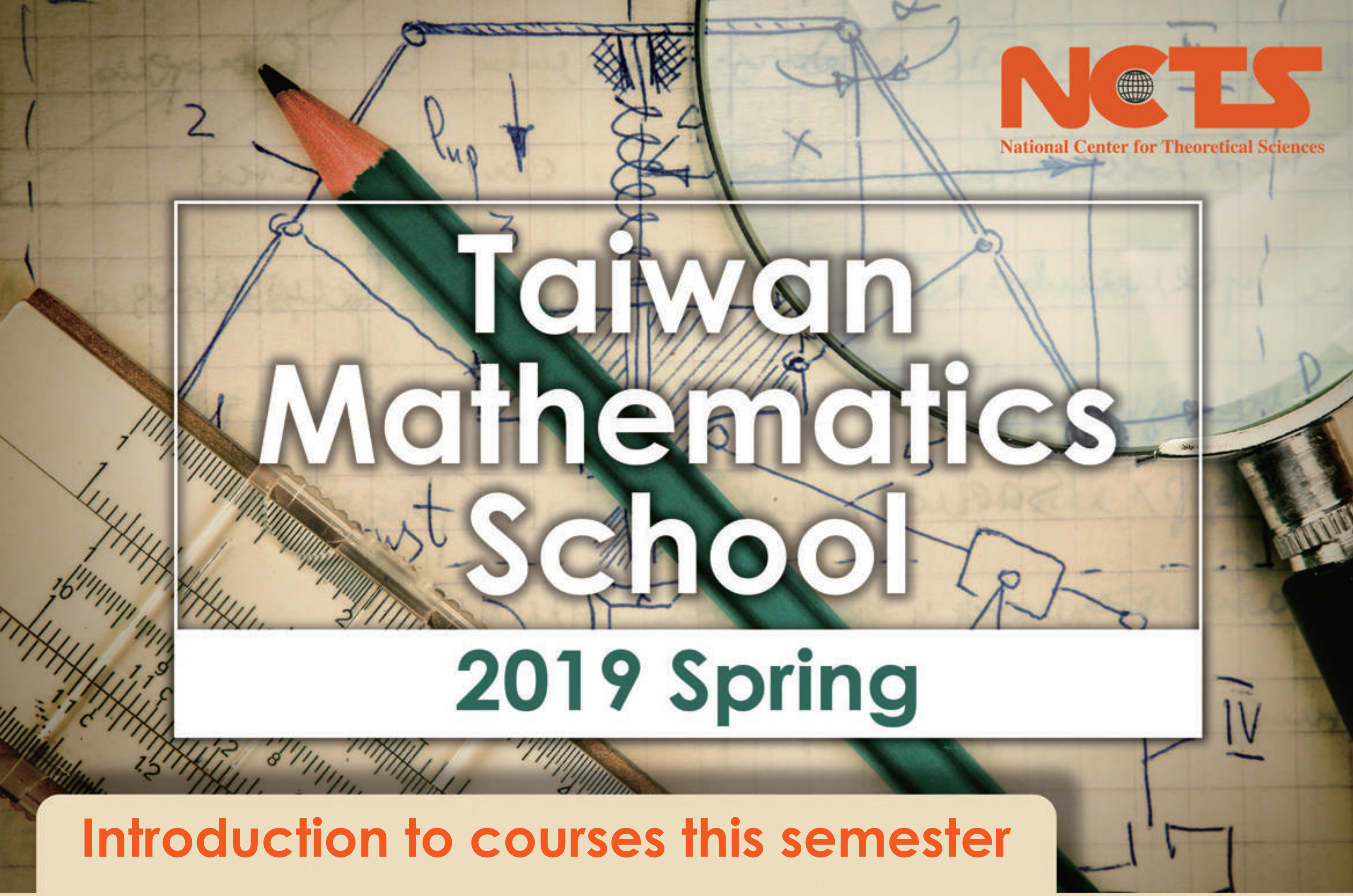

Taiwan Mathematics School 2019 Spring

Introduction to courses this semester

Course **Boltzmann Equations**

Lecturer | Willie Hsia (NTU)

This semester we'll have Prof. I-Kun Chen of NTU and Chun-Hsiung Hsia of NTU to give an advanced course of Boltzmann Equations. This course conducts research ideas and technical machinery for the investigation of Boltzmann equations. We aim to prepare graduate students for the research of the Boltzmann equations.

We shall start with the 89 annals paper of Diperna and Lions to introduce the most important ingredients of the Boltzmann equations. In particular, the basic conservation laws, H theorem and entropy inequalities. There are three useful types solution concepts : distributional solution, mild solution and renormalized solutions. We shall follow Diperna and Lions's idea to show the advantages of these three concepts and see how they go well together to combine with other useful ideas to overcome the subtleties to obtain the L^1 existence theory of cut-off Boltzmann equations. We then connect it to our recent research work on steady state solutions of Boltzmann equations. Unlike the evolution equations on flat domains, we do not have the well-known velocity averaging lemma to help to gain the compactness in the construction of weak solution. Regarding the issue of steady state solution on bounded domain, we need new key features to improve the previous results in literature. We shall explore some new ideas according to our recent work in this course.

Course Algebraic Surfaces

Lecturer | Jungkai Chen (NTU)

In this course, Prof. Jungkai Chen of NTU is going to give an introduction to the theory of algebraic surfaces, for motivated students who have some basic knowledge in algebra and geometry and would like to explore the beauty of algebraic geometry. Our purpose is to give a modern treatment of surface theory with minimal model theory and some other recent technique in mind, and leave the classical material as applications of general theory.

The course outline is as follows:

(1) Review on algebraic curves/ compact Riemann surfaces
(2) Affine varieties, projective varieties (and some commutative algebra)
(3) Cohomology (and some homological algebra)
(4) Vanishing theorems
(5) Divisors and projective embedding
(6) Intersection theory
(7) Riemann-Roch theorem
(8) Cone of curves
(9) Birational maps
(10) Minimal models program (in general and in dimension two)
(11) Canonical bundle formula and pushforward of canonical sheaves
(12) Biraional classification
(13) Pluricanonical maps and surfaces of general type
(14) Elliptic surfaces
(15) Surfaces with Kodaira dimension 0
(16) K3 surfaces
(17) Ruled and rational surfaces
(18) Surface singularities

Course Introduction to Machine Learning and Deep Learning with Python

Lecturer | Yen Lung Tsai (NCCU)

This course is designed for students majoring in math, or for those who are interested in applying math to solve real-world problems through programming. The focus of this course is deep learning, which is a very hot topic recently. Our target students are those who had little programming experiences before, so that we can introduce basic Python programming at the beginning, aiming to help students be familiar with those packages related to both numeric and symbolic computations.

We will introduce machine learning and start with techniques of deep learning in the first few weeks. In the middle of the semester, we will ask students to find a real-world problem as their final project and to solve the problem by applying deep learning method.

After taking this course, we hope this course will inspire students'greater interest and also enhance their curiosity on math since they will find math are both useful and powerful in our daily life. As for Python, they can always use this "in" skill on school work, future-career searching, career-oriented issues or even simply just on math ideas. This handy, lifetime-worthy programming language will help our students dig more deeply and broaden their vision of the beautiful world of math.

Interview with Prof. Leon Simon (Part II)

• The Beginning Stage of Career

Q. Have you ever made serious mistakes in your papers?

As far as I know, I've never had a paper that was really wrong, but of course there had been expository glitches. I think most long papers, long technical papers, have minor defects of some sort or another. The important thing is, can the minor expository glitches be filled in by any competent mathematician? If so, that's merely an expository error. But if it's a genuine gap that even the author doesn't know how to fix, then that is a totally different story. But you can never be sure, of course, you know. There is no such thing as a hundred percent surety on any of those sort of things. It's part of what mathematicians have to live with, right? Someone could walk up to you tomorrow and point out a serious error in one of your papers. It could happen.

Q. Your career seems to start from working on elliptic PDEs. Could you tell us how you entered the field of geometric measure theory (GMT) after that? You published a famous lecture note in GMT based on your lectures around 1983. It seems that there were many exciting activities during the year in Canberra. Why and what did it happen there?

Okay, yes, my thesis work was on PDEs -- gradient estimates for quasilinear elliptic equations. I was lucky I got into this field about the time that De Giorgi, Bombieri, Giusti, and Miranda were looking at the minimal surface equation.

Q. So who was your thesis supervisor?

Jim Michael -- He wasn't really working in this area, but I was fortunate that he was at that time getting interested in PDE. I was very fortunate to be his student -- he was not quick or broad, but had impressive ability to think deeply and originally about fundamental questions. That had a strong effect on my approach to mathematics.

Q. How did you learn about your thesis problem?

Well, I basically learned that, I think, by myself. Somehow I heard about it. It was sort of in the air at that point and everyone was excited about that field. What got me interested initially was that in my undergraduate work -- in the final year of undergraduate -- we have to write a brief thesis. It didn't have to be very original, and, with Jim Michael's encouragement and guidance, I wrote my undergraduate thesis on applications of the topological degree and fixed point theorems to nonlinear problems. And I chose quasilinear elliptic equations as one of the applications, and one of the main things there is the minimal surface equation. So that's how I got into it. I really didn't know much about it at that stage. We didn't have a PDE book like the famous one by Gilbarg or Trudinger. I had to use the textbook by C.B. Morrey, his book "Multiple Integrals in the Calculus of Variations." It is a wonderful book but sort of chaotically written. I just read it by myself. Jim Michael didn't know that material. He was interested in those things but he didn't know that literature. The other text that you could use was the book by Ladyzhenskaya and Uraltseva, the Russians who were very prominent in that field. But both those books were difficult to read. Particularly Morrey's -- a wonderful book but he gave the impression that it was one of his first drafts and he decided just to leave it as it was.

Q. When S.-T. Yau was a student at Berkeley, he also took Morrey's PDE course...

Yes, Yau was a graduate student at the time Morrey's book had just come out, and he went to Morrey's lectures. Morrey wasn't a good expositor unfortunately, apparently. He was a very nice man, by the way, very gentlemanly and kind man, but he wasn't a good expositor. Apparently after about three weeks, there was not a single student left in the class except for Yau. He stayed the whole time because Yau was smart enough to see how deep C.B. Morrey was and how much he could learn from him. So he stayed in that class and learned that. Unfortunately I didn't have that advantage. I used his book, and I tried to learn that material. It took quite a bit of effort. Both Yau and I revere Morrey as a great mathematician.

Q. So this happened as you were a graduate student.

Er, yes, I was struggling and trying to get a thesis. I was maybe in my third year of graduate work.

Q. Did your advisor give you a problem?

No, no, I eventually realized that I should work on gradient problems. Eventually it became clear to me that everything was sort of standardized, and the only thing that was specific to a given problem that made one quasilinear problem harder than another was whether or not you could prove the appropriate gradient estimates for solutions. It took me a while to realize that, but I did eventually realize it, and I remember going into Jim Michael's office one day and said, "I want to work on gradient estimates." And he said, "Oh, okay. Fine, you can work on gradient estimates." But then around that time Bombieri, De Giorgi, and Miranda proved the gradient estimate for the minimal surface equation in arbitrary dimension, finally extending the 2-dimensional work of Finn and others from almost 20 years earlier. A big breakthrough exactly at the time I was starting work for my thesis. I had to read the paper, of course, so I started really working hard, reading their paper, which was written in Italian by the way. I learned a lot from that paper and the slightly later related work of De Giorgi and his coauthors. Of course it became very clear to me, both from his earlier very famous Hölder continuity estimate for solutions of divergence form linear equations with discontinuous coefficients, and his work on minimal surfaces, that De Giorgi was a real genius, an incredible mathematician. He didn't publish that much on minimal surfaces, actually, only his main work on regularity theory and then his joint papers with a few of the other Italians.

Q. Did you meet De Giorgi later?

I did meet him later, yes, but he didn't speak much English, strangely. So he was very old school in that regard -- all of the young Italians mathematicians at that time of course spoke very fluent English. But he was also a very nice man. A very pleasant man.

So I started working on gradient estimates and I eventually got some of the ideas in their paper worked out. But then there was a long struggle. I don't remember exactly how long it went on, but it was quite -- maybe almost a year that I was not getting anywhere. Eventually I realized I could prove the Sobolev inequality -- the general type. It was not quite the Sobolev inequality in the eventual Michael-Simon paper -- it was a bit less precise (involving the root mean square of principle curvatures rather then the mean curvature), but it was fine for the PDE applications that I had in mind. My method -- this is one of the main things in my thesis -- is that you can prove a result like that by slicing with hyperplanes and using an inductive procedure. In order to do that, I also have to learn some differential geometry, of which I knew none of up to that point. For instance, I didn't even know about the Coarea Formula, and, in order to slice, you need to know the Coarea Formula. So I had to work out the Coarea Formula myself. Later I found it was well known, Federer 3.2.22. That's the Coarea Formula. So of course I was happy to see it there because I was a little bit unsure of whether I got the proof of that right because it was a bit tricky, and I saw that in Federer's book it was even more general. He does arbitrary slicing and all the rest of it. I just needed slicing by hyperplanes. So I was very lucky that worked out and I generalized Bombieri-De Giorgi-Miranda to a much bigger class of quasi-linear equations and that was my thesis.

And it was natural then that I got interested in GMT after that, because I was interested in the Italian school -- De Giorgi and all those people. By that stage I had found Federer's book, which had only come out a couple of years earlier. Very lucky for me. It was exactly the right period to be in that field; several important results had just come out, and also Federer's book.

Q. It was published in the '70s.

Yes, 1969, I think, was when his book came out. I found it in the library in I think 1970 or early 1971. I looked at the title: "Geometric Measure Theory." Hmm! That sounds interesting I thought. So I borrowed that book from the library, and of course I couldn't understand much of it not only because it was advanced material, but because it was written in that manner that famously makes it very difficult to read.

Q. After that, you moved to U.S.?

Yes, when I finished my thesis, I briefly got a job at another university in Adelaide. There were two universities in Adelaide at that point. There was the oldest university, where I was: University of Adelaide. And then there was another newer university: Flinders University. I got offered a lectureship at Flinders University, so I went there. But then in late 1972, after I've been there for a year, to my surprise, I got a letter one day -- of course there was no email; the internet wasn't dreamed of then except in science fiction -- we couldn't imagine there would be such a thing. Anyway, I got a letter in the mail from Stanford University, from Gilbarg, saying: Would you like to apply for a three year assistant professor position? I couldn't believe that! I found out later that the reason was that one of the examiners in Australia of my thesis -- you have to have two outside examiners in Australia for your thesis -- one of them was Neil Trudinger. He was back in Australia at that point, but he had just come from Stanford. He got his PhD with Gilbarg at Stanford. So he sent a copy of the thesis to Gilbarg and that's how my offer came about. I was very lucky about that. So with great trepidation, I went off to Stanford University. I was so green. I knew nothing basically at that point -- I mean I had huge gaps in my mathematical knowledge. Because in Australia we were using the English system, which means you focus almost exclusively on the narrow area of the thesis, and no graduate classes. That has changed to some extent now to provide something a bit more in line with the US system, but in those days the focus was very narrow.

Anyway, so I went to Stanford as an Assistant Professor, for about 4 years. Of course I learned a lot of mathematics while I was there. I had to, otherwise I couldn't teach graduate classes! I had to teach. So that worked out okay. I was fortunate that the senior faculty (Gilbarg, Osserman, Phillips in particular) were so supportive, and also that there was a really incredible group of younger faculty, including Yau. I learned a lot from Yau and Rick Schoen (who had just come on the scene as a graduate student), and we had a lot of fun.

• Running into GMT

Q. That was the '70s. My question about this is... You wrote up these lecture notes, so we're curious... knowing now what happened during the time, since you moved back and organized them...

Yes, so I was in Stanford for about four years, then I was offered a job in Minneapolis. A permanent job. A professorship in Minneapolis. In the meantime, I had an offer from Melbourne University. So I only stayed in Minneapolis for one year, but that was an important year. Bob Hardt and I worked, ultimately successfully, on the boundary regularity problem for area minimizing hypersurfaces during that year, so that was a crucial year for me. I probably wouldn't have worked on that problem if I hadn't met Bob in Minneapolis, and during that time Bob also acquainted me with many of the finer points of GMT. I was there for a year and then I went back to take up the Melbourne job, and I was in Melbourne for about three years. Then I headed off to the Australian National University (ANU) in Canberra, so I was in Canberra from '81 to '86 and that's when I started writing the GMT notes. I was giving a course there on GMT and I have several graduate students at that point. It was a good thing to do at that time.

Q. During that period of time, did you get any visitors there?

Yes, a lot. Fortunately, we got support from the federal government. The so-called Centre of Excellence for Mathematical Analysis (CMA) -- the government was funding about twenty centres and one of them was in mathematics, the CMA. We got significant funding for visitors. Way more than we would ever have dreamed of otherwise. We could have lots of visitors and we had an extremely active group there. Lots of mathematics was done during that period. At one point, we had an almost German-speaking department. It was pretty amazing. We had all sorts of activities. The Friday seminars were somewhat of famous. We had this afternoon seminar that used to go from about 2 PM until about 5 PM. It wasn't people talking about their own work. It was a working seminar where we worked through other people's work. And that was very valuable for everyone in that group (usually about 10 or 12 people, sometimes more, including faculty, visitors, and graduate students). I'd really recommend that for anyone who has got a research group to have a seminar like that. It is a lot of hard work, but extremely valuable. You need young people. The old guys are often too lazy to do the hard work to read all those new papers. We used to take in turns to speak and it was challenging work, preparing those things, because we didn't know initially much about that stuff. We spoke a lot on various papers in geometry -- I remember we looked at some of Yau's papers and some of Rick Schoen's papers and... oh, numerous papers we got through.

Q. So there were so many Germans. Why?

That was partly because I formed contacts in Germany, particularly with Claus Gerhardt in Heidelberg and Stefan Hildebrandt in Bonn. I met a lot of Germans there. They were all interested in minimal surfaces and in particular they were interested in learning about elliptic PDE, the same that I was. They were very much interested in that. They were also interested in learning about GMT, though some of them were a bit reluctant. Because at that time the German background was more in classical minimal surfaces, you know, famous people like Heinz and people of that sort had done really basic work. Mainly in two-dimensional classical minimal surfaces, where you think in terms of mappings, like that. I had a lot of contacts, and initially Klaus Ecker and Gerhard Huisken came as postdocs. Then we invited other people -- like Stephan Luckhaus, who came for a few months. We also had Americans. Brian White spent a lot of time there, almost a year there, I think, at one point, and Rick Schoen also spent a couple of months there. And we had numerous visitors. As I said, we had lots of funding. We were very lucky. Otherwise, we could never have done that.

Q. So it seems then that after that the biggest group in GMT is still in Italy.

Well I think the main focus is still in the US, but yes... in Italy they had a new lease of life in recent times with people like De Lellis and so on. And I think that was partly Ambrosio and De Giorgi's legacy -- it was strange. There was sort of a delayed effect of De Giorgi's legacy. Initially, I think he was just too brilliant. Everyone was just sort of in awe of him and somehow didn't quite strike out on their own to the extent they should have, independently working on hard problems. It took a little while, it took a few years, before you really saw the effect. But then they really came into their own, with people like De Lellis and so on really doing incredibly good work.

Q. In other respect, for example like Dal Maso... they did the gamma convergence business... It's also De Giorgi's legacy...

It's also De Giorgi, yes, yes, that's right. He was very original -- not just in minimal surfaces.

They also got an institute in Pisa, and I think that really helps them a lot. Again, traced back to De Giorgi. So De Giorgi, C.B. Morrey, Ladyzhenskaya, and Uraltseva. They were my heroes in mathematics. Tremendously powerful people. There's also Federer, of course. He's very admirable even though his book is extremely hard to read. It's quite amazing that book of Federer actually. There is so much in it, and most of us only know a small fraction of what's in there, really. It's just incredible, and even though you might not necessarily like his style of writing, all the details are there if you've got the patience to weed them out. He doesn't cheat by leaving gaps for the reader to fill in. It's basically all there. Really very impressive.

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(To be continued...)

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