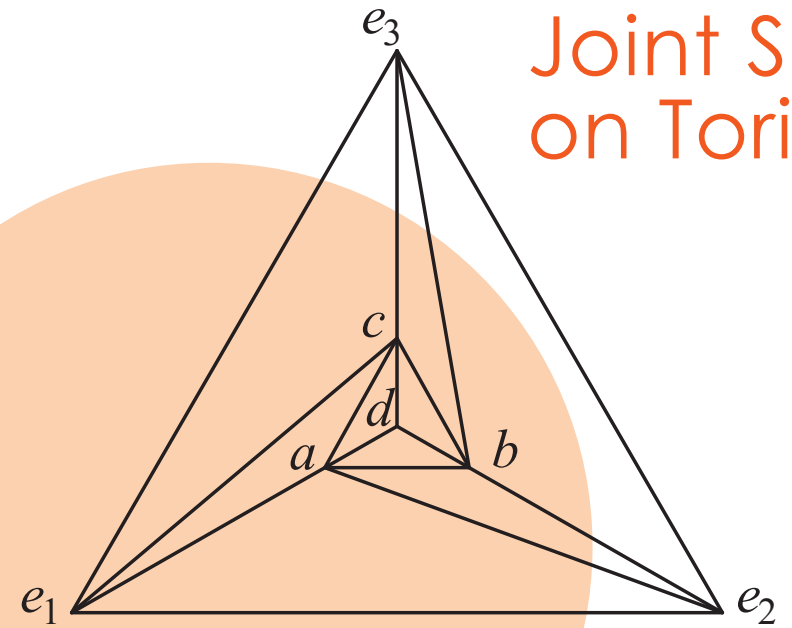
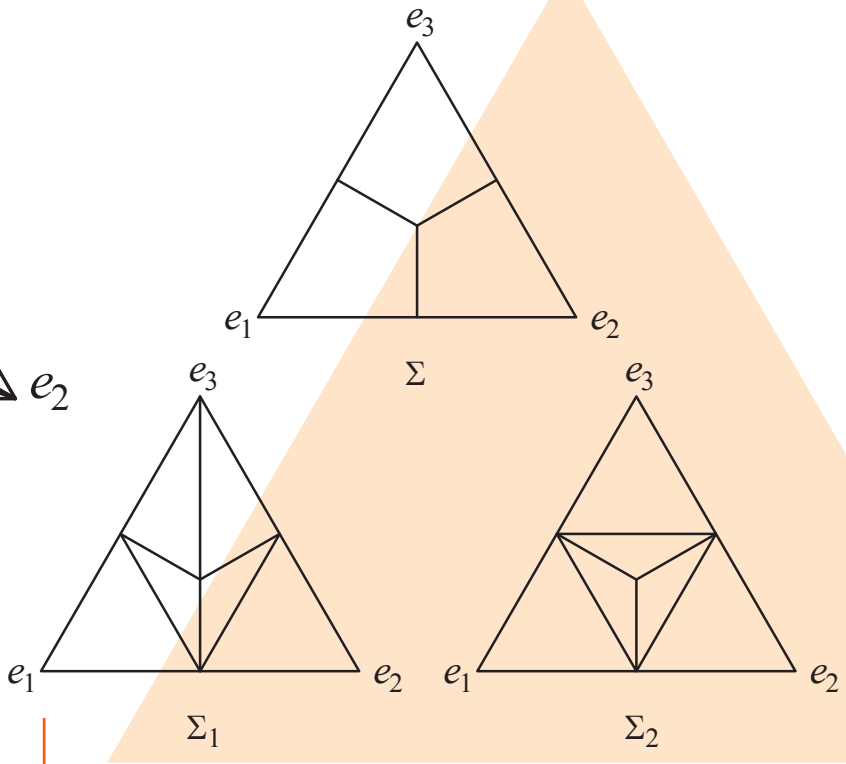
National Center for Theoretical Sciences

Newsletter

Vol. 7 Fall 2019





NCTS National Center for Theoretical Sciences

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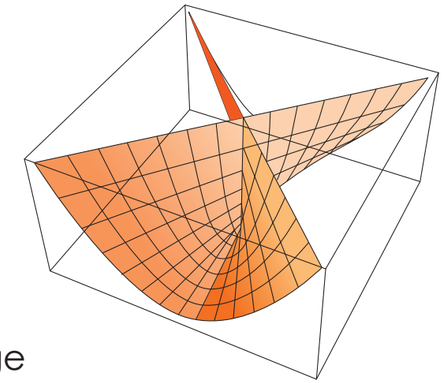
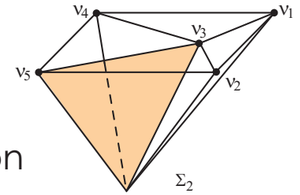
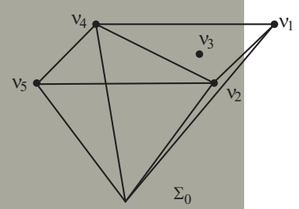
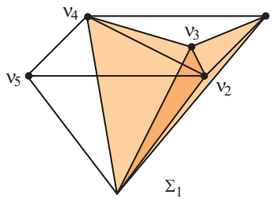
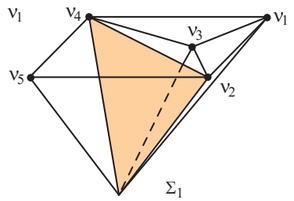










NCTS National Center for Theoretical Sciences

Dear Friends of NCTS,

This summer NCTS hosted the first ever joint summer school with MSRI on toric varieties. There were about 20 selected local students together with 30 selected foreign students (29 from USA and 1 from UK) from abroad in the exciting course. Participating students not only benefited from the elucidating lectures by Cox and Schenck, but also enjoyed the intensive discussion and social events.

Another group of motivated students worked on Undergraduate Summer Research Program aiming for various projects under the guidance of professors and experts. These hand-on project-oriented projects were designed for students who were interested in advanced studies in pure and applied mathematics.

It is also delightful to share the news that NCTS Center Scientist, Chung-Jun Tsai was announced recipient of MOST Ta-You Wu Memorial Award, following former award-winning Center Scientists Fu-Tsun Wei (NCU), Ku-Chien Wu (NCKU) and more.

Having serving the mathematical community for more than two decades, NCTS keeps its commitment to promote frontier mathematical research and provide various opportunities for mathematicians, especially for Taiwanese young talents. It is an active research center with many fun and interesting things happening here. And we welcome more leading mathematicians from Taiwan and from all over the world to join NCTS and have fun!

NCTS Director
Prof. Jungkai Chen

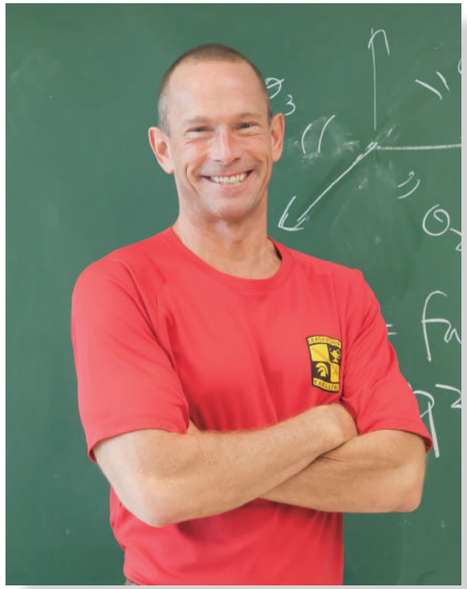
MSRI-NCTS Joint Summer School on Toric Varieties

July 29 -August 8, 2019 @NCTS

For the first time, collaborating with the prestigious Mathematical Sciences Research Institutes (MSRI), National Center of Theoretical Sciences (NCTS) hosted a graduate summer school for a large group of international students. It brought 2 lecturers, 2 teaching assistants, 29 international students, and 18 Taiwanese students all together to have a two-week long period learning in math and enjoying Taiwanese culture. The topic of the summer school was on toric varieties, which were algebraic varieties involving a wonderful interplay between algebra, combinatorics and geometry. Many of the key concepts of abstract algebraic geometry (for example, constructing a variety by gluing affine pieces)

had very concrete interpretations in the toric case, making toric varieties an ideal tool for introducing students to abstruse concepts. The lectures were conducted by two experts on toric geometry, Prof. David Cox and Prof. Hal Schenck, who were also two of authors of the now standard textbook "Toric Varieties, Cox-Little-Schenck." With the help of two teaching assistants, Prof. Kuei-Nuan Lin and Prof. Jen-Chieh Hsiao, students from abroad and Taiwan worked together on assigned exercises and gave presentations on their work. Even though the program ended earlier due to a typhoon strike, the school remained a great success. It provided NCTS an invaluable experience on organizing such a big event and we believed all the participants had a wonderful memory at NCTS.

Mathematical Sciences Research Institutes (MSRI), which was founded by Calvin Moore, Shiing-Shen Chern, and I. M. Singer in 1981 , is one of the prestigious centers of mathematics in the world. MSRI, a place for breeding younger generation, constantly hosts con-



ferences and graduate schools in different areas of mathematics that brings leading experts in the fields to exchange ideas. This makes MSRI as influential as it could be and is the model NCTS aiming for. Prof. Jungkai Alfred Chen, the chair of NCTS with vision to solidify and globalize the Taiwanese mathematical community, not only helped to organize many international conferences, workshops, and short courses, but also made the first advance to collaboratively work with MSRI on running the graduate summer school for a large group of international graduate students. The proposal of running jointly this summer school dated back in mid-2017 and the international students were recruited by MSRI not until early 2019. From this January, NCTS began to manage the local accommodation, entry permit of Taiwan, and the recruitment of Taiwanese students. The format of the summer school was formed by two speakers, two TA's, and finally executed with the help of NCTS staff and the local organizer Dr. Ching-Jui Lai.

The lectures, though quite intense, were delivered excellently by our two passionate lecturers. The material ranged from basic notions of toric varieties, to general theories in algebraic geometry such as sheaf cohomology, differential forms, quotient constructions, etc, and then back to how these theories could be seen and applied in toric geometry. Overall it covered at least eight chapters of the book "Toric Varieties." Even with these wonderful lecturers, it was certainly not easy for all the students to digest all the content. To compensate, the speakers had decided to have the discussion session every afternoon, asking students to work on given problems related to morning lectures. Students were assigned to ten groups of equal size to work together. Moreover, the group members were selected so that each time there was a half international and half local students. With the effort of our diligent TAs, the group members were also exchanged every two or three days so that every student

could find proper partners to talk with. From the students' feedback, the group discussion part of the summer school was where they learn the most, not only for absorbing new material, but also for how to exchange ideas with their international peers. Each group of students were supposed to present their solution to the daily exercise everyday during the summer school. For many students, this was their first time presenting, or first time speaking English in public, or first time talking to a large group of international audience, not to say there were two distinguished professors sitting in. It appeared to be very difficult to many students, but eventually everyone broke their barrier and successfully went on the stage for at least one time. This was even more valuable for Taiwanese students, since in general it required a lot of resource for them to participating an international workshop, and also a very rare chance to have kind audience so

that they can practice presentations in English under less pressure.

Running this summer school earned NCTS tremendous experience for enhancing mathematical maturity of Taiwan students. The local participants came from NTU, NTHU, NCU, NCCU, and NCKU, and ranged from senior undergraduates to postdocs with diverse background in algebraic geometry. To better prepare them for joining the program, a week long bootcamp in fundamental algebraic geometry was held at NCTS with lectures given by Prof. Jiun-Chen Chen and Prof. Shin-Yao Jow. Some students also audited in the summer school and eventually decided to take part in the student discussion. After interviewing with many of our students, they agreed that joining this program did broaden their view not only in mathematics but also on how mathematics can be developed in the international community. We hope this experience can be delivered in Taiwan mathematics community and look forward to hold more similar programs in Taiwan to benefit our students.

Finally, we were happy to hear from our distinguished speakers that "NCTS staff are excellent for working out all the necessities before and during the summer school," from our lovely international guests that "NCTS provides great working environment for learning," and from all of them that Taiwan is a great place to explore and enjoy. Whenever there are chances, surely they will come back to visit and work at NCTS.

The Spring probability workshop is an annual meeting of probabilists in Taiwan. Every year, the organizing committee recruits worldwide active researchers to distribute talks on their recent works and share their research experiences with young probabilists. The aim of this workshop is to create a panel of communications among the domestic and international probabilists and to enhance the international visibility of Taiwan probability group.

The topic of the year of 2019 spring probability workshop was mainly on stochastic PDEs, but it also contained other subjects in probability. The organizing committee invited Krishna Athreya, Tadahisa Funaki, Davar Khoshnevisan, Takashi Kumagai and Carl Mueller as plenary speakers; they were all excellent probabilists. Tadahisa Funaki, Davar Khoshnevisan and Carl Mueller were working on stochastic PDEs and had tremendous contribution in that field. Takashi Kumagai was an expert on potential theory and also many other fields. He was one of organizers of the previous spring workshop and always gave a constant help to probability community in Taiwan. Krishna Athreya, needless to say, was well-known for his achievements in branching processes. Except the above five plenary speakers, the organizing committee also recruited well-reputed speakers worldwide, including Mainland China, Hong Kong, India, Korea, Japan, UK and US. All those talks offered a good chance to learn stochastic PDEs and also new material from other topics in probability. This workshop successfully promoted the Taiwan probability group and created a good atmosphere for all the participants. All local and international scholars had a good time together through conversations, discussions and sharing ideas. Thanks NCTS again for supporting this wonderful workshop.

Highlights of Events

5/3 – 5/5	NCTS Symplectic Expedition: Following Yasha
5/18	The 18th Taiwan Geometry Symposium
5/20 – 5/22	2019 Spring Probability Workshop
5/20 – 5/24	Arithmetic of Function Fields and Diophantine Geometry
5/23 – 5/25	2019 NCTS Workshop on Dynamical Systems
5/27 – 5/31	International Workshop on Geometric and Harmonic Analysis
5/31	The 2019 Symposium for Young Analysts
6/17 – 6/21	NCTS Workshop on Analytic Trends in Complex Geometry and Related Fields
6/21	Theoretical and Numerical Methods for Shape Optimizations
7/9	Mass in General Relativity
7/10 – 7/12	MARCH (Medical AI Research Collaboration Hub) Workshop
9/2 – 9/7	Taiwan-Japan International Symposium: Time Series, Machine Learning and Causality Analysis
9/26 – 10/1	East Asian Symplectic Conference 2019

Upcoming Events

10/17 – 10/18	NCTS Workshop on Mathematical Modelling and Analysis
11/1 – 11/5	The Second Taiwan-Japan Joint Conference on Differential Geometry
11/29 – 12/1	Conference on Algebraic Representation Theory (CART)
12/20 – 12/23	Workshop for Numerical Algebra and Data Science

Date May 20 - 24, 2019

Venue Room 202, NCTS

Arithmetic of Function Fields and Diophantine Geometry

The group working on function field arithmetic in Taiwan had held conferences almost every year over the past ten years. The three organizers of this conference, Dale Brownawell, Chieh-Yu Chang and Matthew Papanikolas, included the topic of Diophantine Geometry this year and invited both established and young number theorists from cross the globe to discuss the state of the art in various areas of number theory and to encourage further research in these areas. They also used this chance to honor Professor Jing Yu, on the occasion of his retirement from National Taiwan University, for his fundamental contributions to the development of function fields over the past four decades.

This conference was mainly hosted by NCTS and was also partially supported by the NSF and the Journal of Number Theory. There were eighteen invited speakers giving 50-minute talks as well as nine junior speakers delivering 25-minute contributed talks. There were about one dozen international graduate students and postdoctors with local expenses supported by NCTS as well as some senior foreign scholars joining this conference.
There were many excellent talks in this conference. The presentations covered a wide range of topics and the contents were deep. For example here were the topics of the most senior invitees, each of which presents some revolutionary work:

a) Wuestholz introduced the idea of approaching some transcendence questions in positive characteristic from his recent point of view of incomplete integrals or periods of 1-motives.

b) Masser introduced the first results on incomplete intersections using the Carlitz structure.
c) Vojta took his very recent work on Diophantine geometry extending Roth's theorem on diophantine approximations over the rational numbers to function fields of arithmetic varieties as the basic step in his project to work toward the analogue of Schmidt's Subspace Theorem.

During the conference questions and discussions between participants and speakers opened up promising directions for further research study, as planned. We believe that this conference brings a lot of benefits to the participants, particularly the junior scholars and graduate students.

There were around 80 participants attending this conference. We arranged enough tea time in the space between speeches to give the scholars and students plenty of time to discuss mathematics and relax. In addition we arranged a half-hour small concert and reception on the first night, and a banquet on the second night. This friendly arrangement allowed the senior scholars an opportunity to relax and interact with each other, regardless of field, and with graduate students and junior scholars.

We consider this a very successful conference and we look forward to having more opportunities to hold similar activities in the future at NCTS to enrich the experiences of students and researchers in mathematics in Taiwan and to maintain its international visibility.

Summer Courses

5/28 – 7/3	Topics on Mathematica Foundation of Machine Learning
6/20 – 7/4	Art and Practice of Regression Trees and Forests
7/1 – 7/5	Dynamical Systems: Combinatorics in Holomorphic Dynamics
7/8 – 7/19	Mathematical Modeling and Analysis of Infectious Diseases
7/15 – 7/23	Spectrally Negative Levy Processes and Applications
7/15 – 7/26	The Mathematics of Gravitational Radiation
7/29 – 8/1	Mathematical Biology
7/29 – 8/9	MSRI-NCTS Joint Summer School on Toric Varieties

Upcoming Courses

9/9 – 2019/1/3	Taiwan Mathematics School: Geometric Measure Theory I
9/9 – 2019/1/3	Taiwan Mathematics School: Dynamical System-Deterministic and Chaotic Behavior of Model Systems
9/9 – 2019/1/3	Taiwan Mathematics School: Topics on Drinfeld Modules and T-Motives

2019 NCTS Workshop on Dynamical Systems

Date 23-25 May, 2019

Venue National Center for Theoretical Sciences

The annual conference "2019 NCTS Workshop on Dynamical Systems" was held at Chee-Chun Leung Cosmology Hall from May 23 to May 25. This conference is initiated by Professor Song-Sun Lin (National Chiao Tung University), and the aim of this workshop is to bring together active researchers from various fields of dynamical systems, to discuss recent and prospective advances in research. This year the organizers arranged two 2-hour mini-courses, which were given by Professor Meng Wu and Professor Zeng Lian. The topics of these mini-courses were centered on fractal geometry and random dynamical systems respectively. Besides, there were also ten 50-minute talks at this conference. The speakers included Yun-Ping Jiang, Lien-Yung Kao, Tomoki Kawahira, Bing Li, Yu-Hao Liang, Ling-min Liao, Russell Lodge, Ruxi Shi, Jui-Pin Tseng and Feng-Bing Wang. The topics of these talks were around complex dynamical systems, ergodic theory, arithmetic dynamics, PDE, ODE and conformal dynamical systems. There were over 50 participants attending this meeting. During the workshop the interactions between the participants were very active. This conference indeed provided a good platform for people working on dynamical systems to share and exchange ideas here. The conference will be continued next year and we believe that there are more and more people working on dynamical system will benefit from it.

International Workshop on Geometric and Harmonic Analysis

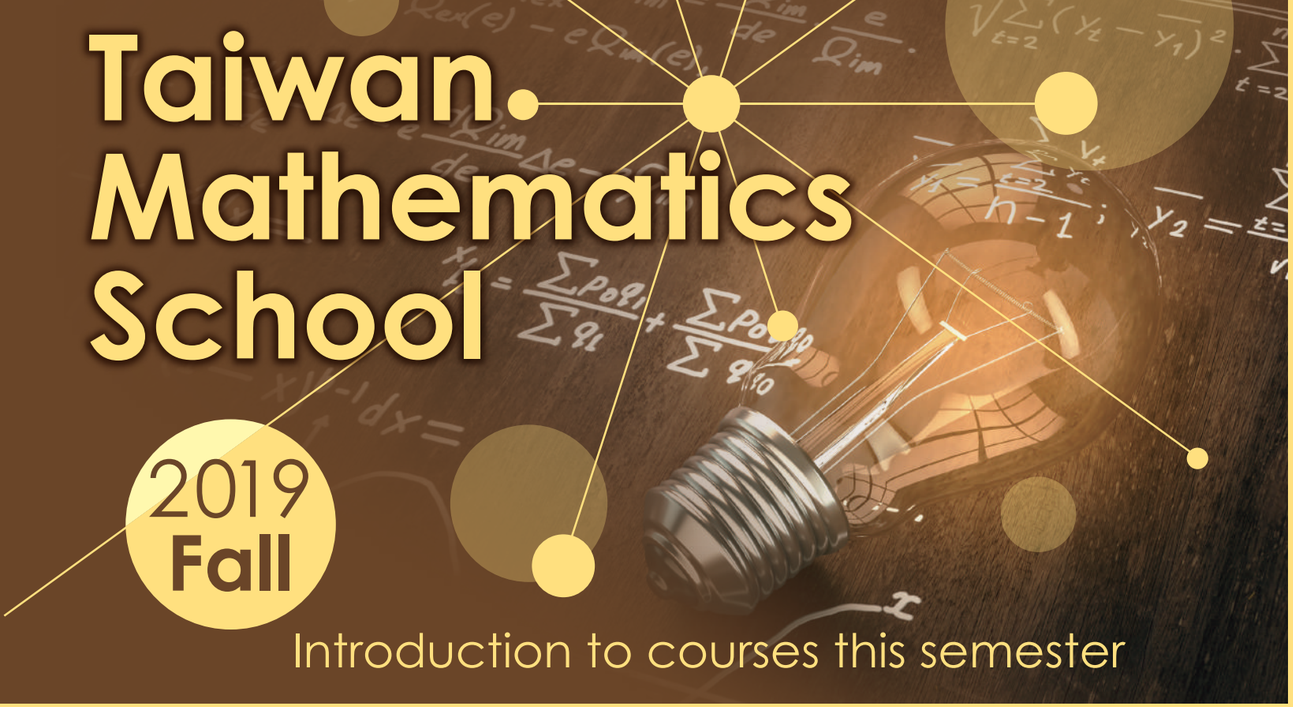
Date 2019.5.27-5.31 Venu R202, NCTS

This activity was initialized by Prof. Der-Chen Chang (Georgetown University) several years ago, and became a tradition at NCTS in recent years. In the past three years, Prof. Chun-Yen Shen (National Taiwan University) joined as a co-organizer. This workshop aims at inviting people who work on modern analysis with focus on Geometric PDE and Harmonic analysis on Riemannian manifolds and Euclidean spaces to share their insights on some active research topics. For example, in this workshop we had Prof. Schultz, Prof. Rung-Tzung Huang, Prof. Markina, Prof. Furutani, Dr. Khalil as well as Dr. Mahmoudi talked about the differential operators on complex manifolds. Prof. Ji Li and Prof. Duong talked about singular integrals and flag singular integrals on manifolds with end but without doubling property. Two speakers from Hong Kong, Prof. Young and Dr. Liu, talked about Hilbert transform along curves and Falconer distances problem. The audiences and speakers benefited greatly by this successful workshop. It is expected such workshop will be held again in the future.

Taiwan-Japan International Symposium: Time Series, Machine Learning and Causality Analysis

Date 2019.9.2-9.7 Venu RNSYSU, NCKU, FCU

This symposium was jointly held by Waseda University and three universities in Taiwan, National Sun Yat-sen University in Kaohsiung, National Cheng Kung University in Tainan, and Feng Chia University in Taichung. The topics included time series analysis, machine learning, and causality analysis. There were six international invited speakers: Masanobu Taniguchi (Waseda University), Takayuki Shiohama (Tokyo University of Science), Yoichi Miyata (Takasaki City University of Economics), Kou Fujimori (Waseda University), Akitoshi Kimura (Waseda University), Mike K. P. So (The Hong Kong University of Science and Technology), and ten local invited speakers: Shih-Feng Huang (NUK), ShengLi Tzeng (NSYSU), Hsiang-Ling Hsu (NUK), Hai-Tang Chiou (NTHU), Ray-Bing Chen (NCKU), Shuen-Lin Jeng (NCKU), Liang-Ching Lin (NCKU), Henghsiu Tsai (AS), Chor-Yiu Sin (NTHU), Edward M.H. Lin (THU). The symposium was held respectively at each university for two days from Sept. 2-Sept. 7, 2019. There were around fifty participates in each university. This symposium provided a platform for deeper and broader communication and discussions between Japanese and Taiwanese statisticians to achieve bilateral mutually beneficial cooperation. The participants in the meeting had a full discussion on their interested topics. The symposium promoted international communication and cooperation between Japanese and Taiwanese statisticians and was successfully concluded.



With the approaching of the new school year, the course plan of Taiwan Mathematics School is as follows:

Geometric Measure Theory I

Lecturer | Ulrich Menne (National Taiwan Normal University)

A very successful strategy for the study of geometric variational problems is to firstly prove existence in an enlarged class of competitors by means of compactness theorems and subsequently study the regularity of the solution therein. For instance, instead of considering only smooth submanifolds, one proves existence in the classes of boundaries of sets of finite perimeter, integral currents, or integral varifolds -- all of which are based on the more basic concept of rectifiable set. In the ensuing regularity theory, the generality of varifolds allows to unify a substantial part of the treatment. The purpose of the course is to develop, after providing the necessary infrastructure, the concept of rectifiable set as well as key elements of the theory of varifolds.
We assume sound familiarity with the concepts of measure, measurable function, Lebesgue integration, and product measure. In the initial part of the one-year course, we focus on developing the relevant concepts from advanced measure theory (Riesz representation theorem, covering theorems, derivatives of measures), functional analysis (locally convex spaces, weak topology), multilinear algebra (exterior algebra, alternating forms) and basic submanifold geometry (second fundamental form, Grassmann manifold). Then, we develop Hausdorff measures and the area of Lipschitzian maps to treat rectifiable sets. Finally, the basic theory of varifolds is developed (first variation, monotonicity identity) to treat the isoperimetric inequality and compactness theorems.

Dynamical System: Deterministic and Chaotic Behavior of Model Systems

Lecturers | Vladimir Zykov (Max Planck Institute)
Masayasu Mimura (Hiroshima University & MIMS, Meiji University)
Chih-Hung Chang (National University of Kaohsiung)

This course has a twofold purpose: In the first part we provide various models which come from biological mathematics, physics and dynamical systems. Interesting problems arise when we investigate the solutions of those models: 1. How about the structure of solutions? 2. What is the asymptotic behavior of solutions? And 3. How to characterize a given chaotic system? The aim of the second part is to answer the above problems. To do this, we first introduce elementary theories of dynamical systems in one dimensional domain. Next, we will introduce the singular perturbation theory which is a useful tool in investigating the qualitative behavior of solutions of a parametrized model when a small parameter involved in the model. Finally, we introduce the chaotic behavior of the well-known Lorenz system. The useful technique, namely, the symbolic dynamics theory, which is an efficient method to characterize the chaotic behavior of a dynamical system, will also be presented.

Topics on Drinfeld Modules and T-Motives

Lecturers | Chieh-Yu Chang (National Tsing Hua University)
Jing Yu (National Taiwan University)
Fu-Tsun Wei (National Tsing Hua University)

In 1974, Drinfeld introduced "elliptic modules," called Drinfeld modules nowadays, to derive the Langlands correspondence for GL(2) over function fields. In the modern developments of the function field arithmetic, Drinfeld module plays a major role in various topics. Using "shtukas," which is a generalized concept of Drinfeld modules, Lafforgue proved the Langlands correspondence for GL(n) over function fields. Meanwhile, Anderson's "t-motives" (the "dual" of the higher dimensional Drinfeld modules) is a very powerful tool for the study of transcendence theory and "special L-values" in the context of equal characteristic.
This course is prepared for graduate students who are interested in the arithmetic of function fields and related topics. We will cover basic theories of Drinfeld modules, Drinfeld modular curves and t-motives etc. Recent developments around these topics will be also described. Students are required to have some backgrounds on algebraic number theory, algebraic function fields and non-archimedean analysis.

We encourage students who are interested in courses mentioned above or want to know more about those theories to join us for further exploration to mathematics world.

Research Spotlight

Prof. Ming-Chih Lai's research interests are mainly on Numerical Methods for PDEs and Computational Fluid Dynamics, especially in developing mathematical models and numerical methods for incompressible fluids with interfaces.

For the past several years, he has continued working on two major fluid interfacial problems; namely, the incompressible flow with surfactant, and the vesicle problem. Mathematically speaking, in additional to solving the Navier-Stokes, the first problem involves solving a convection-diffusion equation on a moving interface (insoluble surfactant case) or even an irregular domain (soluble surfactant case), while the vesicle problem involves a PDE constraint (surface divergence zero) on a moving interface. Those problems pose well-known computational challenges in scientific computing community. For instance, in 3D vesicle simulations, one needs to compute the vesicle boundary forces which involve finding the geometrical quantities of the surface such as mean curvature, Gaussian curvature and even the surface Laplacian of mean curvature in discretization. How to numerically compute those geometrical quantities in 3D is already an important computational issue even in discrete computational geometry and computer graphics. Recently, in additional to study the hydrodynamic effects on these interfacial problems, he also moves forward to study those flow behaviors under electric field; that is electro-hydrodynamics. His research spotlights on these two problems can be summarized as follows.

For the flow with surfactant, first of all, he has simplified the derivation of insoluble surfactant along the interface and made comparisons with previous ones in literature. It turns out that his derivation based on Lagrangian formulation is simpler and more appropriate for the front-tracking approach of the interface. Using the Immersed Boundary (IB) method, he and collaborators were able

to simulate the interfacial flows with insoluble surfactant. Based on the novel reformulation of the surfactant equation, they have developed a new numerical scheme and proved that the total surfactant mass is conserved exactly in discrete sense. Meanwhile, he has developed an equi-arclength parametrization technique for the interface so that the interfacial grid clustering and mesh distortion can be avoided automatically. Nontrivial extensions for the above method to the 3D axis-symmetric interface and 3D doubly periodic interface with insoluble surfactant have also been done. Recently, using coupled grid based particle method to track the interface, the authors have applied to 3D two-phase flows with insoluble surfactant and electrohydrodynamic simulations.

The vesicle dynamics is another interesting problem that Prof. Lai has been working on these years. In additional to the fluid dynamics, the major difficulty arises that the vesicle boundary (or the interface) is surface incompressible (or inextensible in 2D) so that the local interface area (or arc-length in 2D) must be conserved as time evolves. Since the constraint is local, one should introduce an elastic self-adjusted tension on the vesicle boundary which acts as Lagrange multiplier for the local surface incompressibility constraint. The role of this elastic tension defined on the interface is pretty much like the pressure in incompressible fluid. Recently, Prof. Lai has simplified the model and formulated an open problem for the well-posedness of solution that appears to get much attention in the theoretical PDE community. His conjecture is when the problem is stationary (Stokes equations), the elastic tension must be uniquely determined unless the interface is a circle. The numerical results based on IB method also confirm first-order convergence of the elastic tension; however, the rigorous proof of numerical convergence remains open.

Other than different applications to his developed schemes, his major research achievements are listed in the following. He has proved that the spreading operator acting on the tension and the surface divergence operator acting on the velocity are in fact skew-adjoint mathematically in 2D and 3D cases. Furthermore, he has developed numerical schemes based on IB framework so that this skew-adjoint property is also satisfied in discrete sense. As a result, the elastic tension does not do extra work to the fluids in both theoretical and numerical senses which is highly consistent with the physical meaning of tension as a Lagrange multiplier. Using the above property, he has developed a linearly semi-implicit scheme for the unsteady Stokes problem with an inextensible interface and proved the scheme is unconditionally energy stable. His scheme can even evaluate the local inextensibility error which is hard to be estimated from other works in literature. To avoid the complexity of enforcing the local surface incompressibility constraint for 3D simulations, he has developed a nearly incompressible approach by introducing a modified tension energy so that the new elastic force derived from the modified tension energy has the exactly same mathematical form as the original one. Numerical results provide a good agreement with those ones obtained in theory and experiments.

Recently, he has started the simulations of Newtonian vesicle in Oldroyd-B fluid under shear flow. It is surprising to find that the stationary inclination angle can be negative without the transition to tumbling (TB) motion. Moreover, the inertia effect plays a significant role that is able to turn the vesicle back to positive inclination angle through TT-TB-TT transition as the Reynolds number increases. To the best of our knowledge, this is the first numerical work for the detailed investigations of Newtonian vesicle dynamics suspended in viscoelastic Oldroyd-B fluid in literature. Prof. Lai's numerical results can be used to motivate further studies in theory and experiments for such coupling vesicle problems.

Representative works

1. Y. Seol, W.-F. Hu, Y. Kim and M.-C. Lai, **An immersed boundary method for simulating vesicle dynamics in three dimensions**, *Journal of Computational Physics*, Vol 322 (2016), pp. 125-141.

2. Y. Seol, Y.-H. Tseng, Y. Kim, M.-C. Lai, **An immersed boundary method for simulating Newtonian vesicles in viscoelastic fluid**, *Journal of Computational Physics*, Vol 376 (2019), pp. 1009-1027.

3. M.-C. Lai, K.-C. Ong, **Unconditionally energy stable schemes for the inextensible interface problem with bending**, *SIAM Journal on Scientific Computing*, Vol 41, No 4, (2019), pp. B649-B668.

4. S.-H. Hsu, J. Chu, M.-C. Lai, R. Y.-H. Tsai, **A coupled grid based particle and implicit boundary integral method for two-phase flows with insoluble surfactant**, *Journal of Computational Physics*, Vol 395 (2019), pp. 747-764.

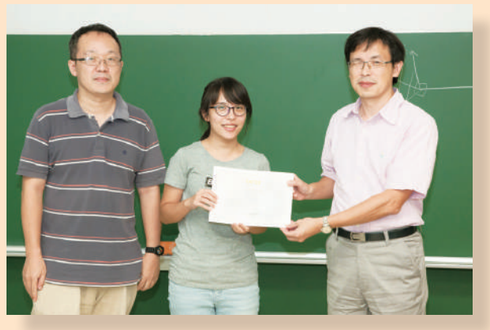
5. S.-H. Hsu, W.-F. Hu, M.-C. Lai, **A coupled immersed interface and grid based particle method for three-dimensional electrohydrodynamic simulations**, *Journal of Computational Physics*, 398 (2019), 08903.

Undergraduate Summer Research Program

From July 1 – August 9, 2019 NCTS hosted the Undergraduate Summer Research Program. This year, NCTS hosted 19 students from universities around Taiwan to research one of six projects and present their findings.

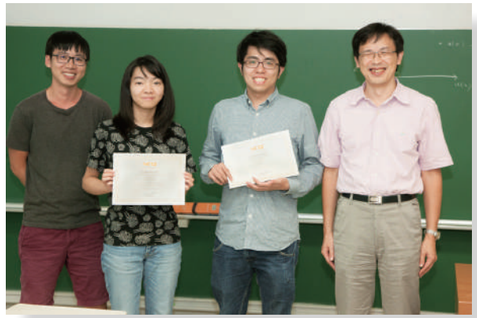
Group 1

which was supervised by Prof. Pengwen Chen of NCHU, looked at *Compressive Sensing and Phase Retrieval*. In this program, participants were interested in the recovery from underdetermined measurements. For instance, the standard compressive sensing problem consisted in the reconstruction of one sparse vector or one low rank matrix. In this program, participants were asked to test a few algorithms in this field and work on some research topics.



Group 2

which was supervised by Prof. Tzyy-Leng Horng of FCU and Prof. Tai-Chia Lin of NTU, looked at *3D High-Performance Computation of PNP Equations in Ion Channels*. The research subject was mathematical modeling and numerical computation of ion channels. This included (1) the basic ODE model for ion channels, Hodgkin-Huxley (HH) model, and (2) the PDE model from 1st principle: Poisson-Nernst-Planck equations (PNP). PNP model could actually be reduced to HH model. In addition, the mathematical modeling of gating current in Shaker channel was included for advanced study.



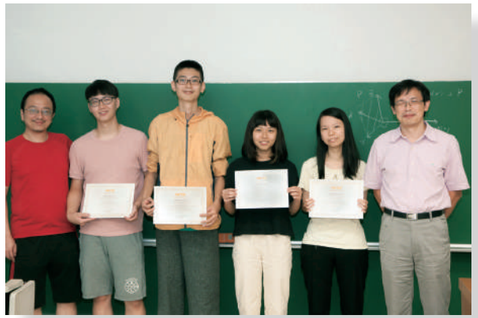
Group 3

which was supervised by Prof. Gi-Ren Liu of NCKU and Prof. Yuan-Chung Sheu of NCTU, looked at *Hidden Markov Models and Its Applications*. The project aimed to let participants understand the mathematical background behind HMMs and learn how to implement it by Matlab or Python.

Group 4

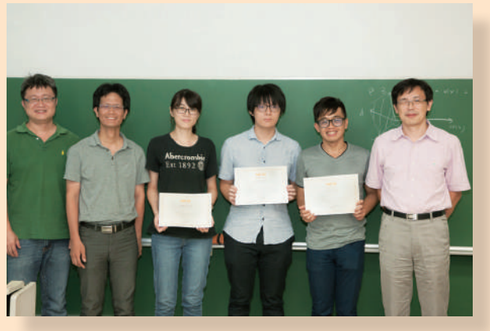
which was supervised by Prof. Tzer-jen Wei of NDHU, looked at *Generating and Detecting Deepfakes*. In this program, participants studied a few deep learning techniques for generating fake contents, such as generative adversarial networks, variational autoencoders, and generative models for text. In addition to investigate the theory and mathematical model behind these techniques, they also reviewed recent advancements and explored its potential applications. It had been shown that the state of the arts generative models could generate realistic, but fake things like faces, Airbnb rooms, articles. On the other hand, these technoledge could be easily used in malicious ways. Thus, techniques for detecting fake contents were equally important.

Undergrads in this program needed to learn and use a wide range of tools to translate theories into real applications.



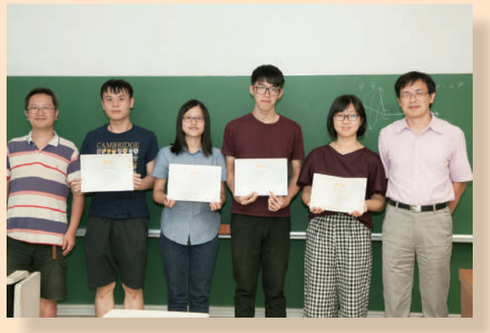
Group 5

which was supervised by Prof. Ming-Cheng Shiue of NCTU and Prof. Chun-Hsiung Hsia of NTU, looked at *Mathematical Analysis and Computation of Synchronization and its Applications*. In this program, mentors demonstrated students how the celebrated model, Kuramoto model, gave hints to the synchronization phenomena. Meanwhile, students were required to write matlab programs to perform numerical simulations. Finally, some applications in numerical weather prediction would be introduced and explored by using this technique.



Group 6

which was supervised by Prof. Chia-Fu Yu of AS, looked at *Structures of Unitary Groups and Applications*. The goal of this project was to study two classical papers by J. Dieudonne -- On the structure of unitary groups I & II. Through working on them participants found answers together to these problems and saw what problems remain open. They also investigated interesting problems of computing the cardinality of unitary groups over finite fields, and computing the determinant of unitary groups over local fields and over its ring of integers.



At the final stage of the research program, the participants gave a presentation to discuss their findings. All in all, the students were benefited greatly from high-quality mentorship and enriching mathematical research experience provided by NCTS Undergraduate Summer Research Program.

Interview with Prof. Leon Simon (Part III)

• Almgren's Legacy

Q. In 1980s, the other big breakthrough is Almgren's work...

Yes, yes, he came to Australia. In fact, he gave a long series of lectures on his work, which I really spent a lot of effort trying to understand, and was sort of almost there but then somehow after he left I let it slip. I didn't really follow up on it, which I should have done. But he was very generous with his time. He talked about it in great detail, for many hours. There was a group of us. At first in Melbourne, and then the center moved to Canberra. That was around the time I was moving, from Melbourne to Canberra, 1980 or so I think. So it started in Melbourne and then later we moved to Canberra, and he gave many lectures. And we sort of had -- we were well on the way to understanding even the difficult stuff. We had mastered the Dirichlet minimizing stuff. We understood all that, but then the Center Manifold thing, that was more difficult. But he explained many of those ideas, and I sort of felt that I was almost there, that I almost understood that, but then as I said, somehow I let it slip. I guess I had the idea that, well, I can do this later, Fred will be around for many years, but tragically he died in his early 60s, so he wasn't around as long as people would have expected. I talked to Bill Allard about that. He had exactly the same attitude. He also hadn't quite mastered all of that material, but he sort of had the attitude that, well, there's plenty of time, I can ask Fred about it, and so on. Tragically, that didn't turn out to be the case.

Q: So when Almgren visited Canberra, he had already finished his big regularity theorem, right?

Yes, he had already finished it. The telephone books were available. In fact, in Melbourne we were all working from handwritten versions that needed to be typed up. And Fred wrote things in a very light pencil, so when you try to photocopy them, it doesn't -- copy machines weren't so good in those days, so it didn't come out so well. It was a bit of a problem, actually. We were working with handwritten notes. Can you believe it? Yes.

Q. It's a very exciting period of time.

Oh yes, that was when it was really taking off. The next big thing was, of course, Hamilton's work in geometric flows. But minimal surfaces was the dominant area of geometric analysis at that time. Minimal surface equation and geometric measure theory -- applications to geometry and the geometric calculus of variations were the big things at that time. And there were plenty of open problems. Now there are only the hard problems. Not really, but you do have that effect in a field, right? What they sometimes refer to as low-hanging fruit. When the field is young, there are many results that are relatively easy to come by. But then, as the field gets older, it's only the really hard central questions that are still there. And if you get to the point where they're settled, then the field starts to age somewhat, typically, and you get into very technical things that only the real specialists would really care about. I think fields typically tend to go that way. Geometry hasn't gone that way because it's such a rich field, I think. There are just so many problems. PDE, too, it's a huge field, so I don't think there's any danger of that aging too much any time soon.

Q. Can you talk a bit more about Almgren... He was in Princeton, also the other center for GMT. Is Brakke his student or not?

Yes, Brakke was his student. One of his PhD students.

Q. Brakke's work seems to be isolated for a while.

That's right! In fact, it's amazing. Brakke's work preceded Hamilton's work on Ricci flow, but no one really appreciated the significance of Brakke's work, I think, until after Hamilton developed Ricci flow. That was so spectacular, and then people once again started asking, well, what about mean curvature flow? It seems there's a parallel theory there, and then they rediscovered Brakke's work so it got a whole new lease on life. But at the time maybe it was too difficult for people somehow, they didn't quite get it, I don't think. That's just how incredibly powerful Almgren was. I think he was really behind -- I mean Brakke did the work, of course, but Almgren was directing that work, it was Brakke's thesis work. It's clear that Almgren must have realized that you can make this sort of mean curvature flow construction. Around that time Almgren also directed the thesis of John Pitts on minimax in codimension~1, building on the machinery that Almgren had set up in arbitrary codimension. So Almgren was not only doing the arbitrary codimension regularity, he was also heavily involved in all these other things. Quite extraordinary, actually.

Q. So you take the notion of varifold for us in your lectures at NCTS...

That was also Almgren! That was another thing -- he had these notes that he had never published. The only thing he actually published was that little book called "An Invitation to Varifold Geometry." But he has these mimeographed notes actually -- I have a copy -- on Varifold theory. The way he set the theory up was a little different. Allard actually refined it and developed it much more, but Almgren had many of the ideas in some form or another in those notes. Also Allard's definition of Varifold was a little bit different. Allard defined varifold to be a Radon measure on the Grassmanian. I think Almgren's definition was equivalent but a little bit different somehow... I forget. But yes, so... Almgren -- you can trace back many important developments to Almgren. Quite a tragedy that he died at a relatively early age.

Q. What do you think the future development of GMT?

Well... I mean there are undoubtedly very many really hard questions still. I mean, we've only just sort of scratched the surface of the structure of the singular set -- things like that. Um, that's been one of my main interests, but you know, there's just so many basic questions which we just don't know how to approach. So I think some new ideas are really needed. Maybe we're at a point where some new ideas are needed to get things moving again, somehow. There was a big breakthrough recently -- Naber and his coauthor Valtorta. They proved that Almgren's tangent cone stratification of a singular set could actually be made in terms of rectifiable subsets of various dimensions. That is perhaps not as you would like it ultimately, that is in terms of manifolds, smooth manifolds -- you can't quite do that yet -- although you might still have the hope to do that. But Naber and Valtorta were able to at least do it at the level of rectifiable set of various dimensions -- a stratification in terms of rectifiable sets. And that was a big breakthrough, I think a lot of people were surprised that that was doable. So that was very original. They got a sequence of papers out. Basically they were doing Reifenberg type theorems, but new ideas involving different approach to covering Lemmas and so on, to make all of that work. So that's been a big development, one of the main developments concerning the singular set in the last ten years or so.

But there's still so much to do, you know, so many open questions that shouldn't still be open. And part of the problem is that you can't be quite sure about what to expect.

What could be true and what is not true. So it would be very valuable to have more examples of singular minimal surfaces and so on. Again, that seems to be difficult, generating examples. There are many examples of singular minimal surfaces, but they're all structurally somewhat like real analytic varieties, stratifying into blocks of various dimensions. And don't really know examples where the singular set is radically different than that -- for example can one have a sequence of isolated singular points with an accumulation point away from the boundary; very basic question, but still open.

Q. Do you think there's any other associated direction or questions that also inspire development in GMT?

Well, I think it's always been true that people have been interested in applications to geometry. That sort of motivated a lot of developments. Like Schoen-Yau's proof of the positive mass theorem, and other applications. I think that's been the big motivation for working with minimal surfaces. And also energy-minimizing maps. There's sort of a parallel theory there. But yes, I'm not really sure other sort of directions, other sort of variational problems that would play such an important role. I'm really not the right person to ask about that. I don't know. It would be nice if, you know, some more progress could be made, really, on some of those basic problems that are still open. That has to be for the next generation. The next few generations, perhaps.

• Miscellaneous

Q. You could come over for several years to help the, uh, the development of this direction. You don't need to give lectures, I know it's too much work, but to organize some working seminars. I think that would be much easier.

I really think the idea -- particularly in an institution like this where you've got good funding for research -- of having working seminars, where you don't just talk about your own work, you talk about other people's work. I think that's very, very good and an important way to help people get into a situation where they have more of a chance to successfully attack fundamental problems.

Q. Before, we had some kind of reading seminar, but maybe we should set up some working seminars. But we are too busy.

Yes -- that seminar in Australia, I had the advantage there that I had no teaching during that period. I had a pure research position. I could put all my effort into my own work and in running the working seminar, in running the research group and getting people to come and visit, and getting them actively involved. People were terrific during that period. They would come and give many hours of lectures. They didn't seem to mind doing that. It was really a very exciting period. But yes that sort of thing is very valuable.

Q. We need to create some position with one semester off. Maybe not a whole year --

You also need to have someone to take responsibility for running that.

Q. We run it, but we need to not do other things --

But you can't just leave it to a nebulous group of people. You got to have someone who is really running the thing and making sure everything is working and making sure, you know, what is the topic next week and who is going to speak and making sure during that discussion that things keep moving along and that the right questions are asked, and so on. And to do that, you really need --

Q. -- Time

Yes, certainly time, but you also need someone that's totally dedicated to it. And the other people have to be really supportive. They have to be actively engaged. We were just very lucky during that period in Canberra. A very good crew, consisting of several people that were there permanently, and we always seemed to be lucky with our visitors. They really got involved in doing that.

Q: We should copy that experience, and we should record your words here so that our director could -- I think what you say is useful, but sometimes we want to do something, but we don't have time.

Yes, I think you really do need to be given a break from teaching if you take your responsibility to run a research group. A serious one. Of course in those days, there was no arXiv on the web. There was no web. We had this preprint thing. We would get people to produce as they were there writing papers and so on. An amazing number of papers came out of that period, actually. And the initial drafts of those papers came out as preprints there. Now it's the arXiv, which is much better, because everyone can see it. But I think it's important to keep track of those things. Even though you should get people to put their work on the arXiv, try to keep track on, you know, what's being produced. That always is good from the point of view of convincing administrators and funders how much is being done, and, you know, that they're getting value for their money.

Q. Are there other factors in setting up an active and successful research group?

It's partly a question of the mix of the people involved. It doesn't always work, but during that CMA period in Canberra, we just happened to have the right mix of people. There were a couple of junior people there who didn't even have permanent jobs, but they were very committed, and they would always get engaged in the discussions during the seminars and so on and... Somehow, the chemistry was right. It really worked well, so that's part of the secret of running a good group. But I had that sense of, you know, okay, what do we need to make this really work? You know, who can I invite that will fit into this group really well? And so on.

Q. So it was mainly organized by you?

Yes, mainly organized by me, right. Well, I must say, they were very generous, because... Neil Trudinger was really the head of the institute and he allowed me to basically invite all the visitors I wanted to invite. He could have invited other people, but I basically had carte blanche to invite whomever I wanted.

Q. So good organization is also very important.

Oh I think so, very much so. Yes, that's one important thing. The other is what I said, the chemistry. You've got to have the right mix of people. If you haven't got the right mix of people, you have to do something about that. You have to try to get another ingredient that would help that work out. Get the right visitors, and so on. Yes, we were very fortunate. People typically liked to come and spend a few months in Australia at that time, you know, a pleasant location generally and it was known that the mathematical group was a very congenial one. Being in a less attractive location, maybe it wouldn't have been so easy, but... I think you've got an advantage here. I think that Taiwan is not really known so well; it's a very pleasant place to visit! You really probably need to advertise that a bit more.

Q. You advertise for us! It's good you say that. Thanks a lot today!

Postdocs all over the world

We have several postdoc fellows joining us in summer, 2019.

Dr. Junsik Bae received his PhD degree at Ulsan National Institute of Science and Technology in 2019. His research interest is partial differential equations.

I am interested in a diverse phenomena of plasmas and studying the mathematical models of them. My PhD thesis concerns the asymptotic behavior and linear stability of small amplitude solitary waves of the Euler-Poisson system. It is my honor to work at NCTS. I am looking forward to discussing and learning math with other researchers together. Based on the wealth of high-quality research environments of NCTS, I sincerely believe that continuing my career at NCTS will afford me the best possible opportunity to realize my research objective.

Dr. Yuki Chino graduated with a PhD degree in 2016 from Hokkaido University and has been a postdoctoral fellow at Leiden University for 2 years before coming to NCTS. His research interests are mainly stochastic processes in random media and probability theory and related areas.

I came from Hokkaido, where we have cold winter and a lot of snow. Now I am in the country with opposite climate. It has passed one month in Taiwan but I still feel hot and have not got accustomed with this weather yet. I believe that I can adapt myself to this hot and humid climate and feel at home in NCTS. Life is always challenging. I want to enjoy so many challenges here. At last, I really appreciate to have this great opportunity to enjoy mathematics and my life here.

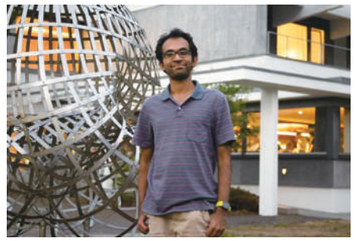
Dr. Bochen Liu got his PhD degree in 2017 from University of Rochester. Before coming to Taiwan, he was a postdoctoral fellow at The Chinese University of Hong Kong. His research interests are Harmonic Analysis, Geometric Measure Theory and Tiling and exponential basis.

I was born in Harbin, China in 1989. After 18 I started moving around the world: four years in Zhejiang University in Hangzhou for bachelor, where I met my wife; four years in University of Rochester in the US for PhD (degree granted in 2017); from 2016 to 2017 I was a RA in Chinese University of Hong Kong; from 2017 to 2018 I was a postdoc at Bar-Ilan University in Israel; before joining NCTS I was a postdoc in Chinese University of Hong Kong. I have worked with different people so I have a wide range of research interests, including harmonic analysis, geometric measure theory, and combinatorics, especially on how ideas/techniques from one area influence the others.

Dr. You-Hung Hsu completed his PhD at University of British Columbia in 2019. His research interests lie in the areas of algebraic and representation theory.

Starting from this fall, I am a first-year postdoc at NCTS. I finished my Ph.D. at the University of British Columbia in 2019, under the supervision of Prof. Sabin Cautis. In my thesis, I constructed a categorical action of the shifted q=0 affine algebra on the derived category of coherent sheaves on Grassmannians. My main research areas are geometric representation theory and categorification. I am also interested in derived algebraic/symplectic geometry. Thanks for giving me this opportunity to continue my research.

Dr. Bharathwaj Palvannan completed his PhD at the University of Washington in Seattle. Before joining NCTS, he was a postdoc at the University of Pennsylvania in Philadelphia. He is interested in questions related to number theory and arithmetic geometry.



The freedom provided by NCTS to devote all my time, in this beautiful and serene campus, to research has been very stimulating. Both the staff (especially Peggy Lee) and my mentor (Ming-Lun Hsieh) have been very generous with their time in helping me settle down at NCTS. In fact, everyone in Taipei has been very friendly. All of this has made life in Taipei very pleasant and enjoyable.

Dr. Arasteh Rad earned his PhD in Dec. 2012 at Universitat Munster under supervision of prof. Urs Hartl. Since then, he had continued to work in his research group as a research assistant. His current research is related to the Langlands program over function fields.

Dr. David Wen completed his PhD at University of California, Santa Barbara in the spring of 2018. Before joining NCTS he was a Limited-Term Assistant Professor at the University of Georgia. His field of research is Algebraic Geometry.

Dr. Sanghyuck Moon got his Ph.D. from Korea Advanced Institute of Science and Technology (KAIST) in February this year. He was affiliated to Yonsei university as a postdoctoral researcher before joining NCTS. His general areas of interest focus on nonlinear elliptic equations and calculus of variation.

Dr. Yoshihiro Sugimoto compelted his PhD at Kyoto University. His primary research interests lie in symplectic geometry, particularly applications of pseudo-holomorphic curve theory (Floer homology theory) to symplectic topology.

Join US

NCTS is calling for 2020 postdoctoral fellows!

Application will be open from Sept. 1, 2019 to Feb. 28, 2020. The appointment starts from August 1, 2020, for 1 or 2 years, extendable up to 3 years. Every talent pursuing academic excellence is welcome. Priority will be given to the following areas: *Algebraic Geometry, Differential Geometry and Geometric Analysis, Differential Equations and Stochastic Analysis, Scientific Computing, Data Science*.

NCTS is calling for Research Pairs!

A Research Pair consists of 2-4 researchers, without the same affiliations. Each member in one pair is expected to spend 2-4 weeks at NCTS for research collaborations, and will be supported as a visiting scholar for NCTS. Application is always open!

Please visit http://www.ncts.ntu.edu.tw/ for position details.

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