

# Newsletter

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## Director's Message

The National Center for Theoretical Sciences (NCTS) was established on August 1, 1997, by the National Science Council (NSC), the predecessor of the current Ministry of Science and Technology (MOST) with the goal of advancing the state of theoretical sciences in Taiwan to a state comparable to those in countries known for their long scientific traditions. The NCTS consists of two divisions: Division of Mathematics and Division of Physics. On January 1, 2021, the NCTS has entered its current fifth phase (2021-2025) with Dr. Yng-Ing Lee as the director of the Mathematics Division and Dr. Shun-Jen Cheng as the deputy director.

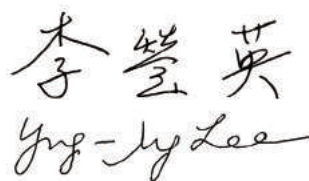
The NCTS aims to become an international center of excellence in theoretical mathematical sciences, where its research staff and visitors find a pleasant environment to conduct first-rate research. Since its establishment it has been serving and promoting theoretical scientists in all of Taiwan who are pursuing cutting edge research directions in pure and applied mathematics. However, at the same time, it also strongly encourages interdisciplinary research endeavors where mathematics plays a fundamental role. The fact that virtually every leading research mathematician in Taiwan today has been involved in the NCTS activities at some point during his or her career indicates the success of the NCTS since its inception. Indeed, as of now, many of the most active research mathematicians in Taiwan are either directly affiliated with, or have kept close connections with the NCTS.

In the near future to come the NCTS faces new challenges. Many of them are due to various external factors. To better prepare itself for these challenges the NCTS in the new phase strives to collaborate even more closely with other research institutions within Taiwan and abroad by sharing and combining resources. The NCTS has already signed MOUs with several world renowned research institutions and we believe that a closer collaboration with research institutions and universities in the long run will have significant benefits for the Taiwanese scientific community and eventually for the society in general. During this phase, we hope to continue to build alliances with research centers and strengthen our international exchange program.

To reach its set goals it is imperative that the NCTS pays special attention to fostering the next generation of theoretical scientists. A broad viewpoint and a solid training in foundational mathematics are necessary requirements for making significant contributions in any field in mathematical sciences today. Located inside the campus of the National Taiwan University in the metropolitan Taipei the NCTS offers a vibrant research environment for a young mathematician: the NCTS has an extensive international post-doctoral program where the majority of the post-docs are from outside of Taiwan. In addition, the NCTS is deeply committed to nurturing the next generation of mathematical scientists in Taiwan and has established the Taiwan Mathematics School with the goal of giving any highly motivated student in Taiwan access to a strong basic educational program in mathematical sciences. At the same time, there are undergraduate research program both in summer and during the semesters at the NCTS, where every professor mentor leads 2-3 students working on a project or a special topic. These programs together with possibility of doing research abroad and attending joint summer schools with MSRI offer a talented youth both an exciting research experience and opportunities to interact with other talented peers in the world.

### Past Directors:

The NCTS during the first phase from 1997-2003 was located on the campus of the National Tsinghua University in Hsinchu. In the initial stages of this phase the director of the Mathematics Division was Dr. Jing Yu who was then followed by Dr. Chang-Shou Lin. The main office of the NCTS remained in Hsinchu during the second and third phase from 2004-2014, however, two additional offices were established at the National Taiwan University in the North and at the National Cheng-Kung University in the South of Taiwan. The directors during these two phases were Drs. Jing Yu and Wen-Ching Winnie Li, respectively. In 2015 the NCTS was relocated to the National Taiwan University in Taipei, and the director of the fourth phase (2015-2020) was Dr. Jungkai Alfred Chen.



NCTS Director  
Prof. Yng-Ing Lee

# 2021 International Day of Mathematics

## Mathematics for a Better World

In many countries, March 14 has already been commonly celebrated as Pi Day because  $\pi$ , one of the world's most widely-known mathematical constants can be rounded to 3.14. Pi day can be observed in many ways; activities such as baking and eating pies, reciting  $\pi$  contest, are carried out. Due to the worldwide celebrations, the International Mathematical Union (IMU) decided to broaden the scope of this day. It was soon approved by UNESCO; March 14 was then proclaimed as the International Day of Mathematics (IDM).

Each year, the official IDM website will announce a new theme to add flavor to the celebration. And this year the theme is "Mathematics for a Better World". As the world celebrates the beauty and importance of mathematics in our everyday lives, Taiwan also responds by holding several events.

Last year, the original plan for celebrating the International Day of Mathematics was to host a series of dramas, magic shows, and puzzle solving activities at Huashan 1914 Creative Park. However, those activities were cancelled due to the outbreak of COVID-19 and a live talk show was held instead. This year, while many countries around the world are still hitting new highs in coronavirus cases, Taiwan's careful compliance with safety rules has kept the pandemic in check. Life



in Taiwan has remained normal for most of the past year and thus National Center for Theoretical Sciences (NCTS) and The Mathematical Society (TMS) decide to continue the Buffon's Needle Challenge and expand the scale of the experiment up to 10 thousand people.

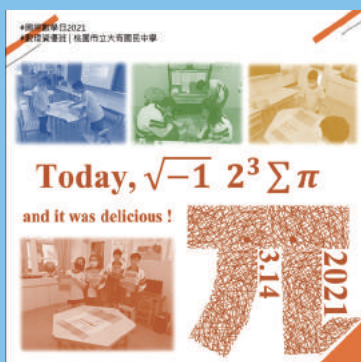
The event "10k Buffon's Needle Challenge" is based on the "Buffon's Needle Problem", named after French mathematician Georg-



Feng Jia Junior High School, Kaohsiung City



Rende Wunsian Junior High School, Tainan City



Da You Junior High School, Taoyuan City



Linkou Junior High School, New Taipei City





Badou Senior High School, Keelung City



Hsinchu Girls' Senior High School, Hsinchu City



Taihsi Elementary School, Hsinchu City

es-Louis Leclerc Comte de Buffon, who first published it in the 18th century. The Challenge aims to answer the question "Suppose a needle is thrown on a parallel-lined paper, what is the probability of it landing on the line?" We invite teachers, students and the general public to participate in this huge event. The Challenge allows teachers to teach the concept of  $\pi$ , and by collecting thousands of results of the experiment, we can roughly estimate its value.

Another achievement we have accomplished this year is that, being invited by the IMU, we take the Challenge global. We call for even more people to join our event. Not only do we create an app for showing the local and global experiment results, but we also add the information of the schools that participate in the Buffon's Needle Challenge to the interactive map in the official IDM website.

Apart from the Challenge, several fun activities will also be held at National Taiwan Science Education Center on March 13 and 14. There will be dramas displaying "The Story of Rabbits and the Fibonacci Sequence" and "Florence Nightingale's Rose Diagram". Speakers such as Prof. Jia-Ming Ying (from National Tsing Hua University) and Wann-Sheng Hong (President of the Taiwanese Society for the History and Pedagogy of Mathematics) are invited to give speeches. We also invite Mr. Wei-Dong Chuang to perform stunning magic tricks.

The International Day of Mathematics is an opportunity to celebrate the essential role that mathematics plays in our everyday life. Our main purpose of organizing all these events is to make Math "fun". Though we can't necessarily make Math easy for everyone, we hope that when the general public starts to realize that Math is indeed helpful and of contemporary use, people will appreciate it more.

## Highlights of Events

10/19 – 10/23	ReaDiNet 2020: An Online Conference on Mathematical Biology
11/7 – 11/8	The 20 <sup>th</sup> Taiwan Geometry Symposium
11/11 – 11/12	L-values and Iwasawa Theory
11/12 – 11/13	NCTS Workshop on K-stability
12/14	2020 NCTS Optimization Day for Young Researchers
12/18	2020 Symposium for Young Analysts
12/18 – 12/19	2020 NCTS South Workshop on Dynamical Systems
1/27	Opportunities and Challenges in Numerical Algebra (I)
2/3	Opportunities and Challenges in Numerical Algebra (II)
2/17	Opportunities and Challenges in Numerical Algebra (III)
2/19	2021 NCTS Young Dynamics Day
2/24	Opportunities and Challenges in Numerical Algebra (IV)
3/8 – 3/9	The 11 <sup>th</sup> Japan-Taiwan Joint Workshop for Young Scholars in Applied Mathematics
3/8 – 3/9	Workshop on Geometric Evolution Equations and Related Fields
3/19 – 3/20	2021 Conference on Advanced Topics and Auto Tuning in High-Performance Scientific Computing
3/20	Opportunities and Challenges in Numerical Algebra (V)
3/20	Meet physicians, Mathematicians and Statisticians, Part 2
3/26	Third NCTS PDE Symposium

## SPONSORS

MATHEMATICS DIVISION, NATIONAL CENTER FOR T  
DEPARTMENT OF MATHEMATICS, NATIONAL TAIWAN

2020.12.14  
FOR  
NCTS  
OPTIMIZATION

This workshop provides a forum for young researchers in optimization community of Taiwan to share their research works using optimization theory or algorithms. In addition, it helps young generations to globally understand the visions and features of optimization. Various aspects and areas of optimization application were presented by young speakers from applied mathe-

CONTA

WEBSITE: <https://www.math.n>

VENUE ADDRESS: M212 at Mathematics Building

THEORETICAL SCIENCES  
N NORMAL UNIVERSITY

# YOUNG RESEARCHERS ATION DAY

mathematics, computer science, industrial engineering, and statistics. Although the community is small, the interdisciplinary interactions are very helpful and successful. We thank NCTS for supporting this activity.

ACT US

ntnu.edu.tw/workshop/2020o/  
ng. 88. Sec. 4. Ting-Chou Road. Taipei. Taiwan

3/27	Opportunities and Challenges in Numerical Algebra (VI)
3/29	Spring Day
4/24	NCTS Student Workshop on Scientific Computing
4/30	NCTS (Central Taiwan) Computational Mathematics Workshop
5/1	The 21 <sup>st</sup> Taiwan Geometry Symposium

## NCTS Short Courses

1/18 – 1/22	NCTS Winter Course: Six Aspects of Combinatorial Mathematics
2/1 – 2/4	NCTS Short Course: Introduction to the Basics of Incompressible Navier-Stokes Equation
4/16 – 5/7	NCTS Short Course on Dynamical Systems
5/17 – 5/31	NCTS Short Course on the Ricci Flow
8/22 – 9/5	Short Course on Advanced Course on Multi-Threaded Parallel Programming using OpenMP for Multicore/Manycore Systems

## Taiwan Mathematics School

2/22 – 6/21	Algebraic Number Theory II
2/23 – 6/17	Mathematical Foundation toward Artificial Intelligence in Medicine
2/25 – 6/17	Algebraic Combinatorics II
2/26 – 6/25	Topics in Geometric Measure Theory II



# 2020 Symposium for Young Analysts

Date

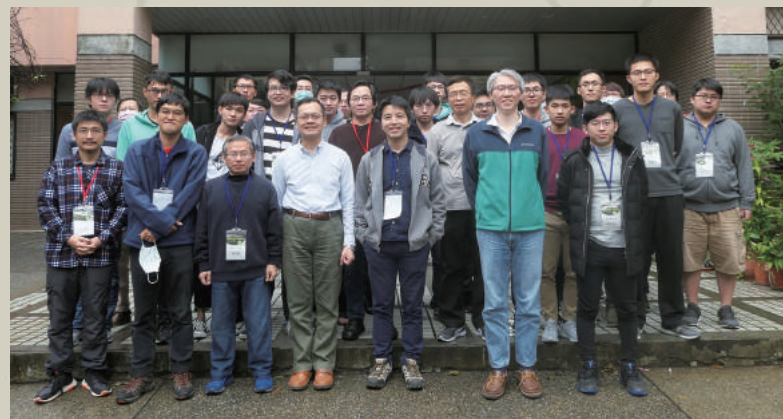
2020.12.18

Venue

M107, Hong-Jing Hall, NCU

This one-day workshop was chiefly to provide young researchers and students opportunities to exhibit academic works, stimulate knowledge growth of mathematics, and acquire recent research tendencies. Based on previous experiences, workshops of this sort have allowed participants a satisfactory environment to refresh up-to-date research progresses, interchange ideas, and place long-term research partnerships. The spectrum of the topics this year principally centered on classical analysis. For instance, two plenary speakers introduced their works in the field of harmonic analysis and PDE. Through sharing the experience of these senior scholars from a macro perspective, this activity has brought young talents to learn popular research themes and future expansion directions. Talks delivered by the junior participants involved harmonic analysis, noncommutative probability theory, and classical probability theory, and these talks were inspiring and promising. A short ceremony was held before the end of the workshop, and a certificate was award-

ed to each speaker to inspire them to sustain researches and retain perseverance on the research road, which left everyone strongly impressed. In summary, the year's workshop was extremely successful, facilitated communication between scholars, and stimulated conversations between all participants. Finally, special thanks to the National Center for Theoretical and Scientific Research and National Central University for their support and assistance to make this meeting operate smoothly.







# Opportunities and Challenges in Numerical Algebra

Numerical algebra has been playing pivotal roles as the pillar of scientific computing for decades. Theories and numerical techniques to their core problems, including linear systems, least squares problems, and eigenvalue problems, are taken very much for granted and used daily in science and engineering without questioning. Today numerical algebra sits at the crossroad of reinventing itself to adapt to a rapidly changing landscape because of the emergence of data science and artificial intelligence, while its original core mission remains. This online workshop series aims to bring you a series of advanced technical talks on the latest developments in numerical algebra and its recent machine learning applications.

So far, we have held four online workshops on 1/27, 2/3, 2/17, and 2/24. Each workshop

is from 9:00 am to 12:00 am. We invite three or four speakers from the USA, Taiwan, or China for each workshop. The speech topics include biological data analysis, machine learning, tensor problem, information retrieval, optimization problems, etc. We hope that through such workshops, Taiwanese scholars in scientific computing and foreign scholars will have more opportunities to communicate about recent machine learning applications. 150 to 260 people participate online in every workshop. Every speech has a heated discussion. As a result, we must reduce the number of speakers from four to three.

The first four online seminars worked well. We will continue this workshop to promote more exchange opportunities.

# Taking an international conference online: challenges, opportunities, and experiences

Ulrich Menne (National Taiwan Normal University & NCTS)

## Abstract

### The NCTS international Geometric Measure Theory seminar.

**Background** Originally, we had planned to run an international conference on *Geometric Measure Theory* in May 2021. Thereby, we had aimed for reporting substantial progress in the field by means of a classical format involving approximately 20 speakers plus 40 participants from many parts of the world. The emerging pandemic however made such an in-person conference unfeasible in the foreseeable future.


**Challenges and opportunities** In going virtual, we noted various challenges. On a basic level, difficulties involve the different time zones of the participants and the fact that virtual presentations are often conceived as more tiring. Furthermore, seamless social interactions and mathematical blackboard discussions in small groups, which are an integral part of any traditional conference, are challenging to recreate. On the other hand, the urgency to move activities online also entails a greater openness of participants worldwide to explore digital technologies. Last but not least, a virtual format can turn out to be more accessible, as it frees the time and money resources usually allocated to travelling.

**Format** For our virtual event, we have chosen a seminar format with one presentation every other month. Thereby, we expect this style to

minimise the impact of time zone differences, whereas the low frequency of presentations should facilitate reaching the target audience by keeping the quality standard high and the necessary time commitment low. In Taipei, the seminar usually takes place in the evening; in fact, its time is largely determined by the locations of *my co-organisers*

- Guido De Philippis (Courant Institute, New York University, US),
- Yoshihiro Tonegawa (Tokyo Institute of Technology, Japan), and
- Neshan Wickramasekera (University of Cambridge, UK).


With our default time, we reach an audience ranging from Japan and Taiwan, all the way to Europe and the east coast of the US. However, to accommodate speakers from Australia or the west coast of the US, we have also employed other times of the day entailing a different geographical distribution of the audience. From the software side, we rely on MS Teams (free for educational use) and Miro to create a virtual venue which is complete with lobby, lecture hall, and separate rooms with boards for discussion. In each of these rooms (channels in MS Teams), the participants can create meetings to interact in small groups. The virtual whiteboards by Miro allow the viewers to navigate and zoom independently; moreover, outside of the lecture hall, every participant can write on these boards.




**NCTS**  
National Center for Theoretical Sciences

Time | **Wednesday, 20 Jan. 2021,**  
10:30-11:30 p.m. (Taipei Time)

Venue | Online (Microsoft Teams)


Registration and more information: 

**International Geometric Measure Theory Seminar:  
Second-Order Estimates for  
Interfaces of Allen-Cahn**



**Juncheng Wei**  
University of British Columbia


In this talk, I will discuss a uniform  $C^{2,\theta}$  estimate for level sets of stable solutions to the singularly perturbed Allen-Cahn equation in dimensions  $n \leq 10$  (which is optimal). The proof combines two ingredients: one is a reverse application of the infinite dimensional Lyapunov-Schmidt reduction method which enables us to reduce the  $C^{2,\theta}$  estimate for these level sets to a corresponding one on solutions of Toda system; the other one uses a small regularity theorem on stable solutions of Toda system to establish various decay estimates, which gives a lower bound on distances between different sheets of solutions to Toda system or level sets of solutions to Allen-Cahn equation. (Joint work with Kelei Wang.)




**NCTS**  
National Center for Theoretical Sciences

Time | **Wednesday, 17 Mar. 2021,**  
8:15-9:15 p.m. (Taipei Time)

Venue | Online (Microsoft Teams)

Registration and more information: 

**International Geometric Measure Theory Seminar:  
Mean Curvature Flow with  
Generic Initial Data**



**Felix Schulze**  
University of Warwick

A well-known conjecture of Huisken states that a generic mean curvature flow has only spherical and cylindrical singularities. As a first step in this direction Colding-Minicozzi have shown in fundamental work that spheres and cylinders are the only linearly stable singularity models. As a second step toward Huisken's conjecture we show that mean curvature flow of generic initial closed surfaces in  $\mathbb{R}^3$  avoids asymptotically conical and non-spherical compact singularities. We also show that mean curvature flow of generic closed low-entropy hypersurfaces in  $\mathbb{R}^4$  is smooth until it disappears in a round point. The main technical ingredient is a long-time existence and uniqueness result for ancient mean curvature flows that lie on one side of asymptotically conical or compact self-similarly shrinking solutions. This is joint work with Otis Chodosh, Kyeongsu Choi and Christos Mantoulidis.

**Development and experiences** By default, our seminar takes place on the third Wednesday of every odd month. So far, we have had presentations by

- Leon Simon (Stanford University), on 18<sup>th</sup> November 2020,
- Juncheng Wei (University of British Columbia), on 20<sup>th</sup> January 2021, and
- Felix Schulze (University of Warwick), on 17th March 2021.

Depending on the time of the day, each presentation was attended by around 50–70 persons in total. The overall number of registered participants is steadily growing and has meanwhile reached around 180 persons. Thus, the seminar seems to be on a promis-

ing way to establish itself in the community. In fact, apart of some initial difficulties with the complex meeting software involved, the feedback from participants was generally very positive. However, concerning social interactions, whereas we do see more than in traditional online seminars, there is still room for improvement. In this regard, NCTS is currently exploring dedicated commercial software alternatives which could give our seminar—and other similar NCTS activities—a further boost.

**Website** All information on the seminar, including the registration procedure, may be found at <http://www.ncts.tw/gmt-seminar.html>.



# 2021 NCTS Young Dynamics Day

Time | **February 19, 2021**

Venue | **Hsu Shou-Chlien International Conference Center HC307,  
Tamkang University**

The one-day workshop “Young Dynamics Day”, which has been hosted every February since 2012 by NCTS, was held on February 19 2021 in Tamkang University. This conference was initiated by Professor Kuo-Chang Chen (National Tsing-Hua University), and the purpose of this workshop is to provide a forum for young generations within the dynamical systems community in Taiwan. In particular, at this time, there are over 45 participants who enjoy this academic banquet in such a hard time.

Speakers are encouraged to present their research interests or projects with ambition, and

contents need not be finished works of their own. In this regard, the workshop is a meeting for people to share their “mathematical dreams”, instead of limited to their research accomplishments.

For 2021 YDD, we invited 8 45-minutes speakers, and the topics revolve around the dynamics of the predator-prey models, traveling wave solutions, pattern formation theory, the mathematical model of the infection of diseases and the fluid dynamics. Finally, National Taiwan Normal University is the tentative venue of the 2022 Young Dynamics Day conference.



# 2021 Third NCTS PDE Symposium

Date/  
**March 26, 2021**

Venue/  
Room 408, College of Science, NUK

The NCTS PDE Symposium was initialized by Professors Chun-Hsiung Hsia (NTU) and Kung-Chien Wu (NCKU) at NCTS in 2020. The main purpose of this symposium is to bring experts and young researchers in PDEs together to introduce their current works and explore new ideas. It offers more free discussion time between talks for participants such that they have more chance to discuss with the speakers and other participants. We expect by joining this series of symposium, our researchers can have chances to talk to each other often to inspire each other.

This time is the Third NCTS PDE Symposium. It was held in National University of Kaohsiung on March 26, 2021. We are glad to invite Prof. Jenn-Nan Wang (NTU), Prof. Hsin-Yuan Huang (NCTU), and Dr. Jia-Yuan Dai (NCTS) as our speakers. The topics of these talks include inverse problems (Prof. Jenn-Nan Wang), Liouville systems (Prof. Hsin-Yuan Huang), and Ginzburg-Landau patterns (Dr. Jia-Yuan Dai). Thanks to the support of NCTS, all participants enjoyed interesting talks and had nice interactions in both lecture time and free discussion time.

# 2021 Spring Day

## March 29, 2021

**Venue** R509 & R515, Cosmology, Bldg, NTU

The 2021 NCTS Spring Day was held on March 29 and was attended by 41 mathematicians. As is traditional by now, during this annual event, each NCTS postdoctoral fellow is invited to give a 30 minute presentation of his/her research work. In addition, this year's NCTS Young Scientist Award was presented to Chih-Whi Chen from the National Central

University for his contribution in representation theory of Lie superalgebras. This year's Spring Day with a total of 18 invited talks created an excellent opportunity for lively scientific discussion among the attendees, and we believe that, just like past Spring Days, this event has been successful and has benefited the mathematics community in Taiwan.



# NCTS Young Theoretical Scientists Award



Chih-Whi Chen received his PhD degree in mathematics from National Taiwan University in 2016. He was a postdoctoral fellow in NCTS (Aug. 2016–Aug. 2017) and Uppsala University (Sep. 2017 to Aug. 2018). He was an assistant professor in Xiamen University in Sep. 2018 and joined National Central University in Sep. 2019.

His research area is the representation theory of Lie algebras and Lie superalgebras with particular interest in related problems of homological algebra and irreducible character for module categories over Lie (super)algebras. Lie superalgebras are generalizations of Lie algebras and are important objects in theoretical physics for they are used to describe the mathematics of supersymmetry. A complete list of complex simple Lie superalgebras was obtained by V. Kac. There are two particularly interesting infinite series called types P and Q for they have no proper Lie algebra

analogue except that one might think the type P Lie superalgebras are purely odd analogue of orthogonal/symplectic Lie algebras and the type Q as the purely odd type A Lie algebras.

Recently, Chi-Whi studies Whittaker modules over quasi-reductive Lie superalgebras. Whittaker modules constitute an important class of modules over semisimple Lie algebras.

He establishes an equivalence between the categories of Whittaker modules and of Harish-Chandra bimodules and provides a classification of simple Whittaker modules for a  $\mathbb{Z}$ -graded quasi-reductive superalgebra.

All in all, Chih-Whi is an industrious and productive young researcher, making steady progress in an important area. We have much to expect of Chih-Whi in the future.

## Chih-Whi's works recognized by YTSA

- Early works on the categories of finite-dimensional modules over the type P Lie superalgebras, including a Bernstein-Gelfand-Gelfand (BGG) reciprocity and the computation of characters of irreducible modules.
- Works on the socle of the tensor product of simple modules of Lie algebras, which reduces the classification of simple modules over the queer algebra  $Q(n)$  to the classification of simple  $sl(n)$ -modules (joint with Coulembier and Mazorchuk).
- A version of Ringel duality for classical Lie superalgebras and a classification of projective-injective modules in the full BGG category  $\mathcal{O}$  for simple classical Lie superalgebras (joint with Cheng and Coulembier).



## NCTS Research Spotlight 1

### Simple Lie Superalgebras and their Representations

**Prof. Shun-Jen Cheng is a Distinguished Research Fellow at Inst. Of Mathematics, Academia Sinica, and the vice-director of NCTS. His research interests lie in the representation theory of Lie algebras and Lie superalgebras. His studies in the past include the relationship between the representation theory of classical Lie algebras and that of Lie superalgebras of classical types. He is especially interested in understanding the characters of modules over Lie superalgebras in general.**

Symmetry, as one of the most abundant and beautiful phenomena appearing in nature, has applications in virtually every branch of sciences. Mathematically, symmetry is described in terms of groups and their actions. Supersymmetry, although not observed in nature, was introduced originally by physicists with the aim of describing physical phenomena that classical symmetry fails to explain. Mathematically, supersymmetry is rigorously described in terms of supergroups and their actions, or in terms of their infinitesimal analogues superalgebras and their modules.

Although Lie superalgebras were introduced to explain physical phenomena, by now they have become objects of substantial mathematical interest of their own. Since their introduction there has been substantial efforts by mathematicians and physicists alike in trying to understand the finite-dimensional complex simple Lie superalgebras. These efforts culminated in the classification of the finite-dimensional complex simple Lie superalgebras by Victor Kac in 1977.

Since then the main attention has shifted to the representation theory of finite-dimensional simple Lie superalgebras. The first step in this direction was to understand the finite-dimensional

representation theory. Of fundamental importance is the problem of computing the finite-dimensional irreducible characters. We recall that for finite-dimensional semisimple Lie algebras the solution of the analogous problem is given by the celebrated Weyl character formula. However, for Lie superalgebras, such a Weyl-type character formula does not hold in general. Indeed, Kac proved that only for a very special class of what is now known as "typical" representations one has such a uniform character formula. Also, in contrast to semisimple Lie algebras, the categories of finite-dimensional modules for simple Lie superalgebras are in general not semisimple.

Progress was slow at first in the computation of the finite-dimensional irreducible characters for the "atypical" modules. It turned out that the problem even for the classical Lie superalgebras, i.e., the general linear and the orthosymplectic Lie superalgebras, is rather challenging. A first breakthrough in this direction is obtained by Vera Serganova who gave an algorithm for computing the irreducible character for the general linear Lie superalgebras in the nineties. This work shows that a closed character formula for finite-dimensional irreducible modules over classical



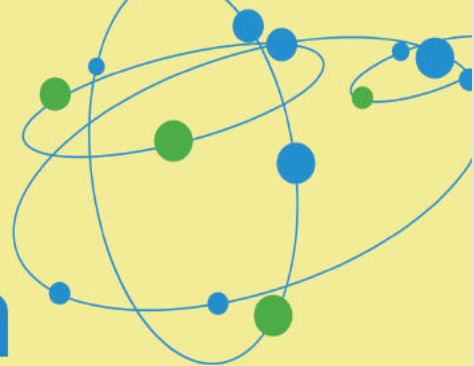
Lie superalgebra of Lie superalgebras was very likely not possible.

A new viewpoint in the representation theory of Lie superalgebras was provided by the work by Brundan in 2004, where he gave a conceptual method to compute the finite-dimensional irreducible characters of the general linear Lie superalgebra. His approach is based on Lusztig's canonical basis, and brings in a completely new perspective to the super Lie theory. Maybe, even more remarkable than his solution to the problem itself is the fact that his approach enabled him to formulate a Kazhdan-Lusztig type conjecture for the irreducible character of the general linear Lie superalgebra in the Bernstein-Gelfand-Gelfand (BGG) category for the first time.

These works motivated the formulation of a super duality conjecture in a subsequent joint work with Weiqiang Wang and Ruibin Zhang in type A. The conjecture gives an explicit connection between representation theories of Lie algebras and Lie superalgebras, and it was proved in more generality in a joint work with Ngau Lam. Eventually, this new viewpoint enabled us to prove Brundan's Kazhdan-Lusztig conjecture in a joint work with Lam and Wang. This super duality approach, enhanced with a newly developed theory of canonical basis for quantum symmetric pairs, enabled Huanchen Bao and Wang to compute the irreducible characters of the orthosymplectic Lie superalgebras in the BGG category. For their work on quantum symmet-

ric pairs and application to orthosymplectic Lie superalgebra, Bao and Wang received the AMS Chevalley Prize in 2020.

Substantial progress in the computation of irreducible characters in the BGG category for other classical Lie superalgebras has also been made in the last few years. We point out here the works for the queer Lie superalgebras by Chih-Whi Chen (reduction methods), Brundan and Davidson (in terms of Webster's orthodox basis), and our joint works with Jae-Hoon Kwon and Weiqiang Wang (further connection with canonical basis). Also there are three exceptional simple Lie superalgebras of classical type that do not belong to any of the infinite series. In this direction, recently our joint works with Weiqiang Wang, Li Luo, and Chih-Whi Chen have essentially completely solved the problem for two of these Lie superalgebras in arbitrary (not necessarily integral) blocks. However, there are still several open problems in the computation of irreducible characters in the BGG category, e.g., the computation of characters in the integral blocks for the queer Lie superalgebras and the computation of the irreducible characters for the periplectic Lie superalgebras.



# Taiwan Mathematics School 2021 Spring

This semester there are three courses which are continuations of the courses of last semester. And there's also a course about Mathematical Foundation and AI in Medicine.

## Algebraic Combinatorics II

**Lecturer:** Eiichi Bannai (Kyushu University)

The purpose of this course is for the audience to explore "algebraic combinatorics" without assuming any previous knowledge of combinatorics. I will try to propose many open problems hoping that some of them are accessible and solvable by the audience. (We encourage and emphasize the discussions on trying to solve these open problems among the audience.) We invited Prof. Eiichi Bannai as our lecturer of this course.

Prof. Eiichi Bannai is a worldwide well-known mathematician working in algebraic combinatorics area. He had been professor in Ohio State University (US), Kyushu University (Japan) and Shanghai Jiatong University (China). He is passionate to supervise students and to teach young generation mathematicians.

## Algebraic Number Theory II

**Lecturer:** Chia-Fu Yu (Academia Sinica)

This is the second semester of algebraic number theory (ANT). The aim is to provide important tools and background knowledge in ANT. We will first complete Lang's exposition of Global Class Field Theory (Artin reciprocity) from the Fall semester. We then develop in details cohomology of groups and Galois cohomology, including Tate's duality theorem. We also plan to cover the local approach of class field theory as well as the Lubin-Tate formal multiplication following Serre's exposition. If time permits, we shall discuss Hilbert symbols, Chebotarev's density theorem and Herbrand function.

## Topics in Geometric Measure Theory II

**Lecturer:** Ulrich Menne (National Taiwan Normal University)

This course provides a thorough introduction of the classical parts of varifold theory including the fundamental compactness theorems and Allard's regularity theorem. Most of the necessary background from locally convex spaces, distribution theory, Grassmann manifolds, and curvature of submanifolds shall be developed in the course. However, we do assume knowledge of real analysis—in particular, concerning the representation of linear functionals by measures and differentiation theory of measures (e.g., covering theorems and densities). The main topics also employ basic properties of Hausdorff measures, the concepts of  $(H^m, m)$  rectifiability of subsets of  $\mathbb{R}^n$ , and some Grassmann algebra (centred around  $m$  vectors and alternating forms). Lecture notes on Geometric Measure Theory (see [Men20]) are made available upon request; they include all afore-mentioned prerequisites. Participants meeting some but not all of these prerequisites are likely to find a suitable topic amongst the preparatory and supplementary ones listed below.

## Mathematical Foundation toward Artificial Intelligence in Medicine

**Lecturer:** Hau-Tieng Wu (Duke University)

Advances in technology have led to versatile and complicated modern datasets, particularly at the forefront of medical advances. With modern data analysis, it has been claimed that soon artificial intelligence (AI) would be generated and human being will be replaced. Toward this goal, the main question is how to analyze modern datasets.

Nonlinearity and nonstationarity are common features of these modern datasets. To quantify these features and hence carry out statistical analyses, there are unprecedented demand for new techniques to model the underlying structure, and develop algorithms to analyze the data. On the other hand, a solid and correct understanding of background knowledge plays a significant role when we apply these new techniques, particularly for the scientific research purpose.

In this lecture, we focus on modern machine learning techniques, particularly unsupervised learning, from both theoretical and practical perspectives, and aim for solving real-world biomedical problems, and hence a possibly ideal AI, or intelligence augment (IA) system. This course is designed to train simultaneously students from different and diverse disciplines with proper background (shown below), including but not exclusively mathematics, statistics, engineering and medicine.

(1) From the theoretical perspective, we introduce statistical theories established based on the differential geometry and random matrix theory framework. We will focus on feature extraction, dimension reduction, data visualization, etc. All introduced theories are needed for real world problems.

(2) From the algorithmic perspective, we will provide algorithm details and implementation tricks. Matlab code will be provided for practicing.

(3) From the practical perspective, we will discuss how to apply the introduced algorithms and established theory to analyze real datasets assuming basic knowledge of supervised learning tools. We will focus on extracting dynamics from biomedical time series, either single channel and multiple channels. We will not focus on biomedical image analysis or pattern recognition unless the temporal information is a concern in the dataset.

Students will be formed into groups, and each group will be assigned a medical problem, a physician mentor, and a hospital. Minimally two hospital tours will be arranged for students to appreciate the importance of clinical problems, and how the data collection and analysis result application are carried out in the clinical environment. Fully/partially solving this medical problem with mathematically rigorous techniques is the goal of this course.

A short summary course of needed background on several topics will be provided in the format of seminar discussion before the semester starts for the sake of self-containedness. Due to the black-box nature of deep learning and the dangers of abuse, it is not the focus of this lecture.





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## NCTS is calling for 2022 postdoctoral fellows!

The NCTS accepts applications for postdoctoral fellows from Oct. 1, 2021 to Feb. 28, 2022. The appointment starts from August 1, 2022, for 1 or 2 years, extendable up to 3 years. Every talent pursuing academic excellence is welcome. Priority will be given to the following areas: *Algebraic Geometry, Differential Geometry and Geometric Analysis, Differential Equations and Stochastic Analysis, Scientific Computing.*

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