The 2020 Annual Report National Center for Theoretical Sciences Mathematics Division

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# 1 Overview

## 1.1 Goals and missions of the Center

Established in 1997, National Center for Theoretical Sciences (NCTS) has been serving the mathematical community of Taiwan for more than twenty years. During the past two decades, the NCTS has been well-recognized as one of the leading research centers in Asia. The main goals of NCTS is to promote fontier research in theoretical sciences and to serve as a platform for collaboration and interaction. To achieve the goals, NCTS has been working very hard on the following specific aspects:

- Foster world class outstanding researchers, and attract top young researchers to do cutting-edge research in NCTS;
- b. Attract worldwide outstanding theoretical scientists to do research in Taiwan;
- c. Develop international and inter-disciplinary scientific research program;
- d. Promote international cooperation and collaboration and aim to become a leading research institution in Asia and worldwide.

## **1.2** Brief summary and highlights of the year

During the past couple years, our visitor program was quite successful. As we encourage our visitors to stay longer in order to have more interaction and possible collaboration with local researchers, the average length of their stay also increased. It is also worth mention that there are increasingly more visitors who come to NCTS to do research using their own travel grants. All these signs show that NCTS has become an important center for mathematical research.

Another highlight of our program was the training program of younger generation. We had joint summer school with MSRI in Taipei for our students to learn and work together with students from the US; the EACDFM (Each Asian Core Doctoral Forum of Mathematics) provides an opportunity for Ph.D. students in East Asia to present their own works and make friends; the international exchange program for young scholars to visit our cooperative research centers, etc. These versatile programs attracted more and more students to do advanced studies in theoretical sciences.

However, the outbreak of Covid-19 made 2020 an extremely difficult year for the NCTS. The border control, quarantine policy and other measures make it almost impossible to travel internationally. Not only our highly-reputed visitor program was stopped almost completely, those abovementioned international programs for students and young scholars were either cancelled or postponed.

Facing the challenges caused by the pandemic of Covid-19, we made the following major adjustments of our operation. First of all, we reallocated more of our budget to student training. To this end, we enlarged our existing programs USRP (Undergraduate Summer Research Program) and our RA program. Other than these, we also initiated two new programs: URP (Undergraduate Research Program) and a special program hosting Taiwanese students who were not able to return to their campus abroad due to the pandemic.

Other than those initiatives, we also spent substantial effort setting up the hardware and software for online meetings and courses. We help our members to keep connected to each other and keep connected with the rest of the world as well. Moreover, we also initiated and played a role in keeping the academic community in the mathematics connected. For example, we started the Geometric Measure Theory network which contains some courses and invited keynote talks by top mathematicians in the field. Also, we have been actively involved in the organizing the Seminar of Algebraic Geometry in East Asia, which is a bi-weekly online seminar.

We hope that our adjustment can help the researchers in Taiwan to keep up with the development of the world, as we have always been doing in the past.

# 2 Operation and Achievement of the Center

## 2.1 Postdoctoral program and cultivation program

## 2.1.1 Graduate and undergraduate program

1. Research Assistants

We allocated a total of 10 Student Research Assistant positions. These positions are designed for students who are in transition to their advanced studies. We open call for applications twice a year. Qualified applicants are interviewed by some of our members. The selection criteria is basically the potential of students to be admitted to prestigious Ph.D. program. Each RA is associated with a topical program and assigned a mentor. Moreover, they are required to take courses and participate in seminars in their fields. Statistics show that about 2/3 of our past RA were admitted to Ph.D. program abroad.

2. Undergraduate Summer Research Program

The purpose of this program is to provide project-oriented studies for undergraduate students during summer. We started to run the USRP program in 2017. In the year of 2020, we received more applications. From an application pool of 52 students, we selected 30 students in 8 programs. It shows that our program has started to receive considerable reputation.

3. Undergraduate Research Program

After the success of the NCTS USRP, we started to consider potential "bridge" programs. It gives rise to the idea of Undergraduate Research Program (URP). The purpose of URP is two-fold. On one hand, some students might want further advanced studies after the exciting USRP. Hence URP provides an opportunities for students to do more solid and advanced studies under the supervision of experts. On the other hand, some students might need some more basic training before they can "get their hands dirty" in research program such as USRP. In this case, the URP provides an opportunities for project-oriented basic training.

We design the URP to start from Oct. 1 to June 30. After URP, students are encourage to apply for USRP or MOST Research Project for Undergraduates.

4. Special Program under Covid-19 Pandemic As we have noticed that earlier some young talents have been affected by the pandemic of Covid-19 substantially. We created a special program for them. In this program, we support two kind of students. The first kind of students are those who have already been admitted to prestigious institutions but are unable to attend due to the visa or border control restriction. The second kind of students are those who have been studying abroad, and came back to Taiwan for vacation or some other reason, but are unable to return to their affiliated institutions.

We provide them support and office space similar as NCTS RA so that they can still learn and work together at NCTS during this difficult period.

### 2.1.2 Postdoctoral program

One of the main goals of the NCTS is the to train our Taiwanese younger generation. Therefore, it is of fundamental importance to have a solid postdoc program. In fact, we aim to be the training camp for postdocs so that they will be able to advance their pursuit of higher advanced research.

We allocated a total of 20 positions for postdoc fellows in our annual budget. The current list of 17 postdocs consists of 7 Taiwanese and 10 international postdocs, and each of them is assigned to a mentor and is associated with a Topical Program. By doing so, each postdoc is involved in various activities of the NCTS. Other than those seminars and events of each Topical Program, we have launched the cooperative project Taipei Postdoc Seminar with Academia Sinica. This seminar is jointly organized by postdoc representatives from NCTS and AS, and the representative of NCTS this year is Peng-Jie Weng. There are talks, which are supposed to be accessible to a general audience of postdocs, every other week. It not only provides an opportunity for postdoc fellows to present their works, but also builds a platform for them for potential collaborations.

## 2.2 International cooperation

## 2.2.1 Cooperation with international institutions

There were several existing cooperation programs with international institutions. We used to call for proposals each year in December. The Executive Committee makes the final selection based on potential of the proposal. Priority will be given to young members. However, due to the pandemic of Covid-19, all these programs have been postponed. We hope that these programs will resume once international travel is feasible.

## a. MSRI, USA

The Mathematical Sciences Research Institute (MSRI) is a successful center with long history. After the previous successful joint summer school on *'Toric Varieties* in Jul 29 (Mon)-Aug 9 (Fri), 2019 held at NCTS, it was decided to hold another summer school *Well-poseness and Related Topics* in 2020. This summer school was postponed until 2022. The MSRI will provide support for lecturers and American students.

## b. RIMS, Japan

We have Exchange Researcher program. We sent young researchers to visit RIMS for one and two months usually.

c. KIAS, Korea

We have Exchange Researcher program. We sent young researchers to visit KIAS occasionally.

d. PMI, Korea

The joint NCTS-PMI workshop has been held annually for years. It was postponed and will be resumed soon.

e. Fields Insttitute, Canada

We have Exchange Researcher program. We sent researchers to visit Fields Institute occasionally.

## 2.2.2 Cooperation with foreign research team

There are several cooperative programs with international teams. Again, due to the pandemic of Covid-19, all these programs have been postponed.

a. East Asian Core Doctoral Forum in Mathematics

This is a forum organized by a team, whose members are 7 main universities: Tokyo, Kyoto, Tohoku, Seoul, Tsinghua, Fudan and National Taiwan University. It was initiated by Kotani, Tsutsumi, Kawahigashi, Ha, Jiou, Wu and Jungkai Chen. Every participating country recommends 8–10 Ph.D. (or postdoc) speakers. The purpose is to provide a platform for young students to present their works in an international meeting and help them build international connections. In the past, it was held in Kyoto, Taipei, and Fudan, Seoul, Tsing-Hua University, Tohoku, Tokyo. It was scheduled to be held at Taipei in 2021. It is now postponed to 2023.

b. Reaction-Diffusion Network in Mathematics and Biomedicine: The GDRI ReaDiNet

This is an international cooperative program led by James D. Murray, and scientific coordinators are from KAIST, Tokyo, Meiji Univ., Paris-Sud, Nice, Univ. Joseph Fourier, and NCTS. Chiun-Chuan Chen (NTU) and Jong-Shenq Guo (Tamkang Univ.) are representatives of the NCTS in this program. The next step of the network is yet to be determined.

## 2.2.3 New international online events

As the international travel was not feasible during this period, we initiated or participated several international online events.

a. Geometric Measure Theory network

This is an international network initiated at NCTS, mainly organized by Ulrich Menne. It consists of invited keynote talks delivered by top experts and lectures.

b. Algebraic Geometry in East Asia seminar

This is a joint effort of many algebraic geometers in East Asia, aiming to create a platform for algebraic geometers and students for further interaction and cooperation. The organizing committee consists of representatives from Taiwan, Japan, Korea, China, Hong Kong, Vietnam, and Singapore. The NCTS provides support and is considered to be the hub of this seminar.

## 2.3 Taiwan Mathematics School

We started the platform of Taiwan Mathematics School in 2017. It was started with a joint effort of Weichung Wang (NTU), Yu-Chen Su (NCKU), Min-Hsiung Lin (NCKU), Tsung-Ming Huang (NTNU), and Wen-Wei Lin (NCTU). The course of "Scientific Computing and Machine Learning on Multi- and Many Core Architectures" was organized and held in different places (NTU and NCKU). Then it was broadcasted to many more different places (NTHU and NCTU). With the assistance of online system and modern internet technology, the inter-university courses was started and served students from all over Taiwan. After that, a lot of efforts was made to tune up the infrastructure and logistic work. This year we had the following courses:

• Topics in Geometric Measure Theory I, Fall 2020, 3 credits.

- Geometric Measure Theory II, Spring 2020, 3 credits.
- Algebraic Combinatorics, Fall 2020, 3 credits.
- Algebraic Number Theory, Fall 2020.
- Symbolic Dynamics on Groups and Countable State Topological Markov Shifts, Fall 2019.
- Dynamics in Network Systems, Spring 2020.
- Mini course on Springer Fibers and Quiver Varieties, Oct. and Nov. 2020.
- Topics on Drinfeld Modules and T-Motives, Spring 2020, 3 credits.
- Statistical Foundations of Medical AI, Fall 2020.

The courses above are either offered by our long-term visitors, possibly with assistance of local members, or jointly by a group of local faculty members in related fields.

# 3 Summary of Data

# 3.1 Workshops and Conferences

2020 Conferences & Workshops				
A-Number Theory and Representation Theory	4			
B-Algebraic Geometry	3			
C-Differential Geometry and Geometric Analysis	1			
D-Differential Equations and Stochastic Analysis	6			
E-Scientific Computing	2			
F-Interdisciplinary Studies	2			
H-Harmonic Analisis	1			
L-Laboratory of Data Science	1			
O-Others	2			
	22			



# 3.2 Seminars

2020 Seminars	
A-Number Theory and Representation Theory	30
B-Algebraic Geometry	53
C- Differential Geometry and Geometrical Analysis	29
D-Differential Equations and Stochastic Analysis	82
E-Scientific Computing	4
F-Interdisciplinary Studies	22
H- Harmonic Analysis	2
L-Laboratory of Data Science	1
O-Others	2
	225



## 3.3 Courses

2020 Courses (including TMS)	
A-Number Theory and Representation Theory	7
B-Algebraic Geometry	0
C-Differential Geometry and Geometrical Analysis	3
D-Differential Equations and Stochastic Analysis	4
E-Scientific Computing	6
F-Interdisciplinary Studies	6
O-Others	0
	26



2020 Courses (without TMS)	
A-Number Theory and Representation Theory	3
B-Algebraic Geometry	0
C-Differential Geometry and Geometrical Analysis	1
D-Differential Equations and Stochastic Analysis	2
E-Scientific Computing	1
F-Interdisciplinary Studies	2
O-Others	0
	9



# 3.4 Visitors

year	2015	2016	2017	2018	2019	2020
visitors	269	407	359	246	197	61
days	3615	5159	4169	5018	2727	2877





## 3.4.1 Visitors from Abroad by Program

Program	Number	%
A-Number theory and Representation Theory	9	20.00%
B-Algebraic Geometry	5	11.11%
C- Differential Geometry and Geometric Analysis	10	22.22%
D-Differential Equations and Stochastic Analysis	4	8.89%
E-Scientific Computing	6	13.33%
F-Interdisciplinary Studies	8	17.78%
H- Harmonic Analysis	0	0.00%
L-Laboratory of Data Science	3	6.67%
0-Others	0	0.00%
	45	100.00%



## 3.4.2 Visitors from Abroad by Country

Country	Number	%
China	1	2.22%
Japan	7	15.56%
South Korea	2	4.44%
Canada	1	2.22%
France	2	4.44%
Germany	4	8.89%
Israel	1	2.22%
Italy	1	2.22%
Netherlands	1	2.22%
Portugal	1	2.22%
UK	1	2.22%
USA	23	51.11%
	45	100.00%



# 3.5 Summary of publication data



# 4 Academic Programs

## 4.1 Number Theory and Representation Theory

## 1. Core Members

- Faculties: Chieh-Yu Chang (NTHU), Ming-Lun Hsieh (AS), Ching Hung Lam (AS), Chia-Fu Yu (AS), Yifan Yang (NTU), Fu-Tsun Wei (NTHU), Ming-Hsuan Kang (NCTU), Yung-Ning Peng (NCU), Chun-Ju Lai (AS), Hsian-Yang Chen (NUT), Shih-Chang Huang (NCKU).
- Postdocs: Yao Cheng (AS), Shi-Yu Chen (AS), Jia-Wei Guo (NTU), Oguz Gezmis (NTHU), Rytaro Harada (NCTS visiting scholar), Changningphaabi Namoijam (NTHU).
- 3. Ph.D. students: Yen-Tsung Chen (NTHU), Hung Chiang (Columbia U. and NCTS visitor).
- Research Assistants and Master Students: Sheng-Yang Ho (NTHU), Pham Lan Huong (NTHU), Tze-Ming Lin (NTHU), Tsu-Chun Dou (NTHU).

## 2. Program Overview

Our program on number theory and representation theory at NCTS covers a wild range of research topics, including Iwasawa theory, automorphic forms and automorphic representation, transcendental number theory and function field arithmetic, arithmetic of abelian varieties and moduli spaces, modular forms and modular differential equations, vertex operator algebras and superalgebras etc. One major role of NCTS is to provide a suitable platform that assists domestic number theorists and algebraists to continue and develop some active and promising research topics, and create some opportunities for international cooperation as well as cultivating young researchers and students. Overall, the research directions of our program can be roughly divided into the following.

- a. Iwasawa theory and *p*-adic methods in algebraic number theory and automorphic forms.
- b. Special values over function fields and related topics.
- c. Arithmetic geometry, modular forms and modular differential equations.

d. Vertex operator algebras, Lie superalgebras and related topics.

We execute this program by organizing research activities. For instance, we cultivate students by offering courses on Taiwan mathematical school, summer/winter schools and supporting travels of some outstanding students for attending important research activities abroad, eg., CMI-HIMR summer school in Bristol, Iwasawa 2019 in Bordeuax.

We run seminars regularly, and half of the speakers are from overseas. The main purpose is to enable our domestic researchers to have more contact with foreign scholars, and catch the latest developments in related important subjects. NCTS Number theory seminars are coordinated by C.-Y. Chang, and they were held in Taipei on Friday and in Hsinchu on Wednesday. Seminars on arithmetic geometry and representation theory are organized by C.-F. Yu and they were held in Taipei. Although many activities were reduced due to COVID-19 in the past year, we will try to conduct online meetings and make it regularly.

#### 3. Research Highlights

Our research results in the past year are very fruitful, especially some papers published/accepted in first class mathematical journals such as Inventiones mathematicae and American Journal of Mathematics. F.-T. Wei of NTHU publishes a solo-authored paper in the Inventiones. His result establishes an analogue of Colmez's conjecture for Drinfeld modules of any rank. C.-Y. Chang of NTHU and Yoshinori Mishiba prove a function field analogue of Furusho's conjecture for multiple zeta values, and their result is just accepted by the Inventiones. It is worthy mentioning that Wei and Chang graduated from NTHU under the supervision of Jing Yu, who ever served NCTS' director for some years, and they were benefited very much from NCTS since graduate students. This shows that NCTS has achieved results on cultivating talents, and it also gives us confidence that NCTS has ability to cultivate domestic scholars to publish papers in top journals.

M.-L. Hsieh of AS has a solo-authored paper accepted by American Journal of Mathematics. He constructs the three-variable p-adic triple product L-functions attached to Hida families of elliptic newforms and proves the explicit interpolation formulae at all critical specializations by establishing explicit Ichino's formulae for the trilinear period integrals of automorphic forms. His formulae perfectly fit the conjectural shape of padic L-functions predicted by Coates and Perrin-Riou.

Below we will sort out important research results in individual topics.

a. Iwasawa theory and *p*-adic methods in algebraic number theory and automorphic forms.

This direction is mainly led by M.-L. Hsieh. There are two major results obtained in these few years. Darmon and Rotger introduced so-called generalized Kato classes in the Selmer group of any p-ordinary elliptic curve E and they further conjectured that these generalized Kato classes are non-zero Selmer classes if and only the rank of E is two (in JAMS 2016). In [CH], Castella and Hsieh give the first examples of this conjecture of Darmon and Rotger. More precisely, they prove that the generalized Kato class is non-vanishing if and only if the *strict* p-Selmer group is one (this implies the p-Selmer group has rank two). In particular, this allows them to give a p-adic construction of a non-trivial Selmer classes for elliptic curves of rank two by using the p-adic deformation of the diagonal cycles. According to literature, this is the first result regarding the construction of non-trivial Selmer classes for elliptic curves of rank two by using the precise for elliptic curves of rank two by using the first result result result results.

In [HY], Hsieh and Yamana complete the construction of the four-variable *p*-adic triple product L-functions attached to Hida families of square-free levels and establish the trivial zero conjecture for the triple product of elliptic curves. This work extends Hsieh's previous work [H] on three variable *p*-adic triple product *L*-functions by including the cyclotomic variable.

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[CH] F. Castella and M.-L. Hsieh, *On the non-vanishing of generalized Kato classes for elliptic curves of rank two*, arXiv:1809:09066.

[H] M.-L. Hsieh, *Hida families and p-adic triple product L-functions*, to appear in American Journal of Mathematics.

[HY] M.-L.Hsieh and S. Yamana, Four variable p-adic triple product L-functions and the trivial zero conjecture, arXiv:1906.10474.

b. Special values over function fields and related topics.

This direction is led by C.-Y. Chang and F.-T. Wei. Our group on this topic currently works on special values arising from periods of *t*-motives and automorphic representations over function fields in positive characteristic.

(1) For special values in positive characteristic, C.-Y Chang currently works multiple zeta values (abbreviated as MZV's). In [CM], Chang

and Mishiba prove a function field analogue of Furusho's conjecture asserting that the *p*-adic MZV's satisfy the same linear relations that the real-valued MZV's satisfy. The key ingredient in the paper is to give logarithmic interpretation for MZV's both in  $\infty$ -adic and *v*-adic cases in the sense that the given MZV can be related to certain coordinate of the logarithm of an explicitly constructed *t*-module at a special point.

There are two continuing works of [CM]. One is to give explicit formulae for other coordinates mentioned above in the  $\infty$ -adic case, and it has been worked out in [CGM]. Another project is to prove the linear map in [CM] is indeed an algebra homomorphism, which was predicted in [CM] from computational evidence. With the joining of Y.-T. Chen, a PhD student of Chang, now this has been proved in the preprint [CCM].

(2) For special values in characteristic zero, Wei currently establishes a Kronecker limit formula over function fields. He considers the Goldman-Iwahori space of rank r over a non-archimedean local field F consists of all the norms on the vector space  $F^r$ , and then introduces an Eisenstein series on Goldman-Iwahori spaces over local fields with positive characteristic, and further proves an analogue of the Kronecker limit formula in [W]. This enables him to derive an analogue of Colmez-type formula in the setting of Drinfeld modules. It is a complete generalization of his previous work in the rank 2 case.

Our group is also interested in arithmetic problems arising from arithmetic geometry over function fields. There is a series joint works by F.-T. Wei and M. Papikian of Penn State University. They currently make use of the Eisenstein ideal of the Hecke algebra on Drinfeld modular Jacobians with square-free levels to compare their rational torsion subgroups with its subgroup generated by the cuspidal divisors. This enables them to produce explicit examples of JacquetLanglands isogenies. Their results are stronger than what is currently known about the analogues of these problems over Q. One of theri present work is to explore the phenomenon in the higher rank case. Papikian and Wei started with the prime level case, and explicitly determine the structure of the corresponding cuspidal divisor groups in [PW]. This is accomplished by examining the Fourier coefficients of the harmonic 1-cochains in higher rank case, with the help of Wei's work on the Kronecker limit formula in [W].

## **References**:

[CM] C.-Y. Chang and Y. Mishiba, On a conjecture of Furusho over func-

*tion fields,* to appear in Inventiones mathematicae.

[CGM] C.-Y. Chang, N. Green and Y. Mishiba, *Taylor coefficients of Anderson-Thakur series and explicit formulae*, to appear in Mathematische Annalen. [CGM] C.-Y. Chang, Y.-T Chen and Y. Mishiba, *Algebra structure of multiple zeta values in positive characteristic*, submitted 2020.

[W] F.-T. Wei, *On Kronecker terms over global function fields*, Inventiones Mathematicae 220 (2020) 847-907.

[PW] Mihran Papikian and Fu-Tsun Wei, *Drinfeld discriminant function and Fourier expansion of harmonic cochains*, submitted.

c. Arithmetic geometry, modular forms and modular differential equations.

This direction is led by C.-F. Yu and Y. Yang. Yu and his coauthors currently have good progress on arithmetic geometry. In [TY], C.-F. Yu and his postdoctor Terakad show that the systems of prime-to-p Hecke eigenvalues arising from automorphic forms (mod *p*) for a good prime p associated to an algebraic group  $G/\mathbb{Q}$  of Hodge type are the same as those arising from algebraic modular forms (mod *p*) associated to an inner form of G. In [KYU], Yu and his authors determine mass formulae for all principally polarized supersingular abelian threefolds defined over an algebraically closed field k of characteristic p. They combine these results with computations of the automorphism groups to study Oort's conjecture; indeed they prove that every generic threedimensional principally polarized supersingular abelian variety over k of characteristic  $\neq 2$  has automorphism group  $\pm 1$ . In [CYZ], Yu and his coauthors study the geometry of reduction modulo p of the Kisin-Pappas integral models for certain Shimura varieties of abelian type with parahoric level structure. They give some direct and geometric constructions for the EKOR strata on these Shimura varieties, using the theories of G-zips and mixed characteristic local G-Shtukas. They establish several basic properties of these strata, including the smoothness, dimension formula, and closure relation. Moreover, They apply their results to the study of Newton strata and central leaves on these Shimura varieties.

Y. Yang and C.-S. Lin recently study quasi-modular forms and differential equations. In [LY], they explore a two-way connection between quasi-modular forms of depth 1 and a class of second-order modular differential equations with regular singularities on the upper half-plane and the cusps. Here they consider the cases  $\Gamma = \Gamma_0^+(N)$  generated by  $\Gamma_0(N)$  and the Atkin-Lehner involutions for N = 1, 2, 3 ( $\Gamma_0^+(1) =$   $SL(2, \mathbb{Z})$ ). Firstly, one notes that a quasi-modular form of depth 1, after divided by some modular form with the same weight, is a solution of a modular differential equation. Their main results are the converse of the above statement for the groups  $\Gamma_0^+(N)$ , N = 1, 2, 3.

#### **References**:

[LY] C.-S. Lin and Y. Yang, *Quasimodular forms and modular differential* equations which are not apparent at cusps: I, arXiv:2103.04890.

[KYU] V. Karemaker, F. Yobuko and C.-F. Yu, *Mass formula and Oort's conjecture for supersingular abelian threefolds*, arXiv:2002.10960.

[TY] Y. Terakado and C.-F. Yu *Hecke eigensystems of automorphic forms* (*mod p*) of Hodge type and algebraic modular forms, arXiv:2006.14342.

[CYZ] X. Shen, C.-F. Yu, C. Zhang EKOR strata for Shimura varieties with parahoric level structure, arXiv:1910.07785.

d. Vertex operator algebras, Lie superalgebras and related topics.

This direction is mainly led by C.H. Lam and Y.-N. Peng. In recent years, Lam continues to study the classification of holomorphic vertex operator algebras of central charge 24. In [L], he proves a conjecture proposed by G. Höhn. In addition, we also discuss a construction of certain holomorphic vertex operator algebras of central charge 24 using a specific orbifold vertex operator algebra. In addition, Lam and his coauthor also have interesting work around this topic. In 1993, Schellekens proved that the weightone space  $V_1$  of a strongly rational, holomorphic vertex operator algebra V of central charge 24 must be one of 71 Lie algebras. During the following three decades, in a combined effort by many authors, it was proved that each of these Lie algebras is realized by such a vertex operator algebra and that, except for  $V_1 = 0$ , this vertex operator algebra is uniquely determined by  $V_1$ . In [ELMS], Lam and his collaborators give a fundamentally different, simpler proof of Schellekens' list of 71 Lie algebras. In fact, they show that every strongly rational, holomorphic vertex operator algebra V of central charge 24 with  $V_1 \neq 0$  can be obtained by an orbifold construction from a Leech lattice vertex operator algebra. This suffices to restrict the possible Lie algebras that can occur as weight-one space of V to the 71 of Schellekens.

Recently, Y.-N. Peng makes good progress on finite *W*-superalgebras in [P]. More precisely, he gives the connection between the finite *W*-superalgebras and super Yangians for type *A*. That is, he explicitly gives a superalgebra isomorphism between the finite *W*-superalgebra associated to an arbitrary even nilpotent element  $e \in \mathfrak{gl}_{M|N}$  and a quo-

tient of a certain subalgebra of  $Y_{m|n}$ , obtaining a super analogue of the previous result by Brundan-Kleshchev (Adv Math 2006) for type A Lie superalgebras in full generality.

In [FLLLW], Lai and his coauthors introduce an affine Schur algebra via the affine Hecke algebra associated to Weyl group of affine type C. They establish multiplication formulas on the affine Hecke algebra and affine Schur algebra. Then we construct monomial bases and canonical bases for the affine Schur algebra. The multiplication formula allows them to establish a stabilization property of the family of affine Schur algebras that leads to the modified version of an algebra  $K_n^c$ . They also formulate several variants of the affine Schur algebra and the (modified) coideal subalgebra above, as well as their monomial and canonical bases. Their result provides a new and algebraic approach which complements and sheds new light on their previous geometric approach on the subject.

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[FLLLW] Z. Fan, C.-J. Lai, Y. Li, L. Luo, and W. Wang) *Affine Hecke algebras and quantum symmetric pairs*, to appear in Mem. Amer. Math. Soc.

#### 4. Highlights of Events

Because of COVID-19, people were not able to travel since the spring of 2020 and it caused that many activities have to be canceled. For instance, we canceled the big conference 'Conference on the non-triviality of arithmetic invariants and its applications' mainly planned and organized by M.-L. Hsieh. In addition, the joint conference on number theory between Taiwan and Japan has to postpone. However, we still hosted (or co-hosted) several workshops listed below.

### Workshops and conferences

1. Conference on Algebraic Representation Theory, Nov. 29-Dec. 1, 2019. Organizers: S.-J. Cheng, M.-K. Chuah, S.-Y. Pan and T. Shoji.

- 2. Japan-Taiwan Joint Workshop on Multiple Zeta Values (held in Okinawa), February 8-11, 2020. Organizers: C.-Y. Chang, M. Kaneko and J. Yu.
- 3. The Taiwan Number Theory Symposium, September 9-11, 2020. Organizer: M.-L. Hsieh.
- 4. *L*-values and Iwasawa theory, November 11-12, 2020. Organizers: A. Burungale , M.-L. Hsieh , B. Palvannan and S. Ramdorai.

We would like to highlight the three-day workshop 'The Taiwan Number Theory Symposium'organized by M.-L. Hsieh. It took place in NTHU and it brought together young and junior number theorists in Taiwan as well as several senior ones. The main purpose of this workshop is to let everyone know each other better and understand the progress of every speaker's current research. Indeed, everyone's comments and feedback are quite positive and we hope to have such a meeting every two years in the future.

#### **Courses and lectures:**

- 1. Taiwan Mathematical School (2019 Fall and 2020 Spring): Drinfeld modules and *t*-motives. Lecturers: C.-Y. Chang, F.-T. Wei and J. Yu. Organizer: C.-Y. Chang.
- 2. 2019 Undergraduate Summer Research Program 'Structure of Unitary Groups and Applications'. Organizer: C.-F. Yu.
- 3. 2020 NCTS Spring Course: Algebraic Tori and Hermitian Forms. Lecturer: C.-F. Yu.
- 4. 2020 Summer Short Course on Local Class Field Theory. Lecturer: C.-F. Yu.
- 5. Mini course on Springer fibers and quiver varieties. Lecturer: Chun-Ju Lai (AS). Organizers: C.-W. Chen and Y.-N. Peng.

### 5. Future Plan

For the future plan on research, our research group continues to broaden and deepen our investigation on new projects. M.-L. Hsieh will continue his joint project with S. Yamana on the *p*-adic Gross-Zagier formula for diagonal cycles. This project touches the core spirit of *p*-adic Kudla program and Iwasawa theory, and will have significant applications. Continuing their previous work, C.-Y. Chang and Y. Mishiba will cooperate to study the precise connection between  $\infty$ -adic MZV's and *v*-adic MZV's. Their ultimate goal is to calculate the precise kernel of the surjective homomorphism from  $\infty$ -adic MZV's to *v*-adic MZV's. F.-T. Wei is currently working on Chowla-Selberg formula for special values of geometric  $\Gamma$ -function over function fields in positive characteristic.

C.-F. Yu will continue to cooperate with his team on the study related to Shimura varieties. One target is to construct admissible integral models for Shimura varieties of abelian type, and to prove the non-emptiness of KR strata for the Pappas-Zhu integral model, the non-emptiness of NP strata as general as possible and the non-emptiness of EO strata for good reduction of Hodge type and abelian type. Y. Yang continues to collaborate with C.-S. Lin on the study of quasi-modular forms and modular differential equations. Their project is quite fresh in the sense that it combines number theory, ODE and complex analysis.

C.H. Lam plans to attack the famous conjecture of Frenkel-Lepowsky-Meurman on the uniqueness of the Moonshine vertex operator algebra. He also plans to study the full automorphism groups of all holomorphic vertex operator algebras of central charge 24.

Below is the list for the upcoming courses and lectures that we plan to run in the upcoming year.

#### **Courses and lectures:**

- 1. Taiwan Math School (2020 Fall): Algebraic Number Theory (I). Lecturer: C.-F. Yu.
- 2. 2020 Winter Short Course on Mass Formulae. Lecturer: C.-F. Yu.
- 3. Taiwan Math School (2021 Spring): Algebraic Number Theory (II). Lecturer: C.-F. Yu.
- 4. 2021 Undergraduate Summer Research Program: Comutational problems of certain algebraic tori. Lecturers: T.-Y. Lee and C.-F. Yu.

In the upcoming year, many seminar talks will be changed to be held online due to COVID-19. Concerning workshops and conferences, it seems difficult to host as epidemic is still series overseas. Since mathematician like to discuss in person, we hope to host conferences in physical meeting if possible. M.-L. Hsieh and F.-T. Wei are planning to organize the Taiwan-Japan joint conference on number theory in the summer of 2021, but we can not confirm if it can be held. It will be based on whether COVID-19 is well-controlled or not.

## 4.2 Algebraic Geometry

## 1. Core Members

- Faculties: Jheng-Jie Chen (NCU), Jiun-Cheng Chen (NTHU), Jungkai Chen (NTU), Wu-Yen Chuang (NTU), Shin-Yao Jow (NTHU), Ching-Jui Lai (NCKU), Jia-Ming Liou (NCKU), Eugene Xia (NCKU).
- 2. Postdocs: Iacopo Brivio, Chih-Wei Chang, Bin Nguyen, Chien-Hsun Wang, David Wen.
- NCTS Scholars: Paolo Cascini (Imperial College London), Yujiro Kawamata (University of Tokyo), Keiji Oguiso (University of Tokyo).

#### 2. Program Overview

Algebraic geometry studies geometric objects defined by algebraic equations. It has a long history and is considered to be one of the most important fields in modern mathematics. Throughout the years, it has developed diverse connection with many other fields, such as complex geometry, number theory, representation theory, mathematical physics, and analysis. The main goal of the Topical Program of Algebraic Geometry is to achieve important advances in algebraic geometry, and to cultivate the next generation of algebraic geometers, through strategic planning of seminars, courses, and workshops, designed for undergraduate and graduate students, postdocs, as well as researchers at various stages of their careers.

Thanks to the consistent and generous support of NCTS, the research group of algebraic geometry in Taiwan has maintained in a good size and been active in various directions of study. One of the most prominent research directions in Taiwan is birational geometry, and many of our core members work in this area, such as Jheng-Jie Chen, Jiun-Cheng Chen, Jungkai Chen, and Ching-Jui Lai. Their work has achieved certain international recognition, and many new international postdocs are attracted to NCTS as a result, including Iacopo Brivio from UCSD (Ph.D. student of J. McKernan), David Wen from UCSB (Ph. D. student of D. Morrison), and Bin Nguyen from Universidade de Lisboa (Ph.D. student of M.M. Lopes).

### 3. Research Highlights

The research highlights of the program consists of the following parts: the Noether inequality for algebraic threefolds; exceptional collection of objects on some fake projective planes; the mapping class group action on SU(3)-character varieties; on Chern number inequality in dimension 3.

1. The Noether inequality for algebraic threefolds, by *Jungkai Chen* (joint with Meng Chen and Chen Jiang).

One of the important problems in birational geometry is to study the "geography" of algebraic varieties, that is, the constraints on the possible values of the various birational invariants. For example, there are two fundamental birational invariants of a projective variety X of dimension n: the geometric genus  $p_g(X)$  defined by

$$p_g(X) = h^0(X', K_{X'}),$$

and the canonical volume vol(X) defined by

$$\operatorname{vol}(X) = \lim_{m \to \infty} \frac{h^0(X', mK_{X'})}{m^n/n!},$$

where X' is a resolution of X and  $K_{X'}$  is the canonical divisor. In 1875, M. Noether proved that

$$\operatorname{vol}(X) \ge 2p_q(X) - 4$$

for any projective surface *X*. This result, now known as the Noether inequality, is a milestone in the history of the surface theory.

Motivated by the study of explicit birational geometry of 3-folds, one naturally asks for the 3-dimensional analogue of the Noether inequality. This has been a very challenging open problem, especially when the 3-folds have non-Gorenstein singularities. In 2020, Jungkai Chen, joint with Meng Chen and Chen Jiang, prove that

$$\operatorname{vol}(X) \ge \frac{4}{3}p_g(X) - \frac{10}{3}$$

for any projective 3-fold X of general type with either  $p_g(X) \le 4$  or  $p_g(X) \ge 11$ . This inequality is optimal due to known examples found by M. Kobayashi in 1992.

2. Exceptional collection of objects on some fake projective planes, by *Ching-Jui Lai* (joint with Sai-Kee Yeung).

A fake projective plane is a smooth compact complex surface M with the same Betti numbers as  $\mathbb{P}^2_{\mathbb{C}}$  but not isomorphic to  $\mathbb{P}^2_{\mathbb{C}}$ . This is a notion introduced by Mumford, who also constructed the first example. They have been classified into 28 classes by the work of Prasad-Yeung in 2007. Most of the fake projective planes have the property that the canonical divisor  $K_M$  can be written as  $K_M = 3L$ , where L is a generator of the Néron-Severi group. The joint work of Ching-Jui Lai and Sai-Kee Yeung studies whether 2L has any nonzero global sections. It is known that  $H^0(M, 2L) = 0$  is equivalent to the sequence  $\mathcal{O}_M, -L, -2L$  being an exceptional collection for the bounded derived category of coherent sheaves on M. Their result shows that 30 fake projective planes support such a sequence, most of which are new.

3. The mapping class group action on SU(3)-character varieties, by *Eugene Xia* (joint with William M. Goldman and Sean Lawton).

Let  $\Sigma$  be a compact orientable surface of genus g = 1 with n = 1 boundary component. The mapping class group  $\Gamma$  of  $\Sigma$  acts on the SU(3)-character variety of  $\Sigma$ . In the joint work of Eugene Xia, William M. Goldman, and Sean Lawton, it is shown that the action is ergodic with respect to the natural symplectic measure on the character variety.

4. On Chern number inequality in dimension 3, by *Jheng-Jie Chen*.

The main goal of birational geometry is to classify algebraic varieties up to birational equivalence and to find a good model inside each birational equivalent class. Based on the work of Reid, Mori, Kollár, Kawamata, Shokurov and others, the minimal model conjecture in dimension three in characteristic zero was proved by Mori. That is, starting from a mildly singular threefold, there exists a sequence of elementary birational maps (divisorial contractions and flips) such that the end product is either a minimal model or a Mori fiber space.

It is thus natural to expect that further detailed and explicit studies of three dimensional birational maps in the minimal model program will be useful in the studies of three dimensional geometry in general. The case of divisorial contractions to points has been intensively studied and classified by Kawamata, Kawakita, Hayakawa and Jungkai Chen, but the cases of divisorial contractions to curves and flips are less studied. Jheng-Jie Chen proves that if  $X \rightarrow X^+$  is a threefold terminal flip, then

$$c_1(X) \cdot c_2(X) \le c_1(X^+) \cdot c_2(X^+),$$

where  $c_1$  and  $c_2$  denote the Chern classes. This gives an affirmative answer to a question of Xie. He also obtains similar but weaker result in the case of divisorial contractions to curves.

### 4. Highlights of Events

This year we are facing a historic global pandemic, which prohibits almost all academic international traveling. Even during this challenging time, we still managed to have in-person domestic activity as well as online international activity, which were well participated and well received.

1. NCTS Summer Workshop on Foliation.

The notion of foliation was introduced by differential geometers and has received considerable attention for decades. Recently there has been increasing interest in applying the various techniques of foliation into algebraic geometry. The decomposition of Calabi-Yau manifolds and foliated MMP are among many of them.

In order to understand the recent development of foliation in algebraic geometry, we organized a working seminar starting in July on foliation. Some basic introduction was given in this working seminar. And then we organized an NCTS Workshop on Foliation, which was held from August 24 to August 26, at ShiTou. We arranged seven 90-minute talks during the workshop. Details of the recent works of Horing-Peternell and Dreul were discussed. The schedule of the workshop were not very tight in order to encourage more discussion. Our participants found the arrangement very helpful for fruitful interaction.

2. NCTS Workshop on K-stability.

The concept of K-stability originated from complex geometry. It was introduced by Gang Tian as a criterion to characterize the existence of Kähler-Einstein metrics on Fano manifolds. Later it was formulated using purely algebraic geometric terms by Simon Donaldson. Recent development has made it become clear that K-stability plays an important role in the study of birational geometry of Fano varieties.

In order to understand the connection between K-stability and the birational geometry of Fano varieties, we organized a working seminar starting in October on K-stability. Some basic introduction was given in this working seminar. And then we organized an NCTS Workshop on K-stability, which was held from November 12 to November 13, at Fo Guang University. We arranged six 90-minute talks during the workshop. We covered a good portion of the recent survey article by Chenyang Xu. The schedule of the workshop were not very tight in order to encourage more discussion. Our participants found the arrangement very helpful for fruitful interaction.

3. NCTS One-day Workshop in Algebraic Geometry.

This was a one-day workshop on October 16 at NCKU. The aim for this workshop is to promote the interaction among algebraic geometers in Taiwan. Five local speakers were invited to each give a 50minute talk on their recent works.

4. Seminar in Algebraic Geometry of East Asia

Due to the global pandemic, most international research activities are switched to be online. NCTS has joined in and supported one of these activities, the seminar of Algebraic Geometry in East Asia (AGEA). AGEA is a collaborative program conducted by China, Japan, Korea, Singapore, Taiwan, and Vietnam, and consists of regular research talks from algebraic geometers around the world. Through participating in the AGEA seminars, researchers in Taiwan can see the research in current trends and keep in contact with the international community. The AGEA seminar is biweekly and started in August 2020.

## 4.3 Differential Geometry and Geometric Analysis

## 1. Core Members

Our basic participants are from AS, NCKU, NCTS, NCU, NCUE, NKNU, NPTU, NSYSU, NTHU, NTNU, and NTU.

- a. Key Members: Chih-Wei Chen, Jih-Hsin Cheng, River Chiang, Nan-Kuo Ho, Chin-Yu Hsiao, Yng-Ing Lee, Chun-Chi Lin, Ulrich Menne, Chung-Jun Tsai, Dong-Ho Tsai, Mao-Pei Tsui, Ye-Kai Wang.
- b. Associates: Shu-Cheng Chang, Hung-Lin Chiu, Chung-I Ho, Rung-Tzung Huang, Ting-Jung Kuo, Kuo-Wei Lee, Chih-Chung Liu, Chiung-Jue Anna Sung, Ryosuke Takahashi, Chin-Tung Wu, Siye Wu, Mei-Lin Yau, Mei-Heng Yueh.
- c. Postdoctoral Fellows: Jui-En Chang, Ting-Hui Chang, Sheng-Fu Chiu, Ser-Wei Fu, Andrea Galasso, Sin-Hua Lai, Yang-Kai Lue, Yoshihiro Sugimoto, Chien-Hsun Wang, Huan Wang, Yi-Sheng Wang, Hung-Yu Yeh.

- d. Long-term visitors: Da Rong Cheng (University of Chicago), Sun-Yung Alice Chang (Princeton University), Paul Yang (Princeton University), Shih-Fang Yeh (Michigan State University), Kuan-Hui Lee (University of California, Irvine), Tang-Kai Lee (Massachusetts Institute of Technology), Chao-Ming Lin (University of California, Irvine).
- e. NCTS scholars: Kaoru Ono (RIMS, Kyoto), Richard Schoen (UC Irvine).

## 2. Program Overview

The Program of Differential Geometry and Geometric Analysis covers a broad spectrum of themes, including a more analytic perspective as well as a more topological incarnation of modern geometry. Integration with applications is also recently introduced. Our current research focuses are:

A. Geometric evolution equations

Geometric evolution equations are important, active research topics where the deformation of geometric objects by nonlinear parabolic equations can bring a new understanding to the geometric or topological problems. We plan to study different geometric flows to understand the structure of manifolds and submanifolds. In particular, we would like to understand the following problems.

- Chung-Jun Tsai and Mu-Tao Wang have made very important progress about the dynamical stability of minimal submanifolds under mean curvature flow. Mao-Pei Tsui and his collaborators have studied the generalized Lagrangian mean curvature flow in cotangent bundles. Yng-Ing Lee and her team have made break-through in proving the stability of the equivariant Lagrangian expanders under Lagrangian mean curvature flow. The investigation to understand the structure of singularities of mean curvature flow will be an important topic in our research group.
- Chun-Chi Lin and his collaborators have made important progress on geometric flows of higher-order, in particular the long time existence of smooth solutions and their asymptotic behaviour for the gradient flows of the elastic energy of curves under various boundary conditions and length controls. Such results have potential applications in particle physics, mechanics, geometric control theory and computer aided geometric design. The plan is to continue exploring higher order geometric flows in the elastic network setting.

- Perelman's non-collapsing theorem and pseudo-locality theorem revealed that, along the Ricci flow, local volume can be controlled by the curvature and vice versa. Since then people have found more delicate relationships between volume and curvatures. We focus on the asymptotic volume ratio of ancient solutions because this is the crucial issue for classifications of finite time singularities.
- B. Geometry of CR and complex manifolds

One direction is to study geometric quantization theory on complex and CR manifolds. The aim of the geometric quantization theory of Kostant and Souriau is to relate the classical observables (smooth functions) on a phase space (a symplectic manifold) to the quantum observables (bounded linear operators) on the quantum space (sections of a line bundle). Our team led by Chin-Yu Hsiao has generalized a particularly efficient Berezin-Toeplitz quantization in several directions. Moreover, by developing a G-invariant complex Fourier integral calculus and G-invariant Szegö kernel asymptotics and spectral theory, we establish "quantization commutes with reduction theorem" on some classes of CR manifolds. The goal is to establish fundamental theorems on the CR manifolds of high codimensions concerning Kohn Laplacian, Szegö projection, embedding problems, and geometric quantization theory using a combination of microlocal analysis and representation theory. This also serves as an important step toward generalizing classical complex geometry theories to complex and CR orbifolds.

Another direction is to study phenomena related to Yamabe problem in CR geometry. The Yamabe problem concerns the existence of metrics with constant scalar curvature, which has many applications in various contexts. Motivated by Robin's volume renormalization for singular Yamabe metrics, Jih-Hsin Cheng and his collaborators defined an ADM-like p-mass with a positive mass theorem in dimension 3 for CR manifolds, and found solutions of the CR Yamabe problem with minimal energy. It is also observed that certain energy functional of CR invariant surface appears as the coefficient of the log term in the associated volume renormalization. Similar to Gromov-Lawson and Schoen-Yau's result in Riemannian geometry, connected sum of CR manifolds with positive CR Yamabe constant admits a CR structure with positive CR Yamabe constant. But in striking contrast to the Riemannian case, examples of compact three-dimensional CR manifolds of posi-

tive Webster class exist with negative p-mass, while the infimum of the CR-Sobolev quotient is not attained. We expect to develop further understanding concerning p-mass, Tanaka-Webster curvature, and CR Yamabe problem.

C. Mathematical general relativity

Einstein's General Relativity models the space-time by 4-dimensional manifolds and gravitational fields by Lorentzian metrics. The field and matter distribution are linked by Einstein's equations—a nonlinear second order partial differential system that involves 10 unknowns in 10 equations. It is not surprising that major breakthroughs in General Relativity require ideas and sophisticated tools from geometry and analysis. Prominent examples include Penrose-Hawking singularity theorem (by comparison geometry), the proof of positive mass theorem by Schoen-Yau (using minimal surface) and Witten (using spinor), and Christodoulou-Klainerman's nonlinear stability of Minkowski spacetime (intricate a priori estimates of connection and curvature components

Among the conserved quantities, only the definition of energy-momentum of isolated gravitating system at spatial infinity (Arnowitt-Deser-Misner) and at null infinity (Trautman-Bondi) is well-accepted. Mu-Tao Wang and S.-T. Yau made the breakthrough in 2008 by defining quasi-local mass for gravitational fields in finite regions. Together with Po-Ning Chen, they further defined a full set of quasi-local conserved quantities and derived many desirable properties of them in the past decade.

Our program investigates Chen-Wang-Yau's quasi-local angular momentum and center-of-mass at null infinity. Recently we witnessed the Nobel Prize winning detection of gravitational waves by LIGO. As more observational data flow in, the theoretical analysis is hindered by an insufficient understanding of angular momentum of gravitational fields. Our study will shed light in this direction.

## D. Symplectic geometry, gauge theory, and mirror symmetry

One of our focuses lies on the geometry and topology of the moduli space of connections with involutions. The moduli space of flat *G*-bundles over a Riemannian manifold has been studied intensively in the past 50 years in different forms: gauge theory, mathematical physics, moduli space of semi-stable holomorphic vector bundles, and geometric invariant theory. Over the years, we have extended the understanding of the moduli space of flat connections to the cases when the underlying manifold is nonorientable, or/and when the structure group G is complex connected. The current interest is to investigate Toda type equations as generalized flat (meromorphic to be precise) connections with involutions/real structures.

Another focus is dedicated to the understanding of the groups of symplectomorphisms. Using techniques of pseudoholomorphic curves developed by Gromov, the homotopy types of symplectomorphism groups have been obtained for some, but a very limited number of, symplectic four-manifolds. In fact, very few symplectomorphisms are known other than the famous Dehn-Seidel twists, which hinted at relation between symplectomorphism groups and Lagrangian embeddings. Following our works on construction of symplectomorphisms of certain types and analysis of their isotopy classes, our goal is to investigate symplectic mapping class groups, subgroups in the symplectomorphism groups, and Lagrangian invariants.

E. Computational geometry, and manifold learning theory

Many data can be viewed as a point cloud in an m-dimensional Euclidean space. In some cases, we expect that the cloud shows certain pattern and thus can be treated as a lower dimensional submanifold when the points are distributed sufficiently dense. Manifold Learning is to learn the submanifold via dimensionality-reduced embedding maps. For example, orthogonal projections are such maps, although they usually fail to visualize nonlinear or non-convex structures. Hence several nonlinear embeddings have been designed for practical usage. People use MDS, ISOMAP, LLE, Laplacian eigenmap, etc., to perform dimensionality reduction for various types of data including marketing surveys, medical diagrams and images, electron microscope images, etc. However, theoretical justification is rather obscure for most of the cases. Manifold Learning is, arguably, an empirical science and lacks for a systematized theoretical foundation. Such foundation shall be built on mathematics including geometry, topology, graph theory, functional analysis, stochastic analysis, and optimization. Our aim is to investigate the embedding maps in Manifold Learning by using geometric analysis.

One of our goals is to create an excellent research environment for differential geometers in Taiwan to work on world class subjects both independently and in collaboration with fellow mathematicians. Our research group has been very active and maintains a high research standard thanks to the consistent support of NCTS. We have formed several working teams
in Taiwan as well as internationally. Another important goal of ours is to integrate our activities to attract excellent undergraduate and graduate students and provide new opportunities for the postdoctoral fellows and junior faculty to enrich their research perspectives and prosper.

#### 3. Research Highlights

Our members have always been working vigorously and have produced many interesting and important results. Here we sample a couple of research highlights in our program:

a. Angular momentum at null infinity by Ye-Kai Wang (NCKU)

J. Keller, Ye-Kai Wang, and S.-T. Yau [1] evaluated the limit of Chen-Wang-Yau quasi-local angular momentum and center-of-mass at null infinity (described by Bondi-Sachs formalism). The result is expressed solely in terms of physical observables. Together with Po-Ning Chen and Mu-Tao Wang [2], they generalize the computation to asymptotically flat null hypersurface and show that the value is independent of O(1)-perturbation of foliations. This distinguishes Chen-Wang-Yau's definition from others and makes it suitable for applications.

b. Shi-type estimates of the Ricci flow based on Ricci curvature *by Chih-Wei Chen (NSYSU)* 

In [3] we prove that the magnitude of the derivative of Ricci curvature can be uniformly controlled by the bounds of Ricci curvature and injectivity radius along the Ricci flow. As a consequence, a precise uniform local bound of curvature operator can be constructed from local bounds of Ricci curvature and injectivity radius among all n-dimensional Ricci flows. Moreover we discuss the behavior of Ricci curvature and its derivative when the injectivity radius is thoroughly unknown. In particular, another Shi-type estimate for Ricci curvature is derivative of scalar curvature.

Our key technique can be seen as a geometrical alternative to Moser's iteration.

c. A strong stability condition on minimal submanifolds and its implications *by Chung-Jun Tsai* (*NTU*)

In [5], we observed that the key mechanism of [4] is a stability condition of minimal submanifolds. With that, we are able to prove a strong local rigidity theorem, and a mean curvature flow stability theorem with only Lipschtiz closeness assumption. In [6] we study a famous example of hyper-Kaehler 4-manifolds. It provides an interesting example to our theorem in [5]. We also confirm a conjecture by Micallef and Wolfson about the stability of minimal sphere in this example.

# References

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## 4. Highlights of Events

The scientific activities consist of three parts: seminar series; workshops and conferences; lectures and courses. As a whole they help our program maintain diversity, keep focus, support working relations, and develop future possibilities.

We summarize the activities in 2020 as follows.

a. Seminar Series: NCTS Differential Geometry Seminar, NCTS Seminar on Differential Geometry, Taipei Postdoc Seminar, NCTS Seminar on Symplectic Geometry and Gauge Theory, AS-NCTS Seminar on Geometry. b. NCTS international Geometric Measure Theory Seminar organized by Ulrich Menne (NTNU), Guido De Philippis (Courant Institute, New York University, US), Yoshihiro Tonegawa (Tokyo Institute of Technology, Japan), and Neshan Wickramasekera (University of Cambridge, UK).

As traditional (in-person) international conferences appear not to be feasible, we have chosen a seminar format with one presentation every other month. Thereby, we expect to minimise the impact of time zone differences, whereas the low frequency of presentations should facilitate reaching the target audience by keeping the quality standard high and the necessary time commitment low. Usually, we reach an audience ranging from Japan and Taiwan in the east all the way to Europe and the east coast of the US. We employ a combination of various software (MS Teams, Miro, and Wonder) to allow social interactions during and after the seminar.

The first speaker we have was Leon Simon (Stanford University) on 18th November 2020, and this series of seminars continues in 2021.

c. The 20th Taiwan Geometry Symposium, November 7-8, 2020

We started in Fall 2010 this series of regular meetings to foster discussions and interactions within the geometry community in Taiwan. We meet once every semester. This 20th symposium took place at NCTS / National Taiwan University with the follwoing speakers: Siao-Hao Guo (NTU), Jesse Madnick (NCTS), I-Hsun Tsai (NTU), Wei-Ting Kao (NTU), Tran The Dung (NTNU).

d. NCTS Winter Short Course: Introduction to Curve Shortening Flow by Chih-Wei Chen (NSYSU), Kuo-Wei Lee (NCUE), and Ye-Kai Wang (NCKU), January 13–17, 2020

Geometric flow is an active field in geometric analysis. It has played a prominent role in several recent breakthrough in geometry, topology and mathematical physics: Perelman used Ricci flow to prove the Poincare conjecture in 2003 (based on Hamilton's work from 1982 to 1997); Huisken-Ilmanen used inverse mean curvature flow to prove the Riemannian Penrose inequality in general relativity. Geometric flow also appears ubiquitously in applications. Applied mathematicians use mean curvature flow to smooth out rough data and study surface tension in fluids and materials. This minicourse introduces students the basic ideas geometric flow through curve shortening flow. We will follow professor R. Haslhofer's lecture notes and learn Huisken's proof of the classical theorem of Gage-Hamilton and Grayson using maximum principle, Harnack inequality and blowup analysis.

e. Taiwan Mathematics School: Geometric Measure Theory II by Ulrich Menne (NTNU), March 6–July 3, 2020

A very successful strategy for the study of geometric variational problems is to firstly prove existence in an enlarged class of competitors by means of compactness theorems and subsequently study the regularity of the solution therein. For instance, instead of considering only smooth submanifolds, one proves existence in the classes of boundaries of sets of finite perimeter, integral currents, or integral varifolds—all of which are based on the more basic concept of rectifiable set. In the ensuing regularity theory, the generality of varifolds allows to unify a substantial part of the treatment. The purpose of the course is to develop, after providing the necessary infrastructure, the concept of rectifiable set as well as key elements of the theory of varifolds.

f. NCTS Mini Course on Manifold Learning by Chih-Wei Chen (National Sun Yat-sen University) and Mao-Pei Tsui (National Taiwan University), July 23–24, 2020

Manifold learning encompasses much of the disciplines of geometry, computation, and statistics, and has become an important research topic in data mining and statistical learning. The simplest description of manifold learning is that it is a class of algorithms for recovering a low-dimensional manifold embedded in a high-dimensional ambient space. The goal of this mini-course is to introduce three of the important topics in manifold learning theory: PCA (Principal Component Analysis), diffusion map and MDS (Multidimensional Scaling).

g. Taiwan Mathematics School: Topics in Geometric Measure Theory I by Ulrich Menne (NTNU), September 18, 2020 – January 15, 2021

This course provides a thorough introduction of the classical parts of varifold theory including the fundamental compactness theorems and Allard's regularity theorem. Most of the necessary background from locally convex spaces, distribution theory, Grassmann manifolds, curvature of submanifolds, and elliptic partial differential equations shall be developed in the course. However, we do assume knowledge of real analysis—in particular, concerning the representation of linear functionals by measures and differentiation theory of measures (e.g., covering theorems and densities). The main topics also employ basic properties of Hausdorff measures, the concepts of  $(H^m, m)$  rectifiability of

subsets of  $\mathbb{R}^n$ , and some Grassmann algebra (centred around m vectors and alternating forms).

## 4.4 Differential Equations and Stochastic Analysis

## 1. Core Members

#### a. Differential Equations

Chiun-Chuan Chen (NTU), I-Kun Chen(NTU), Chun-Hsiung Hsia (NTU), Jenn-Nan Wang(NTU), Hung-Wen Kuo(NCKU), Kung-Chien Wu, Chinghsiao (Arthur) Cheng, Chi Hin Chan(NYCU), Hsin-Yuan Huang(NYCU), Jin-Cheng Jiang (NTHU), Dong-Ho Tsai (NTHU), Jong-Shenq Guo (TKU), Jann-Long Chern (NTNU), Zhi-You Chen(NCUE), Jia-Yuan Dai (NCHU).

#### b. Dynamical System

Jung-Chao Ban (NCCU), Kuo-Chang Chen (NTHU), Chih-Hung Chang (NUK), Cheng-Hsiung Hsu (NCU), Yi-Chiuan Chen (Sinica).

## c. Probability

Lung-Chi Chen (NCCU), Chii-Ruey Hwang (NCCU), Guan-Yu Chen (NYCU), Yuan-Chung Sheu (NYCU), Hao-Wei Huang (NSYSU), Shang-Yuan Shiu (NCU), Gi-Ren Liu (NCKU).

#### d. Harmonic Analysis

Chin-Cheng Lin (NCU), Chun-Yen Shen (NTU), Daniel Spector (NTNU).

#### 2. Overview of the Program

The study of differential equations is one of the most traditional fields of mathematics in Taiwan. The NCTS Topical Program Differential Equations and Stochastic Analysis also aims to incorporate interdisciplinary research as one of its priorities, and its four main directions are: partial differential equations, dynamical systems, stochastic analysis and harmonic analysis. Partial differential equations are one of the most widely adopted mathematical tools in modern sciences. Members of this NCTS Topical Program group are interested in both the developments of the mathematical theory and scientific applications. This includes, for example, synchronization problem, kinetic theory, elliptic partial differential equations, fluid dynamics, inverse problem and reaction-diffusion equations. Interests of the research groups in dynamical systems include bifurcation theory , chaotic systems, ergodic theory, fractal geometry, arithmetic dynamical systems, and complex dynamical systems. The research interests of harmonic analysis group include two weights problems for singular integrals, additive combinatorics and their connections to geometric measure theory. Probability theory plays an important role in many other areas of mathematics, such as partial differential equations, analysis, and combinatorics. It provides the theoretical basis for statistics. In stochastic analysis at the NCTS, focus is placed on statistical mechanics, mathematical biology, finance, theoretical computer science, branching processes and SPDE.

## 3. Research Highlights

## a. Differential Equations

- (1) In a joint work with B.-C. Huang, T. Wakasa, C.-J. Wang and C.-Y. Yu, published in J. Differential Equations (2020), Jong-Shenq Guo studied the structure of stationary solutions of a micro-electro mechanical system with fringing field. It is known that there is a positive critical value such that no stationary solutions exist for the applied voltage beyond this critical value, at least two stationary solutions exist for the applied voltage below this critical value and there is a unique stationary solution at this critical value. In this paper, we verify that there are exactly two solutions below this critical value analytically. Due to the gradient term in our equation, an explicit formula is not available. This is one of the major difficulties in dealing with this exact multiplicity problem. Finally, the stability of the smaller stationary solutions is also derived.
- (2) Chun-Hsiung Hsia and his collaborators studied the synchronized collective behavior of the Kuramoto oscillators with time-delayed interactions and phase lag effect. Both the phase and frequency synchronization are in view. They first prove the frequency synchronization for both semi-delay and full-delay models with heterogeneous time-delays and phase lags. They also prove the complete and partial phase synchronization for both models with the uniform time-delay and phase lag. The results show that the Kuramoto models incorporated with small variation of time-delays and/or phase lag effect still exhibit the synchronization. These support that the original Kuramoto model (i.e., no time-delays and phase lags) is qualitatively robust in the perturbation of time-delay and

phase lag effects. They also present several numerical experiments supporting our main results. This result is published in J. Differential Equations (2020).

- (3) Hsin-Yuan Huang and his collaborators studied the elliptic PDEs arising from Chern-Simons theories. For the Liouville system, they established the existence of one bubble solutions on flat torus and obtained the sharp estimates of multi-bubbles solutions on compact Riemann surface. For the Chern-Simons-Higgs model with its  $\mathcal{N}$ =2 SUSY extension, they obtained the multiple existence results for the radial solutions. For the Non-Abelian Chern-Simons system of rank two, the form is determined by a non-degenerate matrix K. One of the major questions is how the matrix K affect the structure of the solutions to the self-dual equations. The case det(K) > 0 is well-studied so far. They continue to study the case det(K) > 0.
- (4) Hung-Wen Kuo and his collaborators studied the boundary phenomenon caused by the abrupt change of the wall temperature on the basis of the linearized Boltzmann equation. We obtained the quantitative short time behavior of the solution and showed that the short time solution consists of the free molecular flow and its perturbation, which exhibits logarithmic singularities along the characteristic line and on the boundary.

#### b. Dynamical System

- Jung-Chao Ban, Chih-Hung Chang and their collaborators built up the theory to characterize chaotic properties, entropy and topological properties for the dynamics on free (semi) group (JSP 2020, JMP 2019, 2020; JAC 2019; Tran. AMS 2017; Nonlinearity 2017).
- (2) Yi-Chiuan Chen and his collaborators analyzed the topological properties of holomorphic Julia and Mandelbrot sets. They revealed the analyticity and degeneration of Julia and Mandelbrot sets of  $z^2 + c$  for a class of the parameter c. Related paper is published in Trans. Amer. Math. Soc.
- (3) Cheng-Hsiung Hsu and his collaborators examined the existence and stability of the traveling wave solutions of reaction-diffusion equation, include the illustration of periodic wavefronts. Recently, they attained some theorems about the entire solutions for delayed nonlocal dispersal system with monostable nonlinearities. Related papers are published in J. Dynamics Differential Equations, J. Differential Equations, and Nonlinearity.

(4) Kuo-Chang Chen and his collaborators characterize the variational structure for the heteroclitic orbits for the N-body problem and study the strict convexity for 5-body problem. (JDDE 2020; Nonlinearity 2019; JDE 2018)

#### c. Probability

- (1) Lung-Chi Chen and his collaborator consider the long-range models on  $\mathbb{Z}^d$  of random walk, self-avoiding walk, percolation and the Ising model, whose translation-invariant 1-step distribution/coupling coefficient decays as  $|x|^{-d-\alpha}$  for some  $\alpha > 0$ . In our previous work in 2015, we have shown in a unified fashion for all  $\alpha \neq 2$  that, assuming a bound on the "derivative" of the *n*-step distribution (the compound-zeta distribution satisfies this assumed bound), the critical two-point function  $G_{p_c}(x)$  decays as  $|x|^{\alpha \wedge 2-d}$  above the uppercritical dimension  $d_c \equiv (\alpha \wedge 2)m$ , where m = 2 for self-avoiding walk and the Ising model and m = 3 for percolation. In this paper, we show in a much simpler way, without assuming a bound on the derivative of the *n*-step distribution, that  $G_{p_c}(x)$  for the marginal case  $\alpha = 2$  decays as  $|x|^{2-d}/\log |x|$  whenever  $d \geq d_c$  (with a large spread-out parameter *L*).
- (2) Shang-Yuan Shiu and his collaborator studied of intermittency for the parabolic Anderson problem usually focuses on the high peaks of the solution. In this paper we set up the equation on a finite spatial interval, and study the part of the probability space on which the solution is close to zero. This set has probability very close to one, and we show that on this set, the supremum of the solution over space tends to 0 exponentially fast in time. As a consequence, we find that the spatial supremum of the solution tends to zero exponentially fast as time increases. We also show that if the noise term is very large, then the probability of large peaks decreases exponentially rapidly.
- (3) Kyung-Youn Kim and her collaborator considers non-isotropic Markov process and estimates the upper and lower bounds for the transition density of the processes. They consider the *d*-dimensional Lévy process  $L = (L^1, ..., L^d)$  where  $L^i$  are independent processes with jumping kernels  $J_{\phi}(x^i, y^i) = \phi(|x^i y^i|)^{-1}|x^i y^i|^{-1}$ . Here  $\phi(r)$  is an increasing function with weakly scaling condition. Using the jumping kernel *J* comparable to  $\sum_{i=1}^d J_{\phi}$ , they prove the existence of the Markov process *M* corresponding to *J*, and obtain the sharp two-sided transition density's bound for *M*. By Chen and Bass, the

non-isotropic process with the jumping kernel  $|x^i - y^i|^{-d-\alpha}$  is studied at the first time, and then, there is a research on the regularity of transition density. Very recently, there is a result on the sharp two-sided transition density estimates by Kassmann, Kim and Kumagai is a generalized result covering a very large class of Markov processes using the function  $\phi$ .

## d. Harmonic Analysis

- (1) Chun-Yen Shen and his collaborators, in a series of two weights projects, they proved a very general two weight boundedness criteria for any fractional singular integrals, including the Cauchy singular integral in the complex plane, that when one of the measures is supported on a curve and the other measure is arbitrary in terms of T1 conditions. In addition, the ultimate Tb criterion has also been proved to be held for Hilbert transform.
- (2) Daniel Spector and his collaborators, in an on-going series of papers, have established new Lebesgue and Lorentz space estimates for solutions of partial differential equations (PDE) with  $L^1$  data.?The type of PDE considered are Div-Curl, or more generally, Hodge systems, and arise in the study of electricity and magnetism.?The analogous estimates for  $L^p$  data, p > 1, are classical, and can be proved using soft analysis arguments.?However, the assumption the data is in  $L^1$  is the natural physical one, and requires different techniques.

#### 4. Highlights of Events

We briefly summarize some of the most interesting and important activities held in 2020 as follows.

- First NCTS PDE Symposium / 2020-02-13 at NTU / Chun-Hsiung Hsia( NTU), Kung-Chien Wu (NCKU).
- (2) Taiwan Mathematics School: Dynamics in Network Systems / 2020-03-03 – 2020-06-30) / Jung-Chao Ban (NCCU) and Je-Chiang Tsai (NTHU).
- (3) 2020 NCTS Summer Course on Dynamical Systems / 2020-08-07 2020-09-04 / Jung-Chao Ban (NCCU).
- (4) Second NCTS PDE Symposium/2020-08-27/at NCKU/Chen I-Kun ( NTU), Chun-Hsiung Hsia (NTU), Kung-Chien Wu (NCKU).

- (5) Taiwan Mathematics School: Symbolic Dynamics on Groups and Countable State Topological Markov Shifts / 2020-09-18 – 2021-01-15 / Jung-Chao Ban (NCCU).
- (6) 2020 NCTS South Workshop on Dynamical Systems / 2020-12-18 2020-12-19 / Chih-Hung Chang (NUK).
- (7) 2020 Chengda Workshop in Probability (2020/7/14-15), Lung-Chi Chen (NCCU).
- (8) 2020 NCTS PDE Young Scholar Workshop/2020-08-29–2020-08-31/ at NTTU / Nai-Heng Chang(NTTU), I-Kun Chen (NTU), Chun-Hsiung Hsia (NTU), Kung-Chien Wu (NCKU).
- (9) Nonlinear Phenomena in Evolutionary Partial Differential Equations/2020/9/22
   2021/7/31 at NTU/ Shih-Hsin Chen (NTU), Jia-Yuan Dai (NCTS), Shun-Chieh Wang (NTU).
- (10) NCTS-South Taiwan Seminar on Dynamical Systems / 2020/9/24-2021/1/14 at NUK/Jung-Chao Ban (NCCU), Chih-Hung Chang (NUK).

## 5. Future Plan

- (1) NCTS Probability summer school / Lung-Chi Chen (NCCU).
- (2) NCTS PDE seminar/Fall, 2021- Spring, 2022 / I-Kun Chen (NTU), Chun-Hsiung Hsia (NTU).
- (3) The 11th Japan-Taiwan Joint Workshop for Young Scholars in Applied Mathematics/ Spring, 2021/ Chiun-Chuan Chen (NTU) etc.
- (4) 2021 NCTS Young Dynamics Day/2021/2/19 at TKU/Jung-Chao Ban (NCCU), Chih-Hung Chang (NUK), Kuo-Chang Chen (NTHU), Cheng-Hsiung Hsu (NCU), Ting-Hui Yang (TKU)
- (5) Third NCTS PDE Symposium/2021-03-26/at NUK/Chun-Hsiung Hsia (NTU), Ying-Chieh Lin (NUK), Kung-Chien Wu (NCKU), Tsung-Fang Wu (NUK)

## 4.5 Scientific Computing

## 1. Core members

Faculty: Wei-Fang Hu (NCU), Chieh-Sen Huang (NSYSU), Ming-Chih Lai (NCTU), Min-Hsiung Lin (NCKU), Weichung Wang (NTU), Suh-Yuh Yang (NCU), (alphabetic order)

#### 2. Program Overview

In the field of conventional science and engineering, scientific computing is usually regarded as a narrow technical tool that can be applied independently in the traditional topics of the discipline. Drawing on mathematics and computer science, the Scientific Computing Program at NCTS has achieved several goals in terms of research, education, and reach-out in 2020. First of all, we focus on several research orientations as shown below.

- a. The effects of sorption kinetics on a surfactant-covered viscous drop in an electric field (led by Wei-Fang Hu): For this study, over a range of electric conductivity and permittivity ratios between the interior and exterior fluids, we focused on the dependence of deformation and flow on the transfer parameter J and Biot number Bi that characterize the extent of surfactant exchange between the drop surface and the bulk.
- b. Adaptive Runge-Kutta WENO scheme for advection-diffusion equations (led by Chieh-Sen Huang):

For this study, we defined a WENO-like adaptive Runge-Kutta method (i.e. temporal reconstructions) which blends the L-stable, third order, implicit Radau IIA method with the composite backward Euler method using a weighting procedure inspired from spatial WENO methods. The overall scheme is proven to maintain third-order accuracy when the solution is smooth.

c. Simulations of Newtonian vesicle in Oldroyd-B fluid under shear flow (led by Ming-Chih Lai):

For this study, we find that the stationary inclination angle can be negative without the transition to tumbling (TB) motion. Moreover, the inertia effect plays a significant role that is able to turn the vesicle back to a positive inclination angle through TT-TB-TT transition as the Reynolds number increases. Using the developed skew-adjoint property in our previous approach, we have successfully developed a linearly semiimplicit scheme for the inextensible interface with bending in unsteady Stokes flow and proved the scheme is unconditionally energy stable. d. Entanglement qualification among entangled bipartite systems (led by Min-Hsiung Lin):

In this research, we solved the problem of Gauging the distance between a mixed state and the convex set of separable states in a bipartite quantum mechanical system over the complex field. investigate the rank-1 approximation of a bipartite system over the real field where the entanglement happens. We show that this approximation can be recast as a nonlinear eigenvalue problem or a nonlinear singular value problem. Two iterative methods with rigorously convergence analysis are given correspondingly.

e. Artificial intelligence for medical image analysis and clinical workflows (led by Weichung Wang):

In this research, we intended to build a world-leading platform for Artificial Intelligence for Medical Image Analysis (AIMIA Platform). The platform consists of a high-performance Artificial Intelligent Engine (AI Engine) and innovative Augmented Intelligence Workflows (AI Workflows). To analyze MRI, CT, X-ray, PET, Pathological, Neuroimaging images, the AI Engine includes Image Processing, Quantitative Analytics, Deep Learning, Machine Learning, and High Dimensional Data Analysis Toolboxes.

f. Contrast enhancement for partially shaded low-light images (led by Suh-Yuh Yang):

In this research, we proposed a simple and efficient adaptive variational model for contrast enhancement for partially shaded low-light images. The key idea of this adaptive approach is to employ the maximum image of the color RGB channels as a classifier to divide the image domain into relatively bright and dim parts.

Furthermore, in the field of education, we organized several programs and events as shown below.

- a. NCTS Undergraduate Summer Research Program- Variational Models And Numerical Methods for Image Processing, which aimed to introduce some preliminaries for mathematical image processing.
- b. NCTS Student Workshop on Scientific Computing, which aimed to bring young researchers and graduate students in the field of scientific computing and related disciplines together to introduce and share their current works and findings. We hope to enhance our vision and broaden the horizon by joining this workshop.

- c. Winter Course- Parallel Finite Element Method using Supercomputer, which invited Professor Nakajima to discuss the Finite Element Method (FEM) as a numerical solution to the partial differential equations used to describe one- and three-dimensional steady-state heat conduction problems, and how it is widely used in modeling across various scientific and technological fields.
- d. Summer Course- Advanced Course On Multi-threaded Parallel Programming Using OpenMP For Multicore/Manycore Systems, which was targeted on 3D FVM code for Poisson's equation by ICCG Method (Conjugate Gradient (CG) iterative method with Incomplete Cholesky preconditioning), which is widely used in practical applications.
- e. Taiwan Mathematics School- Statistical Foundations of Medical AI, which covered some of the statistical foundations for medical AI. In particular, the topics include linear regression, logistic regression, ROC curve analysis, Poisson regression, and survival analysis.

Lastly, as for reach-out, while COVID-19 was raging in 2020, the international community tried to use AI technology to propose solutions to jointly fight the epidemic. Weichung Wang, as an active member of NCTS, has cooperated with NVIDIA and 20 hospitals around the world to conduct federal learning and train an AI model that is applicable to the world and has an accuracy of up to 94

#### 3. Research highlight

Many significant research highlights in the field of scientific computing have been delivered during the past year.

a. An immersed boundary projection method for simulating the inextensible vesicle dynamics, conducted by Kian Chuan Ong, and Ming-Chih Lai.

We develop an immersed boundary projection method (IBPM) based on an unconditionally energy stable scheme to simulate the vesicle dynamics in a viscous fluid. Utilizing the block LU decomposition of the algebraic system, a novel fractional step algorithm is introduced by decoupling all solution variables, including the fluid velocity, fluid pressure, and elastic tension. In contrast to previous works, the present method preserves both the fluid incompressibility and the interface inextensibility at a discrete level simultaneously.

- b. Spectrally accurate algorithm for points redistribution on closed curves, conducted by Yunchang Seol, and Ming-Chih Lai.
  We present a novel numerical method that redistributes unevenly given points on an evolving closed curve to satisfy equi-arclength (-like) conditions. Without substantial difficulty, it is also capable of remeshing or employing adaptive mesh refinement. The key idea is to find the discrete inverse of the arclength (-like) function in the framework of the Fourier spectral method to obtain overall spectral accuracy. Both equiarclength and curvature-dependent redistributions are extensively studied, and their spectral accuracy is verified by application to smoothly perturbed points on various curves. We further confirm that our method converges even for the points being perturbed nonsmoothly and randomly.
- c. Adaptive variational model for contrast enhancement of low-light images, conducted by Po-Wen Hsieh, Pei-Chiang Shao and Suh-Yuh Yang. The existence and uniqueness of the minimizer for the variational minimization problem are established in this study. The split Bregman method is used to accomplish an efficient numerical implementation of the adaptive variational model. Moreover, a number of numerical experiments and comparisons with other popular enhancement methods are conducted to demonstrate the high performance of the newly proposed method.
- d. A direct-forcing immersed boundary projection method for simulating the dynamics of freely falling solid bodies in an incompressible viscous fluid, conducted by Po-Wen Hsieh, Suh-Yuh Yang, and Cheng-Shu You. We develop a new direct-forcing immersed boundary approach combined with the Choi–Moin projection scheme for simulating the dynamics of freely falling solid bodies in an incompressible viscous fluid. The most advantageous feature of the proposed method is that it is conceptually simple and rather easy to implement without involving any discrete Dirac delta functions or post interpolations for accuracy like most immersed boundary methods in the literature.
- e. A third order, implicit, finite volume, adaptive Runge-Kutta WENO scheme for advection-diffusion equations, conducted by Todd Arbogast, Chieh-Sen Huang, Xikai Zhao, and Danielle N. King.
  In this research, an adaptive Runge–Kutta method is developed that blends the L-stable, third order, implicit Radau IIA method with the composite backward Euler method using a weighting procedure inspired by spatial WENO methods. The overall scheme is proven to

maintain third-order accuracy when the solution is smooth. Moreover, the resulting scheme is shown to be unconditionally L-stable for smooth solutions to the linear problem.

- f. Chun-Yueh Chiang. On the semigroup property for some structured iterations, conducted by Min-Hsiung Lin. Nonlinear matrix equations play a crucial role in science and engineering problems. However, solutions of nonlinear matrix equations cannot, in general, be given analytically. One standard way of solving nonlinear matrix equations is to apply the fixed-point iteration with usually only the linear convergence rate. To advance the existing methods, we exploit in this work one type of semigroup property and use this property to propose a technique for solving the equations with the speed of convergence of any desired order.
- g. Nonlinear power-like and SVD-like iterative schemes with applications to entangled bipartite rank-1 approximation, conducted by Min-Hsiung Lin.

Gauging the distance between a mixed state and the convex set of separable states in a bipartite quantum mechanical system over the complex field is an important but challenging task. This study offers insight into and might serve as the building block for the more complicated multipartite systems and higher-rank approximation problems.

h. Effects of surfactant solubility on the hydrodynamics of a viscous drop in a DC electric field, conducted by Herve Nganguia, Wei-Fan Hu, Ming-Chih Lai, and Y.-N. Young.

Over a range of electric conductivity and permittivity ratios between the interior and exterior fluids, we focus on the dependence of deformation and flow on the transfer parameter J, and Biot number Bi that characterizes the extent of surfactant exchange between the drop surface and the bulk. Our findings suggest solubility affects the electrohydrodynamics of a viscous drop in distinct ways as we identify parameter regions where (1) surfactant solubility alters both the drop deformation and circulation of fluid around a drop and (2) surfactant solubility affects mainly the flow and not the deformation.

Chaotic swimming of phoretic particles, conducted by Wei-Fan Hu, Te-Sheng Lin, Salima Rafai, and Chaouqi Misbah.
 The swimming of a rigid phoretic particle in an isotropic fluid is studied numerically as a function of the dimensionless solute emission rate (or Péclet number Pe). The particle sets into motion at a critical Pe.

Whereas the particle trajectory is straight at a small enough Pe, it is found that it loses its stability at a critical Pe in favor of a meandering motion. When Pe is increased further, the particle meanders at a short scale but its trajectory wraps into a circle at a larger scale. Increasing even further, Pe causes the swimmer to escape momentarily the circular trajectory in favor of the chaotic motion, which lasts for a certain time, before regaining a circular trajectory, and so on. The chaotic bursts become more and more frequent as Pe increases, until the trajectory becomes fully chaotic, via the intermittency scenario. The statistics of the trajectory are found to be of the run-and-tumble-like nature at a short enough time and of diffusive nature at a long time without any source of the noise.

j. Deep Learning to Distinguish Pancreatic Cancer Tissue From Non-cancerous Pancreatic Tissue: a Retrospective Study With Cross-racial External Validation, conducted by Kao-Lang Liu, Tinghui Wu, Po-Ting Chen, Yuhsiang M. Tsai, Holger Roth, Ming-Shiang Wu, Wei-Chih Liao\*, Weichung Wang.

The diagnostic performance of CT for pancreatic cancer is interpreterdependent, and approximately 40

k. Radiomic Analysis of Magnetic Resonance Imaging Predicts Brain Metastases Velocity and Clinical Outcome After Upfront Radiosurgery, conducted by Che-Yu Hsu, Furen Xiao, Kao-Lang Liu, Ting-Li Chen, Yueh-Chou Lee, and Weichung Wang.

Brain metastasis velocity (BMV) is a reliable measurement of the kinetics of distant brain failure (DBF) and predicts outcomes after initial DBF following upfront stereotactic radiosurgery (SRS). Based on pre-SRS information and magnetic resonance imaging, we developed a machine learning-based clinico-radiomic (CR) model to predict BMV and, in turn, clinical outcomes. This model may help customize optimal radiation treatment strategies and DBF monitoring frequencies for brain metastases patients in this era of precision medicine.

1. Particle Swarm Stepwise (PaSS) Algorithm for Information Criteriabased Variable Selections, conducted by Ray-Bing Chen, Chien-Chih Huang, and Weichung Wang.

A new stochastic search algorithm is proposed for solving informationcriterion-based variable selection problems. The idea behind the proposed algorithm is to search for the best model for the previously specified information criterion using multiple search particles. These particles simultaneously explore the candidate model space and communicate with each other to share search information. A new stochastic stepwise procedure is proposed to update the model during the search for the best model by adding or deleting variables. The proposed algorithm can also be used to generate variable selection ensembles efficiently.

m. An Efficient Contour Integral Based Eigensolver for 3D Dispersive Photonic, conducted by Tsung-Ming Huang, Weichien Liao, Wen-Wei Lin, and Weichung Wang.

An efficient contour integral (CI) based eigensolver is developed to overcome the difficulties of applying existing methods to solve eigenvalues in designated regions. This efficient method combines the contour integral, the fast matrix-vector multiplication, and an efficient linear system solver. The numerical results illustrate the efficiency of our algorithm.

## 4. Future Plan

In the future, we will continue the following plans and studies:

- a. NCTS/NCU Seminar on Computational and Data Sciences Computational and data science is an important interdisciplinary research field among statistical science, computer science, and mathematics. Specifically, compressive sensing is an optimization technique to efficiently acquire or reconstruct sparse information from few measurements. It has been applied successfully in signal/image processing, data analysis, medical imaging, machine learning, etc. We organize this regular seminar to pursuit possible applications of compressive sensing in the following areas: (i) sparse dictionary learning, (ii) image processing, and (iii) numerical methods for multi-scale partial differential equations.
- b. Conference on Advanced Topics and Auto Tuning in High-Performance Scientific Computing, 2022
   The conference encourages interdisciplinary communications between researchers from applied mathematics, statistics, computer science, physical sciences, engineering, and industry to prompt innovations and breakthroughs in this exciting field. The main themes include, but not limited to, simulations, numerical methods, applications, hardware, and particularly software and algorithm auto-tuning via statistical methods.
- c. NCTS Student Workshop on Scientific Computing Keep on bringing more young researchers and graduate students in the

field of scientific computing and related disciplines together to introduce and share their current works and findings. We hope to enhance our vision and broaden the horizon by joining this workshop.

- d. Research in computer science, mathematics, artificial intelligence for healthcare, statistical foundations, and its application for medical AI.
- e. Summer camps and courses for high school students about applied mathematics, medical AI, computer science, and other related fields.

## 4.6 Interdisciplinary Research

## 1. Core members

- a. Committee: Jein-Shan Chen (NTNU), Tzyy-Leng Horng (FCU), Chih-Hao Hsieh (NTU), Tai-Chia Lin (NTU), Je-Chiang Tsai (NTHU), and Fen-Bing Wang (CGU).
- b. Postdoc Fellow: Chun-Wei Chang (NCTS).

#### 2. Program Overview

The purpose of the interdisciplinary studies program at NCTS is to assist domestic mathematicians whose research expertises are the combination of mathematics and the other academic disciplines to develop scientific topics, which in turn stimulate the birth of new mathematical theory or problems. Our research topics are motivated by various scientific processes and phenomena. In the past year, our members focus on the following areas:

- a. Optimization and big data: Jein-Shan Chen.
- b. Mathematical modeling and analysis of ion channels: Tzyy-Leng Horng and Tai-Chia Lin.
- c. Time series analysis for nonlinear dynamical systems: Chih-Hao Hsieh & Chun-Wei Chang.
- d. Complex systems: Je-Chiang Tsai.
- e. Mathematical ecology: Feng-Bin Wang.
- 3. Research highlight

a. Optimization and big data, by J.-S. Chen.

The  $l_1$ -norm regularized minimization problem is a non-differentiable problem and has a wide range of applications in the field of compressive sensing. We adopt six smoothing functions to approximate the  $l_1$ -norm. Then, we recast the signal recovery problem as a smoothing penalized least squares optimization problem, and apply the nonlinear conjugate gradient method to solve the smoothing model. The algorithm is shown globally convergent. In addition, the simulation results not only suggest some nice smoothing functions, but also show that the proposed algorithm is competitive in view of relative error.

#### References

- 1. Signal reconstruction by conjugate gradient algorithm based on smoothing *l*<sub>1</sub>-norm, Calcolo, **56** (2019), 42.
- b. Mathematical modeling and analysis of ion channels, *by T.-L. Horng and T.-C. Lin.*

1. We developed a one-dimensional model for electric potential generation of electrocytes in electric eels. The model is based on the Poisson-Nernst-Planck system for ion transport coupled with membrane fluxes including the Hodgkin-Huxley type. Using asymptotic analysis, we derived a simplified zero-dimensional model, which is denoted as the membrane model, as a leading order approximation. Our analysis provides justification for the assumption in membrane models that electric potential is constant in the intracellular space. This is essential to explain the superposition of two membrane potentials that leads to a significant transcellular potential [1].

2. The classical Poisson-Nernst-Planck (PNP) theory as well as its various modifications have been widely studied in describing the dynamics of mobile charges in electrolytes. The PNP theory assumes the physical system to be isothermal and described using a constant temperature, which might work well for those systems with negligible temperature variations. However, the electric current is usually accompanied with the Joule heating effect, resulting in considerable temperature changes. Moreover, the thermoelectric effects indicate there exists direct conversion between the temperature gradient and the voltage gradient. The coupling between the temperature evolution and the electric currents is very important in understanding the dynamic properties of the electrolyte systems and has attracted extensive interests in recent years. We use the energetic variational approach to

derive a non-isothermal electrokinetic model. The charge transport is described through the Poisson-Nernst-Planck equations with variable temperature, and the heat flux satisfies the Fourier's law. This Poisson-Nernst-Planck-Fourier model satisfies both the first law and second law of thermodynamics as well as the Onsager's reciprocal relations, thus it is thermodynamic-consistent. Finally, we prove the global wellposedness for this model under the smallness assumption of the initial data by the energy method [2].

## References

- 1. X. Cao, Z. Song, T.-L. Horng, H. Huang, *Electric potential generation of electrocytes: modelling, analysis, and computation,* Journal of Theoretical Biology, **487** (2020), 110107.
- C.-Y. Hsieh, T.-C. Lin, C. Liu and P. Liu, *Global existence of the non-isothermal Poisson-Nernst-Planck-Fourier system*, J. of Differential Equations, 269 (2020), 7287-7301.
- c. Time series analysis for nonlinear dynamical systems, *C.-H. Hsieh and C.-W. Chang.*

1. Causal effects of population dynamics and environmental changes on spatial variability of marine fishes.

Populations with homogeneous distributions have better bet-hedging capacity than more heterogeneously distributed populations [1]. Both population dynamics and environmental factors may influence the spatial variability of a population, but clear empirical evidence of such causal linkages is sparse. Using 25-year fish survey data from the North Sea, we quantify causal effects of age structure, abundance, and environment on nine fish species [2]. We use empirical dynamic modelingan approach based on state-space reconstruction rather than correlationto demonstrate causal effects of those factors on population spatial variability [3]. The causal effects are detected in most study species, though direction and strength vary. Specifically, truncated age structure elevates population spatial variability. Warming and spatially heterogeneous temperatures may enhance population spatial variability, whereas abundance and large-scale environmental effects are inconclusive. Fishing may affect population spatial variability directly or indirectly by altering age structure or abundance. We infer potential harmful effects of fishing and environmental changes on fish population stability, highlighting the importance of considering spatial dynamics in fisheries management.

2. Long-term warming destabilizes aquatic ecosystems through weakening biodiversity-mediated causal networks

Understanding how ecosystems will respond to climate changes requires unravelling the network of functional responses and feedbacks among biodiversity, physicochemical environments, and productivity. In this project, we argue that to understand how ecosystems respond to climate change it is necessary to take a larger network view; a view that includes interactions and feedbacks between biodiversity, productivity, and physicochemical factors [4]. We address this problem by analyzing long-term time series (16-39 years) data from ten aquatic ecosystems located worldwide. We applied a novel approach, convergent cross-mapping (CCM [5]), to reconstruct the causal networks quantifying the interactions between system-level properties, including phytoplankton diversity, biomass, and physicochemical factors. We determined that individual quantities (e.g., total species richness or nutrients) were not significant predictors of ecosystem stability (quantified as long-term fluctuation of phytoplankton biomass); rather, the integrated causal pathway in the ecosystem network, composed of the interactions among species richness, nutrient cycling, and phytoplankton biomass, was the best predictor of stability. Interestingly, this causal pathway is weakened for ecosystems experiencing intensive warming; a feature that was found in ten monitoring systems, and in a larger global-scale ocean dataset. Thus, there is compelling evidence for a significant negative relationship between climate warming and ecosystem stability in the global oceans. The study calls for a more holistic network view to understand and predict climate impacts on ecosystem stability.

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- d. Complex network systems, by J.-C. Tsai.

1. Spreading phenoma in the Neolithic transition in Europe.

The Neolithic transition began the spread of early agriculture throughout Europe through interactions between farmers and hunter-gatherers about 10,000 years ago. Archeological evidence produced by radiocarbon dating indicates that the expanding velocity of farming is roughly constant all over Europe. However, the expanding velocity determined by existing modeling approaches is faster than the observed velocity [1]. For understanding this difference, we propose a three-component reaction-diffusion system which consists of two different types of farmers (sedentary and migratory) and hunter-gatherers from the viewpoint of the influence of farming technology. Our purpose is to study the relation between the expanding velocity of farmers and the farming technology parameter. The propagating velocity derived from our model can be compatible with the observed velocity when farming technology is developed. Our results suggest that the reason for the slowdown of the Neolithic transition might be related to the increase in the development of farming technology. Moreover, the bifurcation structure of waves in our proposed model was investigated [2]. The results indicate that there is rich structure of traveling waves in the model. Finally, we also compute the spreading speed of the model when the farmer population is initially with localized support and the hunter-gatherer is uniformly distributed [3].

2. Structural bifurcation analysis in chemical reaction networks.

In living cells, chemical reactions form complex networks, and thus give a very large ODE system. Dynamics arising from such networks are the origins of biological functions. Also the crucial processes are easily obscured by numerical approaches due to the complexity of the underlying networks. We propose an analytical approach, not numerical one, to analyze bifurcation behaviors of network systems using their structures alone [4]. This approach gives an efficient analytical way to determine the bifurcation point, and a satisfactory understanding of the essential mechanism underlying the biological process. In particular, it might be employed to predict the unknown pathways in the chemical networks or biological processes. In fact, the framework of this approach has been applied to our recent experimental work of the E2F6 regulation on miR-193a in the ovarian carcinogenesis [5].

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## e. Mathematical ecology, by F.-B. Wang.

1. The ecological models with variable quotas in a spatially variable habitat are important and significant. However, such topics have received very little attention, perhaps due to the complexities and difficulties in modeling as well as mathematical analysis. One more tractable approach is the reaction?diffusion system or the reaction-diffusion-advection system, which describes the dynamics of dissolved nutrient concentration, the total concentration of stored nutrient by a species at a given point, and the corresponding population density. We point out that the main difficulties in mathematical analysis for the model systems are caused by the singularity in the ratio at the extinction steady state. Thus, standard techniques such as linearization and bifurcation are not applicable. Intraguild predation is added to a mathematical model of competition between two species for a single nutrient with internal storage in the unstirred chemostat. Existence for positive steady state solutions of intraguild prey and intraguild predator is established in terms of the principal eigenvalues of associated nonlinear eigenvalue problems by means of the degree theory in a special cone [1].

2. West Nile virus (WNv) is a mosquito-borne disease caused by flavivirus. To investigate the combined effects of vertical transmission, temperature-dependent incubation periods and seasonality on the transmission of WNv, we develop a delay differential system with stagestructure and time-varying delays. We then derive the mosquito reproduction number  $R_0^V$  and basic reproduction number  $R_0$ , and show that these two numbers serve as threshold parameters that determine whether WNv will spread.As an application, we conduct a case study for the WNv transmission in Los Angeles County, California. We also carry out numerical simulations to identify the situations that require time-periodic delays. Moreover, we find that rising temperatures may potentially increase the risk of disease outbreaks [2].

3. There has been a dramatic increase in the number of countries with reported dengue outbreaks during the past years. Therefore, dengue fever can be regarded as one of the most rapidly spreading diseases in the world, and it is natural to incorporate the spatial variations into the model system because of its remarkably growing spatial spread. On the other hand, seasonal or daily fluctuations in temperature also have a significant influence on the maturation rates of the aquatic population and biting rate of mature female mosquitoes. Our study is devoted to the investigation of dengue spread via a time-space periodic reaction-advection-diffusion model. We establish the existence of the spreading speeds and its coincidence with the minimal speed of almost pulsating waves [3].

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#### 4. Highlights of Events

We run this program through regular seminars and summer courses, as well as workshops and conferences. We briefly describe them below.

#### Seminar

We organize regular seminars to facilitate research discussions and help students broaden their horizons. Seminars on optimization are organized by J.-S. Chen at Taipei. Seminars on analysis and numerical computation of PDE are run by T.-L. Horng and T.-C. Lin a at Taipei. Seminars on applied mathematics analysis are run by C.-W.Chang and J.-C. Tsai at Hsinchu. Seminars on mathematical biology are organized by J.-C. Tsai and F.-B. Wang at Hsinchu.

#### Workshops and conferences

During this past year, we have held some academic activities at NCTS. In particular, the two-day conference on Oct., NCTS Workshop on Mathematical Modelling and Analysis, is a meeting forum for scientific researchers to exchange their ideas from various disciplines such as physics, chemistry, and biology, and to initiate innovative research fields within mathematics itself. Also after this conference, the primary speaker, Prof. Masayasu Mimura, gave several seminar lectures on the development of reactiondiffusion systems and commented some open problems and research directions. The following is the list of workshops and conferences we hosted during the past year.

- a. NCTS Workshop on Mathematical Modelling and Analysis, Oct. 17-18, 2019. Organizers: C.-C. Chen, M.-C. Lai, and J.-C. Tsai.
- b. International Conference on Biomathematics and Biostatistics, Oct. 24-25, 2019. Organizers: J.-S. Guo, C.-T. Lin, and T.-H. Yang.
- c. 2019 Winter Workshop on Optimization, Dec. 12-13, 2019. Organizers: G.-X. Lin, J.-Y. Lin, H.-Q. Nguyen, and R.-L. Sheu.
- d. Workshop on analysis and its applications in biology and physiology, Jan. 2-4, 2020. Organizers: T.-L. Horng, M.-C. Lai, T.-C. Lin, C. Liu.
- e. Mini-workshop on Membrane Protein: Its Biology and Mathematical Modeling, Jan. 10, 2020. Organizers: T.-L. Horng and T.-C. Lin.

#### **Courses and lectures**

To cultivate students, we organize lectures and tutorial courses. One highlight is to have a course on "Dynamics in Network Systems", which

is lectured by mathematicians as well as ecologists and chemist. The diversity of speakers is baed on the fact that physical mechanisms/biological functions arise from the dynamics within complex networks arising from physical systems, chemical reactions and biological process. In this course, we provide some new approaches for the study of network systems, and give the application of these theories for problems from ecosystem, the central carbon metabolism of the E. coli and system biology. In addition, we introduce some useful techniques developed in discrete dynamical systems that can be used to analyze the topological behavior of the derived network system. The following is the list of lectures and courses for the past year.

- a. 2019 NCTS Summer Course on Mathematical Biology, Jul. 29 Aug. 1, 2019. Organizers: J.-C. Tsai and F.-B. Wang.
- b. Taiwan Mathematics School: Dynamics in Network Systems, Mar. 3-Jun. 30, 2020. Organizers: J.-C. Ban and J.-C. Tsai.

## 5. Future Plan

In order to broaden and deepen our study on current projects, we plan to invite our long-term collaborators such as

- a. Luigi Catacuzzeno (University of Perugia, Italy).
- b. Robert S. Eisenberg (Rush University, USA).
- c. Nir Gavish (Technion-Israel Institute of Technology, Irael).
- d. Quentin Griette (University of Bordeaux, France).
- e. Chun Liu (Illinois Institute of Technology, USA).
- f. Atsushi Mochizuki (Kyoto university, Japan).
- g. Ronen Talmon (Technion, Isreal).
- h. Uwe Thiele (University of Munster, Germany).
- i. Yuan-Nan Young (New Jersey Institute of Technology, USA).

and so on. During their visit, we will hold workshops or conferences to provide a platform for the research discussion between foreign visitors and local researchers. In particular, Prof. Hau-Tieng Wu will organize workshops which facilitated the discussion between physicians, mathematicians, and statisticans. We look forward to having more research cooperations between mathematical and scientific communities. The following is the list of the proposed workshop or conference.

#### Workshops and conferences

- a. Meet physicians, mathematicians and statisticans, Aug. 2020. Organizers: Hau-Tieng Wu.
- b. Meet physicians, mathematicians and statisticans, Mar., 2021. Organizers: Hau-Tieng Wu.
- c. NCTS Optimization Day for Young Researchers, Dec., 2020. Organizers: J.-S. Chen.
- d. NCTS Symposium on Applied Mathematics, Jun., 2021. Organizer: C.-C. Chu, T.-S. Lin, and J.-C. Tsai.

## **Courses and lectures**

In the upcoming year, there will be two courses. The first one is the stochastic process and it application to cellular functions. The other one is the dynamical systems and its application to mathematical ecology. The below is the list of courses in the coming academic year.

- a. NCTS Winter Course on stochastic process and it application to cellular functions, Feb., 2021.
- b. NCTS Summer Course on Mathematical Biology, Aug., 2021.

#### Seminar

We continue to hold regular seminars to facilitate research discussions and help students broaden their horizons. Seminars on optimization are organized by J.-S. Chen at Taipei. Seminars on applied mathematics focus on Poisson-Nernst-Planck (PNP) model and its variants. Seminars on mathematical biology will focus on model-free approaches based on the embedding theorem by Floris Takens.

## 4.7 Laboratory of Data Science

## 1. Core members

- a. Faculties: Meihui Guo (NSYSU); Ray-Bing Chen (NCKU); Chieh-Sen Huang (NSYSU); Chih-Wei Chen (NSYSU); ShihFeng Huang (NUK); Liang-Ching Lin (NCKU); Hsiang-Ling Hsu (NUK), I-Chen Lee (NCKU).
- b. Post-doctor: Ping -Yang Chen (NCKU)
- c. Ph.D. students: Cheng Han Chua Chua (NSYSU); Giap van Su (NSYSU); Saga Li (NCKU); Wei-Ting Lai (NCKU.)
- c. Master students and Research Assistants: Hsin Kai Wang (NSYSU); Zi Xuan Wang (NSYSU); I-Hsiao Chen (NSYSU); Hsuan-Tung Wang (NCKU); Yi-Xuan Wang (NCKU); Quan-En Li (NCKU)

#### 2. Program Overview

Data science is an inter-disciplinary field that uses techniques and theories drawn from many fields including mathematics, statistics, computer science, and information science to extract knowledge and insights from structural and unstructured data. Fueled by big data and AI, demand for data science skills is growing exponentially. However, the supply of skilled persons is growing at a slower pace. For this topical program, we aim in promoting three important research topics in Data Science and cultivation of young researchers in data science. We provide a platform for research exchanges and discussions and help domestic young researchers to to build international cooperations. Some of the visiting scholars become co-advise of the PhD students and provide opportunity for the students to visit their institute to broaden their horizons.

#### 3. Research highlight

Our program includes several research groups that address specific issues, but not limited to, such as experiment design, high frequency financial data, scientific computation, biostatistics and etc. In the past year we focus on the following research topics:

- a. Novel Experimental Design and Uncertainty Quantification.
- b. Bayesian Computing and its Applications on Time-Spatial Complex Data Analysis.
- c. Financial and Complex Data Analysis.

Details are given below.

a. Novel Experimental Design and Uncertainty Quantification While modelbased experiments and simulations are extremely successful in a wide range of applications, the successes also suggest the increasing importance of high-consequence prediction and analysis tools. This is because all kinds of variables do involve a certain uncertainty which provide flexibilities and could play a key role in experiments. The needs to develop prediction and analysis tools for dealing with uncertainties lead to the emergence of uncertainty quantification (UQ) naturally. Here we consider the statistical modeling for UQ and use this surrogate model to capture the prediction uncertainty. Currently the problem structure is more and more complex. For example, consider the design problem for the cylindrical pinheatsink. After getting the responses based on Computational Fluid Dynamics (CFD) simulations, our goal is to construct a surrogate model for these different types of variables. There are one continuous variable, two integer variables and two categorical variables. The current surrogate modeling approaches are not proper and would have the numerical problems, like the inverse matrix problems, especially when we consider more and more qualitative variables. In addition to the surrogate modeling for qualitative and qualitative variables, we also consider the multi-fidelity responses. How to properly integrate the effects of qualitative and qualitative variables with multi-fidelity responses should be a challenge problem. In UQ, the sequential design approach is commonly used because we would identify the next explored points based on the current surrogate model. Due to the different targets, the sequential selection principles and criteria are different. For example, when we consider the global optimization problem, the expected improvement criterion is a widely used one. How to adjust the criteria with respect to the different scenarios should be a key problem in the UQ procedures. To identify the next explored points in UQ can be transferred as an optimization problem. Under some conditions, a gradient based approach is used to identify the points. In addition, metaheuristic optimization approaches have been used for searching the best experimental designs for a pre-specified design criterion. For example, we did modify the particle swarm optimization (PSO) to find the best space-filling designs and proposed modified PSO to deal with the optimal designs for the mixture experiments. Due to bigger and bigger data size, optimal design problems are increasingly high dimensional with many covariates. This means that there is a need to construct optimal designs for models with multiple covariates plus interactions. Thus we would target on the extremely high dimensional optimization problems based on the

big data design issues and want to propose the efficient metaheuristic algorithm to solve them. Finally one of the interesting applications is the optimum test planning for an accelerated life test (ALT). In ALTs, the model is non-linear and it involves the numerical integration for the corresponding criterion. Thus to identify the best design, more computational cost is necessary. In this part, it would be more challenge to have an efficient optimization tool to solve the corresponding problems.

b. Bayesian Computing and its Applications on Time-Spatial Complex Data Analysis

Data obtained from the space and time aspects are ubiquitous and increasingly collected in various domains such as neuroscience, epidemiology, climate science, social sciences and so on. For example, in neuroimaging data, BOLD signals measured from the human brain is stored along with the spatial location from which the activity was measured and the time at which the measurement was made. In addition, in epidemiology data, electronic health record data that is widely stored in hospitals provide demographic information pertaining to patients as well diagnosis made on patients at different time points. The presence of spatial and temporal attributes introduces additional challenges to transitional statistical tools so that a new type of tool for computational inference that dramatically reduces the dimension and size of the raw data while capturing its essential aspects is needed. Here we consider a data mining approach using spatio-temproal clustering to group objects based on the spatial and temporal similarity. In this process, we make use of the geographic information sciences in terms of the pervasiveness of all kinds of location-based or environmental devices that record position, time or/and environmental properties of an object or set of objects in real-time. One important application is the data analysis for Functional Magnetic Resonance Imaging (fMRI). fMRI is used to measure the neural activity in millions of locations in every two seconds so it produces the spatial and temporal resolution data sets to allow researchers to investigate brain activity resulting from a stimulus by observing the blood oxygenation level dependent signals. For example, by applying Bayesian spatio-temporal model to fMRI analysis for brain activities related to the eye blinking, we can demonstrate that activity in multiple brain areas is stimulated by eye blinking which may suggest that eye state has a significant effect on multiple brain networks. Here we are interested to propose a spatial-temporal generalized mixed-effects time-varying coefficients model to analyze this kind

of data. The proposed model can be used to study the spatial patterns, locally, remotely and dynamically, for the spatial and temporal data. For example, in brain imaging analysis, the proposed model should be able to study the brain activity simulated by a test and investigate the brain connectivity through the spatial model. Moreover, we consider a perspective analysis for joint analysis of brain imaging and genetics data.

## c. Financial and Complex Data Analysis

The recent developments in Data Science provide many interesting and potentially useful directions to analyze financial and complex data in a more innovative and effective way. In finance, investors are now able to obtain real-time data including the latest price, trading volume, first several (5 or 10) bid-ask quoted prices with the associated sizes, daily high, low and percentage change for each stock in a major world stock market from many websites. Investors can also learn the latest news and articles about current events on global stock markets from social media and social networks (twitter, facebook, LinkedIn, instagram, YouTube, Google+, etc.). These development trends did improve market transparency and have changed styles of trading and decision making based on the rapidly obtained data. Therefore, how to identify useful information from the real-time massive data and how to integrate different types of information (quantify and qualify) effective are important topics and need more extensive and deep exploration. This part of our project is devoted to developing doable recipes for these directions. In this part, we focus on overcoming the aforementioned challenges when facing multiple, high-frequency, and different types of financial time series data. Meanwhile, the applications on market trend prediction, portfolio selection, investment trading strategies and risk management will also be carefully investigated with empirical data. Several recently developed approaches will be considered. One direction is to employ functional time series to properly depict the dynamics of multiple processes as well as achieve the goal of dimension reduction with acceptable information loss. In this direction, we expect to face the issues of basis selection, knots determination, and multi-resolution effects. The team members of this project have some experience and ideas on handling these issues but more deep explorations are needed. Another direction is to modelling the massive and complicated relationships among different types time series by network models. Timevarying mixed graphical model is one possible way to handle different types (quantify and qualify) of time series data. One can also combine the techniques of filtering out the autocorrelation and GARCH effects inherient in single time series at first, and adopting the mixed graphical model to model the joint effects among the filtered processes. The team members have conducted some preliminary studies and found its high potential in capturing important network effects for trend prediction and the associated applications. The 3rd direction is to include the information contained in the latest news and articles into the classical time series framework. To achieve this objective, text mining is a key technique to draw text information about our target of interest. Nevertheless, how to draw the text information effectively and strengthen its utility for our target of interest still need more extensive investigation. The 4th direction is to propose a new trading strategy by using daily closing prices as well as the daily opening, high, and low prices. The conventional trading strategies are usually constructed based on only daily closing prices. In this project, the team members will develop the new methodologies to obtain the more accurate estimators of the model parameters and the corresponding portfolio weights by using almost all information of daily opening, high, low, and closing prices. Besides, many trading strategies have some tuning parameters, the Bayesian method will be conducted to have better choices for them. Furthermore, by using the predicting high and low prices, an early existing mechanism can be constructed to prevent the unpredictable losses. For instance, if the traders hold a long position and the intra-daily stock price trigger the estimated high price, the traders are suggested to sell this portfolio immediately by using this early existing mechanism and vice versa. The team members have develop new models for characterizing the daily high and low prices, obtain the corresponding model parameters, and investigate the distributions of the tuning parameters from some trading strategies.

#### 4. Highlights of Events

Due to the ongoing pandemic, we can only host video activities. Details are introduced below.

 Workshops and seminars We organized three workshops in this year. Several internationally renowned scholars were invited as keynote speakers of the workshops. The topics of these workshops and talks are related to the Data Science; Machine Learning; Statistics and so on.

- (a) 2021 YCU-NCKU Student Workshop on Data Science, 2021-03-18: This was a joint workshop among NCKU and Yokohama City University. The graduate students from both sites joint the workshop and shared their current researches.
- (a) Prof. Dennis Li (Purdue University): If Deming Were Born Today - Quality, Statistics in Modern Data Science, 2021-04-09.
- (b) Prof. Samuel Kou (Harvard University): Statistical Inference of dynamic systems via manifold-constrained Gaussian processes, 2021-06-04.
- (c) Prof. Regina Liu (Rutgers University): A comprehensive framework for prediction, 2021-06-10.
- 2. Academic research exchange and PhD student advising We encourage domestic young researchers to attend international conferences and have short term visits and assist them to apply financial support from NCTS. Our visitors not only target on academic research exchange but also serve as the co-advisor of our PhD students. Prof. Weng Kee Wong has visited NCKU several times. In addition to work with Prof. R.-B. Chen on developing efficient numerical algorithms for the design generators, he was also the co-advisor for Dr. Ping-Yang Chen and offered him an opportunity to be UCLA as an exchange student. Following the same idea, the visitor, Prof. Chih-Li Sung did agree to advise one PhD student, Mr. Saga Li, in NCKU. In addition, a PhD student, Ms. Wei-Ting Lai, is preparing her visiting to National University of Singapore under the supervision of Prof. Ying Chen.
- 3. Build international research cooperations The followings are the ongoing international research cooperations.
  - (1) The joint work on numerical design generators: Prof. Weng Kee Wong (UCLA) and Prof. R.-B. Chen have several joint projects related to how to efficiently generate optimal designs with respect to the different models and criteria. In the last year, they had a paper published in Plos One. Currently they do have several joint projects on their hands.
  - (2) New Surrogate Model with Complex Variable Structures for Uncertainty Quantifications: Prof. Chih-Li Sung (Michigan State University) was invited to visit NCKU from 2019.05.20-2019.06.30. Prof. R.-B. Chen; the PhD student, Saga Lin and Prof. Sung

started a new joint project on statistical modeling in the analysis approach for Uncertainty Quantifications (or Computer Experiment in Statistics). This project is motivated from a simulator of the cooling system. In this system, there are many categorical variables with few continuous variables. The current modeling approach cannot deal with too many categorical variables. Thus we want to propose a new modeling approach by integrating the tree structure with Gaussian process. In this year, Dr. Ping-Yang Chen and Prof. Ying Hung (Rutgers University) also joint this project and there is a new project for the modeling problems for the multiple fidelity response and branching factors.

# 5 Host institution commitment

The commitment of Host institution consists of the following aspects: budget, space, and logistic support. From the point of view of budget, the Host institution: National Taiwan University completely fulfill its commitment as we described in the chapter of budgets. Below we will focus on describing the space and other logistic supports.

#### a. Space of NCTS

Thanks to the generous support of the host institution National Taiwan University and the effort of many supporting staffs, we moved into the newly built Cosmology Building in the summer of 2020. According to the original commitment, there are 200 ping free of charge for NCTS Math to use. The rent of the extra space is also reduced for NCTS Math. In total, we use the whole 5F of the building as the common area, administrative area, lecture rooms, visitor's offices and discussion areas. Also we have some spaces in the 4F which are designed as the offices for post-docs and research assistants.

The previous space in Astro-Math. Building provided by the Department of Mathematics has been returned to the Department.

b. Logistic support of NTU

We believe that the logistic support of NTU during the past couple years has become smoother. At the beginning, we spent considerable effort to explain the special mission of NCTS and to seek for possible solution for some unexpected difficulties. After a couple years of trialand-error, most of the unexpected difficulties have be solved.

We started the proposal to build up "Taiwan Mathemtics School". The idea was greatly supported by NTU. It is now possible for the NCTS to offer courses with credits. It is also possible for students in other cooperative universities or departments to take courses of NCTS with credits. The list of cooperative universities consists of: National Taiwan Normal University, National Taiwan University of Technology, and National Taipei University of Education. The list of cooperative departments consists of the Departments of (Applied) Mathematics of the following universities: National Cheng-Chi University, National Central University, National Tsing-Hua University, National Chiao-Tung University, National Cheng University, National Cheng Kung University, National Sun Yat-sen University, National Dong-Hwa University, and more. Students from other universities or department not on the above list might be able to receive credits for NCTS courses depending on the regulations of his/her own institution.
#### 申請機構配合事項同意書

計畫名稱:國家理論科學研究中心第四階段運作計畫(2015.1.1-2020.12.31) 計畫主持人姓名/職稱:陳榮凱/教授

申請機構配合措施:本計畫業經單位內部審查,同意提供下列配合事項。

- 一、配合款:本機構同意提供2000萬之配合款,於執行期間優先使用於計畫所需各項經費(含中心人員薪資、學術活動費用、使用空間的場租、軟硬體設備、裝修維護費、水電雜支等等)。本計畫執行期滿後,收支報告表內需詳細註明配合款支用情形。
- 二、員額:提供\_5名供中心延聘中心主任、中心講座、特約中心科學家。
- 三、管理費:依本校「建教合作計畫管理費分配處理細則」辦理,以科技部計畫15% 管理費計,分配至計畫主持人所屬學院2%,分配至校級中心之管理費約30%。
- 四、結餘款:依本校「建教合作計劃結餘款分配、運用及管理要點」辦理,當年度 結餘款總額扣除個別使用款項後之餘額,校級中心以分配50%為原則。

五、中心空間:

- 1. 現況:
- a. 專屬空間共約\_361\_坪:

天文數學館	二樓 (約183坪)	中心行政區,小型研討室(30人)及大型研討室(120人) 各1間,4間訪問學者辦公室與交誼區;走道公共空間 設有沙發及茶水區,為公告區及休息討論區。
	四樓	5間訪問學者辨公室、1間視訊會議室、2間討論室和
	(約76坪)	1間辦公室(供研究助理使用)。
數學研究中心	二樓 (約 66 坪)	5 間辦公室(供博士後研究員使用)及休息區。
	三樓。 (約36坪)	6間訪問學者辦公室。

b. 共同使用空間共約\_255\_坪:

天文數學館	一樓 (約180坪)	3 間中小型教室(80 人、80 人、20 人)和1 個國際會議 廳(198 人)。
	九樓 (約75坪)	接待、交誼、會議或相關學術活動使用空間。

#### 2. 宇宙學大樓於2016年落成之後:

宇宙學大樓	4 樓(200 坪) 5 樓(200 坪) (2 層樓 400 坪為數學組與 物理組共用)	辦公室、小型會議室與研究室。
	其他樓層 (數學組另行租借 200 坪)	辦公室、小型會議室與研究室。
宇宙學大樓 數學研究中心	二樓	5 間辦公室(供博士後研究員使用)及休
	(約66坪)	急區。
数字研充十心	三樓	C. 明
宇宙學大樓 數學研究中心	(約36坪)	0间动向字有辨公至。

a. 專屬空間共約\_502\_坪:

b. 共同使用空間共約\_150\_坪:

宇宙學大樓	一樓 (150 坪)	大型演講廳(130人)。
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- 六、其它相關配合措施:請詳細說明所提供之各項設備、學人宿舍、裝修維護費、 水電雜支、停車、行政支援...等。
  - 中心主任與執行主任因推動中心業務,同意減免教學課程,依本校「教師核 減授課時數」規定辦理。
  - 校方提供至少10名博士後研究員名額,以招募優秀年青研究人員。(薪資含 配合款中)
  - 3. 提供客座學人宿舍給國內外長期訪問學者。
  - 4. 國內外學生至中心訪問期間的住宿得申請本校學生宿舍。
  - 5. 提供一個月(含)以上中心訪客使用學校體育健身設施收費優惠。



# 6 Appendix

Throughout the Appendix, the following abbreviations are used for the Academic Programs:

- A=Number Theory and Representation Theory
- B=Algebraic Geometry
- C=Differential Geometry and Geometric Analysis
- D=Differential Equation and Stochastic Analysis
- E=Scientific Computing
- F=Interdisciplinary Research
- H=Harmonic Analysis
- L=Laboratory of Data Science

# 6.1 Workshops, Seminars and Courses

## Workshops and Conferences

Workshop Title	Dates Venues		Organizers	Group	
L-values and Iwasawa theory	2020-11-11 / 2020-11-12	Online	Burungale Ashay (Caltech) Hsieh Ming-Lun (AS) Palvannan Bharathwaj (NCTS) Ramdorai Sujatha (UBC)	A	
Quantum Information Science (QIS) and Math	2020-10-17 / 2020-10-17	NCTS	Hsieh Ming-Lun (AS) Lee Yng-Ing (NTU) Lee RayKuang(NTHU)	А	
The Taiwan Number Theory Symposium	2020-09-09 / 2020-09-11	NTHU	Hsieh Ming-Lun (AS)	А	
One day combinatorics workshop	2020-07-21 / 2020-07-21	NTU	Yu Wei-Hsuan(NCU)	А	
NCTS Workshop on K-stability	2020-11-12 / 2020-11-13	Fo Guang Uni.	Chen Jungkai(NTU & NCTS)	В	
One-day Workshop in Algebraic Geometry	2020-10-16 / 2020-10-16	NCKU	Chen Jungkai(NTU & NCTS) Lai Ching-Jui (NCKU)	В	
NCTS Summer Workshop on Foliation	2020-08-24 / 2020-08-26	Xitou	Chen Jungkai(NTU & NCTS) Lin Hsueh-Yung ( Kavli IPMU.)	В	
The 20th Taiwan Geometry Symposium	2020-11-07 / 2020-11-08	NCTS	Chiang River (NCKU) Ho Nan-Kuo (NTHU) Tsai Chung-Jun (NTU) Tsui Mao-Pei (NTU)	С	
2020 NCTS South Workshop on Dynamical Systems	2020-12-18 / 2020-12-19	NPTU	Ban Jung-Chao (NCCU) Chang Chih-Hung (NUK) Chen Kuo-Chang (NTHU) Cheng Chang-yuan(NPU) Hsu Cheng-Hsiung(NCU)	D	
2020 NCTS Optimization Day for Young Researchers	2020-12-14 / 2020-12-14	NTNU	Chen Jein-Shan (NTNU)	D	
ReaDiNet 2020: An online conference on mathematical biology	2020-10-19 / 2020-10-23	Online	Giletti Thomas (Uni. of Lorraine) Guo Jong-Shenq (Tamkang Uni.) Kim Yong-Jung (KAIST ) Matano Hiroshi (Meiji Uni.)	D	
2020 NCTS PDE Young Scholar Workshop	2020-08-29 / 2020-08-31	NTTU	Chang Nai-Heng (NTTU) Chen I-Kun (NTU) Wu Kung-Chien (NCKU & NCTS) Hsia Chun-Hsiung (NTU)	D	
Second NCTS PDE Symposium	2020-08-27 / 2020-08-27	NCKU	Chen I-Kun (NTU) Hsia Chun-Hsiung (NTU) Wu Kung-Chien (NCKU & NCTS)	D	
First NCTS PDE Symposium	2020-02-13 / 2020-02-13	NCTS	Hsia Chun-Hsiung (NTU) Wu Kung-Chien (NCKU & NCTS)	D	
Distinguished Lecture by Ning Ning Yu	2020/10/26	NCTS	Chen Jungkai(NTU & NCTS) Tsui Mao-Pei (NTU) Wu Chun-Chieh(NTU)	Е	
2020 Workshop for Young Scholars on Applied Math	2020-09-03 / 2020-09-03	NCTU	Huang Tsung-Ming (NTNU) Lai Ming-Chih (NCTU) Lin Wen-Wei (NCTU) Yang Suh-Yuh (NCU)	Е	

Mini-workshop on Membrane Protein: Its Biology and Mathematical Modeling	2020-01-10 / 2020-01-10	Feng Chia Uni.	Horng Tzyy-Leng (Feng Chia Uni.) Lin Tai-Chia (NTU)	F
Workshop on analysis and its applications in biology and physiology	2020-01-02 / 2020-01-04	NCTS	Horng Tzyy-Leng (Feng Chia Uni.) Lai Ming-Chih (NCTU) Lin Tai-Chia (NTU) Liu Chun (Illinois Institute of Tech.)	F
2020 Symposium for Young Analysts	2020-12-18 / 2020-12-18	NCU	Huang Hao-Wei (NSYSU) Lin Chin-Cheng(NCU) Shen Chun-Yen (NTU)	Н
Meet physicians, mathematicians and statistcians	2020-08-01 / 2020-08-01	NCTS	Lo Yu-Lun (Chang Gung Memorial Hospital & Chang Gung Uni.) Wu Hau-tieng (Duke Uni.)	L
2020 NCTS Orientation	2020-09-14 / 2020-09-14	NCTS	Chen Jungkai (NTU & NCTS) Hsieh Ming-Lun (AS)	All
2020 NCTS Spring Day	2020-06-05 / 2020-06-05	NCTS	Chen Jungkai (NTU & NCTS)	All

Seminar Title	Dates	Group
NCTS Seminar on Arithmetic Geometry and	1/6, 1/7, 3/24, 4/13, 4/20, 4/27,	
Representation Theory	12/21	А
	2/26, 3/11, 3/26, 4/22, 5/22,	
	6/12, 9/16, 10/28, 11/11, 11/25,	
NCTS Seminar on Number Theory	12/11	A
Seminar on Combinatorics	7/3	А
	3/4, 3/11, 3/18, 3/25, 5/13,	
Taipei Postdoc Seminar	5/27, 9/30, 10/7, 11/18	А
Topics on Drinfeld Modules and T-Motives II	5/27,6/3	А
	3/9, 3/27, 4/10,4/17, 5/1, 5/8,	
	5/15, 5/22, 5/29, 6/12, 6/19,	
	7/10, 7/17, 7/24, 7/31, 8/7,	
	8/14, 8/21, 9/18, 9/18, 9/25,	
	9/25, 10/23, 10/30, 11/6, 12/11,	
NCTS Seminar in Algebraic Geometry	12/18	В
	8/7, 8/14, 8/14, 8/28, 828, 9/11,	
	9/11, 9/25, 9/25, 10/9, 10/9,	
	10/23, 10/23, 11/6, 11/6, 11/20,	
Seminar of Algebraic Geometry in East Asia	11/20, 12/4, 12/4, 12/18, 12/18	В
Taipei Postdoc Seminar	4/22, 4/29, 11/4, 12/2, 12/16	В
AS-NCTS Seminar on Geometry	9/18, 10/16, 10/30, 11/13	С
	1/6, 3/4, 4/7, 6/9, 9/24, 10/8,	
	10/29, 11/5, 11/19, 11/26,	
NCTS Differential Geometry Seminar	12/10, 12/17, 12/24	С
NCTS Distinguished Lectures	1/2	С
NCTS International Geometric Measure Theory		
Seminar	11/18	С
NCTS Seminar on Differential Geometry	3/16, 3/23, 3/30, 4/13	С
NCTS Seminar on Symplectic Geometry and Gauge		
Theory	6/22	С
Taipei Postdoc Seminar	5/20, 9/23, 10/14, 10/21, 12/30	С
	3/11, 4/22, 5/6, 5/13, 5/20,	
NCTS PDE & Analysis Seminar	5/27, 6/3, 6/10, 6/17, 6/24	D

NCTS Seminar on Differential Equations	1/6	D
	9/23, 10/7, 10/14, 10/21, 10/28,	
	11/4, 11/11, 11/18, 11/25, 12/2,	
NCTS Seminar on Functional Analysis	12/9, 12/16, 12/23	D
NCTS Seminar on Probability	1/8	D
	8/5, 8/5, 8/6, 8/6, 8/12, 8/12,	
NCTS Student Seminar in Dynamical Systems and	8/13, 8/13, 8/19, 8/19, 8/20,	
Neural Networks	8/20, 8/26, 8/26, 8/27, 8/27	D
	9/24, 10/8, 10/15, 10/22, 10/29,	
	11/5, 11/12, 11/19, 11/26, 12/3,	
NCTS-South Taiwan Seminar on Dynamical Systems	12/10, 12/17, 12/24, 12/31	D
	3/10, 3/17, 3/24, 4/7, 4/14,	
	4/21, 4/28, 5/5, 5/12, 5/19,	
	5/26, 6/2, 6/9, 9/22, 9/29, 10/6,	
	10/13, 10/20, 10/27, 11/3,	
Nonlinear Phenomena in Evolutionary Partial	11/17, 11/24, 12/1, 12/8, 12/15,	
Differential Equations	12/22, 12/29	D
Taipei Postdoc Seminar	4/15, 5/6, 6/4, 6/17	D
NCTS Seminar on March Events	3/10, 3/17	Е
NCTS Seminar on Scientific Computing	1/6	Е
MeDA Lab Seminar	11/17, 12/29	F
NCTS Data Sciences Forum	9/8	F
	9/24, 10/8, 10/22, 11/5, 11/19,	
NCTS Seminar on Applied Mathematics	12/3, 12/17	F
NCTS Seminar on Dynamics of Network System	4/24, 5/1, 5/8, 5/22	F
NCTS Seminar on Mathematical Biology	1/10, 2/20, 3/13, 3/20, 4/10	F
Taipei Postdoc Seminar	6/10, 10/28	F
NCTS Seminar on Data Science	4/30	L
2020 International Day of Mathematics 國際數學日	3/14	Others
Distinguished Lecture	10/26	Others

#### NCTS course, 2020

Dates	Venue	Gp.	Title	Organizers
2020-12-02 /				
2021-01-29	AS	A	NCTS Winter Course on Mass Formulae	Yu Chia-Fu (AS)
2020-10-16 /			Taiwan Mathematics School: Representation Theory:	Peng Yung-Ning (NCU)
2020-11-13	NCTS	A	Mini course on Springer Fibers and Quiver Varieties	Chen Chih-Whi (NSYSU)
2020-09-17 /				
2021-01-14	NCTS	A	Taiwan Mathematics School: Algebraic Combinatorics	Yu Wei-Hsuan (NCU)
2020-09-14 /				
2021-01-19	AS	A	Taiwan Mathematics School: Algebraic Number Theory	Chang Chieh-Yu (NTHU)
2020-07-06 /			2020 NCTS Summer Course on Local Class Field	
2020-07-10	NCTS	A	Theory	Yu Chia-Fu (AS)
2020-03-10 /				
2020-07-03	NCTS	A	NCTS Course on Algebraic Tori and Hermitian Forms	Yu Chia-Fu (AS)
2020-03-04 /			Taiwan Mathematics School: Topics on Drinfeld	
2020-07-01	NCTS	A	Modules and T-Motives II	Chang Chieh-Yu (NTHU)
2020-09-18 /			Taiwan Mathematics School: Topics in Geometric	
2021-01-15	NCTS	С	Measure Theory I	Lin Chun-Chi (NTNU)
				Chen Chih-Wei (NSYSU)
				Chiang River (NCKU)
				Lin Matthew M. (NCKU)
2020-07-23 /				Shu Yu-Chen (NCKU)
2020-07-24	NCKU	С	NCTS Mini Course on Manifold Learning	Tsui Mao-Pei (NTU)
2020-03-06 /			Taiwan Mathematics School: Geometric Measure	
2020-07-03	NTNU	С	Theory II	Lin Chun-Chi (NTNU)
2020-09-18 /			Taiwan Mathematics School: Symbolic Dynamics on	
2021-01-15	NCTS	D	Groups and Countable State Topological Markov Shifts	Hsia Chun-Hsiung (NTU)
2020-08-07 /			2020 NCTS Summer Course on Dynamical Systems:	Ban Jung-Chao (NCCU)
2020-09-04	NCTS	D	Dynamics and Thermodynamics on Expanding Maps	Chang Chih-Hung (NUK)
				Chen Chih-Wei (NSYSU)
				Lai Ching-Jui (NCKU)
2020-01-13 /				Lee Kuo-Wei (NCUE)
2020-01-17	NCKU	D	Introduction to Curve Shortening Flow	Wang Ye-Kai (NCKU)
2020-03-03 /			Taiwan Mathematics School: Dynamics in Network	Ban Jung-Chao (NCCU)
2020-06-30	NCTS	D,F	Systems	Tsai Je-Chiang (NTHU)

2020-09-07 /			Taiwan Mathematics School: Imaging Principles in	
2020-09-11	NCTS	Е	Medical AI	Wang Weichung (NTU)
			Short Course on Advanced Course on Multi-Threaded	
2020-08-22 /			Parallel Programming using OpenMP for	
2020-09-05	Webex	Е	Multicore/Manycore Systems	Wang Weichung (NTU)
2020-02-21 /			Taiwan Mathematics School: Parallel Finite Element	
2020-02-25	NTU	Е	Method Using Supercomputer	Wang Weichung (NTU)
2020-12-10 /			Taiwan Mathematics School: Statistical Foundations of	
2021-01-14	NTU	E,F	Medical AI: Survival Analysis	Wang Weichung (NTU)
2020-10-29 /			Taiwan Mathematics School: Statistical Foundations of	
2020-12-03	NTU	E,F	Medical AI: Generalized Linear Model	Wang Weichung (NTU)
2020-09-17 /			Taiwan Mathematics School: Statistical Foundations of	
2020-10-22	NTU	E,F	Medical AI: Linear Regression	Wang Weichung (NTU)
2020-07-27 /				
2020-08-05	NTHU	F	2020 NCTS Summer Course on Mathematical Biology	Wang Feng-Bin (CGU)
2020-01-06 /				Lin Tai-Chia (NTU)
2020-01-06	NCTS	F	2020 NCTS Mini Course — Cancelled	Horng Tzyy-Leng (FCU)

### 6.2 List of Visitors

NCTS visitors, 2020 (台灣以外國家)

Arr.Date	Dep.Date	Days	Name	Gp.	Affiliation	Country	Title
2020/1/1	2020/12/31	365	Ryotaro Harada	А	JSPS	Japan	Stu
2020/1/1	2020/1/31	31	Ignaizo Longhi	А	Xi'an Jiaotong-Liverpool University	China	Prof
2020/1/1	2020/1/12	12	Pei-Chun Su	L	Duke University	USA	Stu
2020/1/1	2020/1/12	12	Da Rong Cheng	С	University of Chicago	USA	Prof
2020/1/1	2020/1/17	17	Sun-Yung Alice Chang	С	Princeton University	USA	Prof
2020/1/1	2020/1/17	17	Paul Yang	С	Princeton University	USA	Prof
2020/1/1	2020/1/13	13	Hau-tieng Wu	L	Duke University	USA	Prof
2020/1/1	2020/1/2	2	Horng-Tzer Yau	D	Harvard University	USA	Prof
2020/1/1	2020/1/19	19	Jungin Lee	А	Pohang University of Science and Technology	Korea	Stu
2020/1/1	2020/1/3	3	Albert Chern	F	Technische Universität Berlin	Germany	PD
2020/1/1	2020/1/8	8	Maria Pia Gualdani	Е	University of Texas at Austin	USA	Prof
2020/1/1	2020/1/8	8	Yen-Hsi Richard Tsai	D	University of Texas at Austin	USA	Prof
2020/1/1	2020/1/7	7	Thomas Yizhao Hou	Е	California Institute of Technology	USA	Prof
2020/1/1	2020/1/3	3	Chun Liu	F	Illinois Institute of Technology	USA	Prof
2020/1/1	2020/1/4	4	Yuan-Nan Young	F	New Jersey Institute of Technology	USA	Prof
2020/1/1	2020/1/5	5	Robert S. Eisenberg	F	Rush University	USA	Prof
2020/1/1	2020/1/5	5	Min-Jhe Lu	F	Illinois Institute of Technology	USA	Stu
2020/1/1	2020/1/8	8	Nir Gavish	F	Technion-Israel Institute of Technology	Israel	Prof
2020/1/1	2020/1/14	14	Luigi Catacuzzeno	F	University of Perugia	Italy	Prof
2020/1/3	2020/2/10	39	Long Lee	Е	University of Wyoming	USA	Prof
2020/1/7	2020/1/16	10	Mu-Tao Wang	С	Columbia University	USA	Prof
2020/1/9	2020/2/7	30	Anthony B. Costa	Е	Icahn School of Medicine at Mount Sinai	USA	Prof
2020/1/12	2020/1/16	5	Jason Lo	В	California State University Northridge	USA	Prof
2020/1/12	2020/1/18	7	Daisuke Kawagoe	D	Kyoto University	Japan	Prof
2020/2/2	2020/2/8	7	Pak Tung Ho	С	Sogang University	Korea	Prof
2020/2/16	2020/3/1	14	Cécile Armana	А	University of Franche-Comté	France	Prof
2020/2/20	2020/2/26	13	Kengo Nakajima	Е	University of Tokyo	Japan	Prof
2020/2/20	2020/3/4	13	Takao Yamazaki	А	Tohoku University	Japan	Prof
2020/3/1	2020/4/30	61	Bin Nguyen	В	Universidade de Lisboa	Portugal	PD
2020/3/2	2020/3/31	30	Alejandro López Nieto	D	Freie Universität Berlin	Germany	Stu
2020/3/2	2020/8/28	180	Jui-Ting Lu	Е	École normale supérieure de Lyon	France	Stu
2020/3/4	2020/3/21	18	Yoshinori Mishiba	А	University of the Ryukyus	Japan	Prof
2020/3/11	2020/12/31	296	Miranda Chih-Ning Cheng	В	University of Amsterdam	Netherlands	Prof
2020/4/5	2020/7/14	41	Shih-Fang Yeh	С	Michigan State University	USA	Stu
2020/4/16	2020/5/31	46	Albert Chern	F	Technische Universität Berlin	Germany	PD
2020/4/20	2020/9/18	29	Kuan-Hui Lee	С	University of California, Irvine	USA	Stu
2020/4/28	2021/1/6	254	Hsueh-Yung Lin	В	Kavli Institute for The Physics and Mathematics of The Universe	Japan	PD

Arr.Date	Dep.Date	Days	Name	Gp.	Affiliation	Country	Title
2020/4/29	2020/8/19	113	Hung Chiang	А	Columbia University	USA	Stu
2020/5/2	2020/7/1	61	Shen-Ning Tung	А	University of British Columbia	Canada	PD
2020/6/19	2020/8/16	59	Hau-tieng Wu	L	Duke University	USA	Prof
2020/7/20	2020/7/31	12	Ting Yu Lee	В	Technische Universität Dortmund	Germany	PD
2020/10/19	2020/12/31	74	Tang-Kai Lee	С	Massachusetts Institute of Technology	USA	Stu
2020/12/9	2020/12/31	23	Eiichi Bannai	А	Kyushu University	Japan	Prof
2020/12/9	2020/12/31	23	Etsuko Bannai	А	Kyushu University	Japan	Prof
2020/12/11	2020/12/31	21	Albert Wood	С	Imperial College London	UK	PD
2020/12/14	2020/12/31	18	Chao-Ming Lin	С	University of California, Irvine	USA	Stu

#### NCTS visitors (Taiwan), 2020

Arr.Date	Dep.Date	Days	Name	Gp.	Affiliation	Country	Title
2020/1/1	2020/1/31	31	Chern-Shuh Wang	Е	National Cheng Kung University	Taiwan	Prof
2020/1/1	2020/1/5	5	Chia-Yu Hsieh	D	NCTS	Taiwan	PD
2020/1/1	2020/1/31	31	Feng-Bin Wang	F	Chang Gung University	Taiwan	Prof
2020/1/12	2020/1/22	11	Kazuo Aoki	С	NCTS	Taiwan	Prof
2020/5/11	2020/7/21	72	Chao-Jung Huang	Е	National Taiwan University	Taiwan	PD
2020/6/22	2020/8/6	46	Chi-Heng Lo	А	National Taiwan University	Taiwan	Stu
2020/7/6	2020/8/31	57	Feng-Bin Wang	F	Chang Gung University	Taiwan	Prof
2020/7/15	2020/8/31	48	Chern-Shuh Wang	Е	National Cheng Kung University	Taiwan	Prof
2020/8/5	2020/9/4	31	Chih-Hung Chang	D	National University of Kaohsiung	Taiwan	Prof
2020/8/28	2021/1/28	32	Jann-Long Chern	D	National Central University	Taiwan	Prof
2020/9/1	2021/1/31	153	Jin-Cheng Jiang	D	National Tsing Hua University	Taiwan	Prof
2020/9/1	2021/1/31	153	Jenn-Nan Wang	D	National Taiwan University	Taiwan	Prof
2020/9/2	2020/10/8	37	Chia-Fu Yu	А	Academia Sinica	Taiwan	Prof
2020/10/12	2020/12/31	81	Chia-Fu Yu	А	Academia Sinica	Taiwan	Prof
2020/11/23	2020/12/31	39	Weichung Wang	Е	National Taiwan University	Taiwan	Prof

## 6.3 Publication data

## 6.3.1 Summary of publications of key members 2019-2020

## 6.3.2 Publications of Key Members

In umal Title	2019	2019	2020	2020	То
journar nue		AFF(42)	ACK(66)	AFF(48)	Appear(28)
Acta Arithmetica		2			
Acta Mathematica Sinica		2			
Adv. Calc. Var.		1			
Adv. Theor. Math. Phys				1	
Advances in Mathematics	2	1	1	1	1
AIMS Mathematics			1	1	
American Naturalist			1	1	
Ann. Scuola Norm. Sup. Pisa Cl. Sci		1		1	
Ann. Inst. H. Poincaŕe Anal. Non Lińeaire.	1				
Annales de l'Institut Fourier (Grenoble)					1
Annali della Scuola Normale Superiore di Pisa	1		2		1
Annals of Mathematical Sciences and Applications			1		
Applicable Analysis			1		
Applied and Computational Harmonic Analysis	1	1			
Asian Journal of Mathematics		1			
Aquaculture.		1			
Biomaterials	1	1			
Biomedical Signal Processing and Control			1		
BMC Complementary and Alternative Medicine	1	1			
Bull. Inst. Math. Acad. Sin.				1	
Calculus of Variations and Partial Differential	1				
Equations	T				
Calc. Var. Partial Differential Equations	1	1			
Communications in Computational Physics		1			
Communications in Contemporary Mathematics	1	1			
Communications in Pure and Applied Analysis		1			
Computational Statistics and Data Analysis	1		1		
Computer Methods in Applied Mechanics and			1		
Engineering			1		
Contemp. Math. series of AMS	1				1
Discret Contin Dyn S -A			3	2	1
Discrete and Continuous Dynamical Systems - Series	1	1	1		1
В	1	1	1		1
Duke Math. Jour.			1	1	
Electronic Transactions on Numerical Analysis			1		

European J. Appl. Math.			1	1	
Fisheries Oceanography		1			
Frontiers in Microbiology			1	1	
Geometriae Dedicata		1			
Global Change Biology			1	1	
IMRN 2020					1
Indiana Univ. Math. J.			1	1	
Information Fusion	1	1			
Integers		1			
International Journal of Mathematics			2	1	
International journal of number theory 2020					1
International Mathematical Research Notices	1		1	1	1
Inventiones Mathematicae			1		
ISME Journal	1	1			
Japan Journal of Industrial and Applied Mathematics					1
J. Math. Pures Appl.			1	1	
J. Math. Soc. of Japan				1	
J. Reine Angew. Math.			1		
J. Stat. Mech			1	1	
J. Theor. Nombres Bordeaux				1	
Journal d'Analyse Mathmatique 2020					1
Journal of Algebra			1		
Journal of Algebraic Combinatorics	1	1	1	1	
Journal of Clinical Monitoring and Computing	1	1			
Journal of Clinical Sleep Medicine			1		
Journal of Computational and Graphical Statistics	1				
Journal of Computational Physics			1	1	
Journal of Computational Mathematics		1			
Journal of Differential Equations	2	1	4	3	3
Journal of Ecology		1			
Journal of Evolution Equations					1
Journal of Functional Analysis				1	
Journal of Global Optimization			1		
Journal of London Math. Soc,	1				
Journal of Mathematical Analysis and Applications	1				
Journal of Mathematical Biology			1	1	1
Journal of Mathematical Physics			2	2	

Journal of Mathematical Sciences			1		
Journal of Mathematical Study				1	
Journal of Neural Engineering	1	1			
Journal of Number Theory			1		
Journal of Scientific Computing			1		
Journal of Statistical Physics			1	1	
Journal of the European Mathematical Society	2	1			
Journal of Theoretical Biology				2	
Journal of Topology and Analysis			1		
Limnology and Oceanography			1	1	
Manuscripta Mathematica		1			1
Mathematical Biosciences	1		1		
Mathematical Biosciences and Engineering			1		
Mathematical Proceedings of Cambridge				1	
Philosophical Soc.				T	
Mathematical Research Letters			1		1
Mathematische Zeitschrift		1			2
Memoires de la Societe Mathematique de France	1	2			
Michigan Mathematical Journal	1	1			1
mSystem			1	1	
Nature Communications			1	1	
Nonlinear Anal.	1	1	2	2	1
Nonlinearity				1	1
Oikos		1			
Optimization Letters	1				
Pacific Journal of Mathematics			1	1	
Phys. D,			1	1	
PLOS ONE		1	1		
Proc. London Math. Soc.			1	1	
Proceedings of AMS	1	1			
Proceedings of RIMS Conference				1	
Progress in Oceanography			1	1	
Q. Appl. Math.					1
Ramanujan Journal		1			
Sensors			1		
SIAM Journal on Applied Dynamical Systems	1				
SIAM Journal on Applied Mathematics	1				

SIAM Journal on Imaging Sciences			2		
SIAM Journal on Mathematical Analysis			1	1	
SIAM Journal on Scientific Computing			1		
Statistics in Medicine			1		1
Studia Math			1		
Taiwanese Journal of Mathematics			1	2	1
Transactions of the American Mathematical Society	1	3			
Transformation Groups	1			1	1
WIREs Computational Statistics			1		1

Author	Title	Journal	Year
		Asian Journal of	
		Mathematics, Volume 24	
	Equivariant asymptotics of Szegö kernels	(2020), Number 3,	2020
Andrea Galasso	under Hamiltonian SU(2)-actions	Pages: 501 – 532, DOI:	2020
		https://dx.doi.org/10.43	
		10/AJM.2020.v24.n3.a6	
	Codimension two cycles in Iwasawa		
Bharathwaj	theory and tensor product of Hida	Math. Ann.	2020
Palvannan	families.		
		Manuscripta	
Din Name	Some unlimited families of minimal	Mathematica, Volume	2020
Bin Nguyen	surfaces of general type with the	163, Issue 1-2, pp 13–25	2020
	canonical map of degree 8	(2020)	
Din Mauron	Some examples of algebraic surfaces		nuonvint
bin nguyen	with canonical map of degree 20		preprint
Din Mauron	Some algebraic surfaces with canonical		nuonvint
Bin Nguyen	map of degree 10,12,14		preprint
Chih Wai Chang	Asymptotic orders of vanishing along	Pacific J. Math.	2020
Chin-wei Chang	base loci separate Mori chambers	304(2020), no. 1, 55–64	
Chih Wai Chang	The Kodaira Maps of Toric Vector		proprint
Chini-wei Chang	Bundles		preprint
Chun-Woi	Long-term warming destabilizes aquatic	Clobal Change Biology	
Chang	ecosystems through weakening	26(11), 6412, 6422	2020
Chang	biodiversity-mediated causal networks	20(11). 0415-0425	
	Body size, light intensity and nutrient		
Chun-Wei	supply determine plankton	American Naturalist	2020
Chang	stoichiometry in mixotrophic plankton	195(4):E100-E111	2020
	food webs		
Chun-Wei	Functional diversity promotes	Journal of Ecology	2010
Chang	phytoplankton resource use efficiency	107:2353-2363	2019
	The paradox of the re-oligotrophication:		
Chun-Wei	the role of bottom-up versus top-down	Oilcos 129,1666 1677	2010
Chang	controls on the phytoplankton	OIKOS 128.1000-1077	2019
	community		
Chun-Wei	Vertical structure of heterotrophic	Philipping Journal of	
Chang	bacterioplankton communities in the	Science 149.155 165	2019
Chang	western Pacific Ocean	Science 140:155-105	

## 6.3.3 Publications of NCTS postdocs, 2019-2020

	Higher Dimensional Elliptic Fibrations	Comm. in Contemporary	2020	
David wen	and Zariski Decomposition	Mathematics		
	Automorphism of Minimal Surfaces of			
David Wen	General Type with K_S^2 = 1 and p_g =		preprint	
	2			
	Projectively equivalence for the roots of			
Hang Fu	unity		preprint	
	On the PGL2-invariant quadruples of			
Hang Fu	torsion points of elliptic curves		preprint	
		Journal of London Math.		
Hsin-Ku Chen		Soc,	2019	
	Invariance of plurigenera fails in positive			
lacopo Brivio	and mixed charateristic		preprint	
	b-semiampleness for fiber spaces with			
lacopo Brivio	small Albanese dimension		preprint	
	Ginzburg-Landau patterns in circular			
Jia-Yuan Dai	and spherical geometries: vortices,		preprint	
	attractors and spirals			
	Small amplitude limit of solitary waves			
Junsik Bae	for the Euler-Poisson system	J. Differential Equations	2019	
<b>17</b> 1.	Affinoids in the Lubin-Tate perfectoid	Mathematische Annalen,		
Kazuki	space and special cases of the local	377, 1339-1425, 5-year	2020	
Tokimoto	Langlands correspondence	IF=1.659, 2 citations		
	Unconditionally energy stable schemes			
Kian Chuan Ong	for the inextensible interface problem	SIAM Journal of Scientific	2019	
	with bending.	Computing		
T ' TAT	Perfect Numbers and Fibonacci Primes	1.1	2010	
Liuquan wang	(II)	Integers 19	2019	
1	New proofs of Ramanujan's identities on	Demonstructure I	2010	
Lluquan wang	false theta functions	Kamanujan Journai	2019	
1	Some inequalities for Garvan's birank		2010	
Lluquan wang	function of 2-colored partitions	Acta Arithmetica	2019	
	Modular forms and breakand	Transactions of the		
Liuquan Wang	generalized Frehenius partitions	American Mathematical	2019	
	generalized Frobenius parutions	Society 371		
Liuguan Wang	Fractional powers of the generating	Acta Arithmetica	2010	
Liuquan wang	function for the partition function	Acta Antinnetica	2019	

	Moments of ranks and cranks, and			
Liuquan Wang	quotients of Eisenstein series and the		preprint	
	Dedekind eta function			
1	The smallest parts function associated			
Liuquan Wang	with w(q)		preprint	
Liuquan Wang	Representations of mock theta functions		preprint	
1	Arithmetic properties of odd ranks and			
Liuquan wang	k-marked odd Durfee symbols		preprint	
Liuquan Wang	Some Hecke-Rogers type identities		preprint	
1	New congruences on multiple harmonic			
Liuquan Wang	sums and Bernoulli numbers		preprint	
	On Drinfeld modular forms of higher	W: 0404 44040		
Oguz Gezmiş	rank and quasi-periodic functions	arxiv:2101.11819	2020	
	Deformation of multiple zeta values and	De sum ente Methemetice		
Oğuz Gezmiş	their logarithmic interpretation in	Documenta Mathematica	2020	
	positive characteristic	25, 235524211(2020)		
		Research in the		
Oğuz Gezmiş	Taelman L-values for Drinfeld modules	Mathematical Sciences,	2019	
	over late algebras	(2019) 6: 18		
Ožus Cassis	The de Rham isomorphism for Drinfeld	Journal of Algebra, 525,	2010	
Oguz Gezmiş	modules over Tate algebras	454496	2017	
		Transactions of the		
Dang Lie Wang	Cyclicity and exponents of CM elliptic	American Mathematical	2020	
Peng-Jie wong	curves modulo p in short intervals	Society 373 (2020)		
		8725-8749		
		Journal de Théorie des		
Peng-Jie Wong	Dirichlet I, for stiene	Nombres de Bordeaux 32	2020	
	Differiet L-functions	(2020) 685-710		
Dana Ka Mana	On the moments of torsion points	International Journal of	2020	
Peng-Jie wong	modulo primes and their applications	Number Theory (2020)	2020	
	On generalizations of the Titchmarsh	Acta Arithmetica 193	2020	
Pelig-Jie wong	divisor problem	(2020) 321-337	2020	
Dana Ka Mana	Bombieri-Vinogradov theorems for	Mathematika 66 (2020)	2020	
Pelig-Jie wong	modular forms and applications	200-229	2020	
		Proceedings of the		
Dong Lig Marro	The least prime ideal in the Chebotarev	American Mathematical	2019	
reng-jie wong	density theorem	Society 147 (2019),		
		2289-2303		

	Zeros of Dedekind zeta functions and	Journal of the Ramanujan		
Peng-Jie Wong	below or bedeking zeta functions and	Mathematical Society 34	2019	
	noromorphy of Artin L-functions	(2019), 253-261		
	On the Chabatanan Sata Tata	Journal of Number		
Peng-Jie Wong	on the Chebotarev-Sato-Tate	Theory 196 (2019), 272-	2019	
	phenomenon	290		
D I 14	Counting zeros of Dedekind zeta			
Peng-Jie Wong	functions		preprint	
Peng-Jie Wong	Applications of the Sato-Tate conjecture		preprint	
D I 14	Selberg's orthogonality conjecture and			
Peng-Jie Wong	symmetric power L-functions		preprint	
	Spike layer solutions for a singularly			
Sanghyuck	perturbed Neumann problem:	Commun. Pure Appl.	2212	
Moon	variational construction without a	Anal., 18 (2019), 1921-	2019	
	nondegeneracy	1965		
	Cusp excursions for the earthquake ow	Geom. Dyn. 23 (2019),		
Ser-Wei Fu	on the once-punctured torus	251-261	2019	
a	Cylinder curves in nite holonomy at			
Ser-Wei Fu	metrics		preprint	
0	Flat grafting deformations of quadratic			
Ser-Wei Fu	differentials on surfaces		preprint	
	The second-order evolution of			
Yang-Kai Lue	inextensible and elastic planar curves		preprint	
	with clamped ends			
V 17. * 1	Uniqueness of regular shrinkers with 2			
Yang-Kai Lue	closed regions		preprint	
	Variation of a theme of Landau-Shanks in			
Yen-Liang Kuan	positive characteristic		preprint	
	Numerical irreducible criteria for	Topology Appl. 284,		
Yi-Sheng Wang	handelbody links	107361 (2020)	2020	
		J. Knot Theory		
Yi-Sheng Wang	A complete invariant for connected	Ramification, 29, 1,	2020	
	surfaces in the 3-sphere	1950091 (2020)		
	Topological K-theory with coefficients	Rocky Mountain J. Math,		
11-Sheng Wang	and the e-invariant	50, 1 (2020), 281–318	2020	
	The Becker-Gottlieb Transfer: a			
Yi-Sheng Wang	Geometric Description Oberwolfach		preprint	
	Preprints;			

Yoshihiro Sugimoto	Note on (super)heavy subsets in symplectic manifolds	CMA Proceedings, Vol 48 Apr 2019	2019
Yoshihiro Sugimoto	Applications of square roots of diffeomorphisms	Axioms, 2019, 8(2), 43	2019
Yoshihiro Sugimoto	Spectral spread and non-autonomous Hamiltonian diffeomorphisms	manuscripta mathematica, November 2019, Vol 160, 483-508	2019
Yoshihiro Sugimoto	The sharp energy-capacity inequality on convex symplectic manifolds	Journal of Fixed Point Theory and Applications, 21, 13, (2019)	2019

# 6.4 Undergraduate Research and Summer Research Program

主題名稱	指導老師	學員 1	學員 2	學員 3
1. Denoising the EEG signal using optimal	吳浩榳	邱能泰	郭倍誠	
shrinkage to correct biased singular value	(Duke U.)	(陽明醫)	(陽明醫)	
	余家富	梁珮欣	楊新怡	
2. Tamagawa numbers of CM tori	(中研院)	(中山應數)	(台大數學)	
3. Improvement of generalization of Larman-	俞韋亘	高暐竣	葉政叡	
Rogers-Seidel's theorem	(中央大學)	(台大數學)	(中央數學)	
5. Computation of irreducible characters and	彭勇寧	林明駿		
branching rules for A_n	(中央大學)	(中央數學)		
	曾昱豪	陳重凱	郭昱婕	
6. Finite difference method for PDEs	(高雄大學)	(高雄應數)	(高雄應數)	
	班榮超	古賀琳		
7. Classification theory on Markov tree shift	(政治大學)	(政大應數)		
8. Flux-based Bifurcation Analysis in	蔡志強	森田展宏		
Reaction Network Systems	(清華大學)	(清大數學)		
	郭鴻基	凌文海	黃懷逸	李琦文
9. Vortex Dynamics in Numerical Models	(台灣大學)	(台大大氣)	(台大大氣)	(台大大氣)
10. Mathematical models of COVID-19 with	陳建隆	陳彥嘉		
diffusion effects and their data forecast	(師範大學)	(師大數學)		
	黃信元	許哲瑋		
11. Application of Fokas's Method on PDE	(交通大學)	(交大應數)		
	黃皓瑋	盧德倫	馬宗儀	李柏駿
12. Topics on Free Probability	(中山大學)	(台大數學)	(台大數學)	(政大應數)

#### NCTS URP 2020 (2020.10.1-2021.6.30)

#### NCTS USRP 2020

	指導老師	助教	學員 1	學員 2	學員 3	學員 4
1.傳染疾病傳播的數學模型						
探究與數據預測-及其應用						
於 COVID-19 的傳播探討						
Mathematical Modelling and						
Data Prediction in Infectious						
diseases Spread and COVID-	陳建隆		師大數學三	師大數學三	中央數學二	中央數學三
19	(中央大學)	汪漢鈞	陳彥嘉	洪翊誠	陳俊昇	林柏全
2.使用分離核來談邊界元素						
法/邊界積分方程之秩降問						
題						
On the rank deficiency of			海大河海工	海大河海工	海大河海工	
BIE/BEM using degenerate	陳正宗		程三	程三	程三	淡大土木三
kernels	(海洋大學)	莊博揚	戴暐宸	呂政軒	周彥廷	張致唯
3.非交換機率論中的組合與						
分析						
Analysis and combinatorics						
in non-commutative	黃皓瑋		台大數學三	台大數學三	政大應數三	中央數學三
probability theory	(中山大學)		盧德倫	馬宗儀	李柏駿	葉政叡
4.散射變換及其應用						
Scattering Transform and Its	劉聚仁		交大應數三	成大光電四	成大數學三	
Applications	(成功大學)	陳柏穎	呂書緯	劉智皓	張恒宇	
5.動力系統中高維度交互網						
絡的重建						
Reconstruction of high						
dimensional interaction					彰師大數學	
networks in dynamical	蔡志強		清大數學三	清大數學三	四	東吳數學三
systems	(清華大學)	張俊偉	森田展弘	吳昇原	閻天立	林凌豪
6.影像處理的變分模型和數						
值方法						
Variational models and						
numerical methods for	楊肅煜		師大數學四	政大應數三	交大應數一	
image processing	(中央大學)	魏辰晏	廖家緯	陳俊憲	戴晨洋	
7.複乘代數環面算術性質之						
探討	余家富		台大數學三	中正數學三	台大數學二	中山應數三
Arithmetic of CM tori	(中研院)	林浚沂	楊新怡	張宏彬	蔡子暘	梁珮欣

8.離散幾何與最佳化						
Discrete Geometry and	俞韋亘		台大數學二	中央數學四	台大數學三	政大應數四
Optimization	(中央大學)	陳佑銘	王健安	林育愷	高暐竣	王姿云