NCTS Midterm Report, 2015

National Center for Theoretical Sciences,
Mathematics Division

October, 2015

Contents

I. Overview

A. Report of Director

1. Goals and missions of the Center

Theoretical sciences are undoubtedly the foundation of sciences and technology of our daily life. In fact, it is generally believed that theoretical science will play more and more important role in the next few decades. Cultivating young generation with solid training of fundamental sciences and fostering frontier academic research in theoretical sciences are thus essential to further development of our society. Therefore, in 1997, National Center for Theoretical Sciences (NCTS) was established by National Science Council. Since its establishment, NCTS has completely changed the mathematical society. The achievement of NCTS during the past 18 years was remarkable. It is generally believed that NCTS is the key factor for the great improvement of the mathematical research in Taiwan.

NCTS has set its objective to promote researches of theoretical sciences in Taiwan. The aim consists of the following specific aspects:

a. Fostering world class outstanding researchers, and attract top young researchers to do cutting edge research in NCTS;
b. Attracting worldwide outstanding theoretical scientists to do research in Taiwan;
c. Developing international and inter-disciplinary scientific research program;
d. Promoting international cooperation and collaborations, aiming to become a leading research institution in Asia and in the world.

Toward the pursuit of being a global center of excellence, we would like to emphasize "global" and "excellence". The globalization should be realized in two aspects: by consolidate further international cooperation, and by cooperate with scholars in Taiwan not necessarily from the host institute. Therefore, we proposed the idea of OPEN NCTS in our proposal. To realize OPEN NCTS, we invited and welcome active mathematicians (possibly none of them are Taiwanese) to form Research Pairs to do their research in NCTS. We open calls for program proposals from research teams. All of our programs, events, activities, and supports are designed to serve all excellent mathematicians and students around Taiwan, not just for people in the campus of NTU. The only one and the most important criterion of above mentioned programs and events is "seeking excellence".

2. Brief summary of the year

To realize the above objectives, important indicators other than quality and quantity of research articles are:

a. Quality and quantity of foreign scholars: A world leading center is a place where researchers would choose to spend their sabbatical time, or a place where researchers from different places to meet and collaborate together. Some extra effort was made to attract long-term visitors. We encourage visitors to stay longer for further and deeper academic cooperation. Hence, we extended the existing NCTS Scholar Program to recruit more excellent researchers for longer visit. We have 4 more NCTS Scholars and Distinguished Scholars: Fan Chung Graham (UC San Diego), Gunther Uhlmann (University of Washington), Paolo Cascini (Imperial College London), and Nikolaos Zygouras (University of Warwick) who will visit NCTS for a longer period. Moreover, we inaugurated the Research in Pairs Program to recruit two or more scholars from different places to collaborate in NCTS. With these efforts, the statistics shows that various visitor programs of NCTS attracted more than 200 foreign visitors of duration of almost 3000 days in 2015. The average duration of visitors has been increasing notably.

b. Quality and quantity of postdoctoral fellows: Another important feature of world leading center is that it is a place where new Ph.D. would love to start their academic career there. Therefore, we extended our program for postdoc fellows. Starting from August of 2015, we have 16 postdoc fellows. Eleven of those postdoc fellows are international postdocs from Japan, Korea, China, Hong Kong, Vietnam, and Romania respectively. Each one of them are associated to a Topical Program and also associated to a senior member. We also encourage postdoc fellows to build up their own international academic connection. In fact, by cooperating with other prestigious center or institute, we promoted exchange or visiting program for postdoc fellows. For example, according to the MOU between RIMS and NCTS, we will be able to send our postdoc fellow to visit RIMS for a period of one to three months.

c. Cultivate the youth: One of the foremost missions of NCTS is to cultivate young talents. Hence, the Center will not only put lots of efforts on student-training in each program, such as student seminars, summer schools, but also will it play a more active role in inter-university cooperation, so that it serves as a platform that outstanding scholars from different home institute can have joint student-training program. As a result, we held about 20 short courses and summer/winter schools, 3 courses and 3 symposiums designed for young generations.

   NCTS also designed some programs for undergraduate and graduate students to work in the Center or to visit cooperating foreign institutes. The Program of Research Assistants and short-term Research Assistants are designed for this purpose. The one and the only one selection criterion of RA is the potential to be admitted to Ph.D. program of prestigious department or institutions. We hope that NCTS produce many good theoretical scientists of next generation as she did before. However, even though we allocated 10 positions of RAs, we only recruited 4 qualified RAs this year. Perhaps the rapid shrinking of student numbers of recent two years is one of the key factors.

Restructure of Topical Programs.

Starting from the year 2015, NCTS was relocated to the campus of National Taiwan University. Not only the operating structure has been reformulated, but also new vision and missions are introduced. To this end, our Topical Programs had been reformulated and regrouped. Currently, we have 6 Topical Programs and 2 Initiatives. The 6 Topical Programs consist of: A. Number Theory and Representation Theory, B. Algebraic Geometry, C. Differential Geometry and Geometric Analysis, D. Differential Equation and Stochastic Analysis, E. Scientific Computing, F. Interdisciplinary Studies. The Topical Programs are structured following the advice of the International Scientific Advisory Committee held in July of 2014. The 2 Initiatives are G. Big and Complex Data Analysis and H. Harmonic Analysis.

We understand that the rapidly changing world produces new problems and new challenges in mathematical sciences. In particular, these newly developed topics usually require excellent researchers with broader view. Therefore, we encourage cooperation and interactions among various disciplines. The previous programs of PDE, Dynamical System and Probability were therefore merged into a larger program: Differential Geometry and Stochastic Analysis for this purpose. The newly created program: Interdisciplinary Studies was created in order to promote further theoretical research on newly developed area in mathematical sciences. Through the Open Calls for Proposals of 2015, we selected two new initiatives: Big and Complex Data Analysis and Harmonic Analysis. These two programs reflect important new research topic and emerging new research group respectively.

B. Summary of Statistics

1. Summary of activities

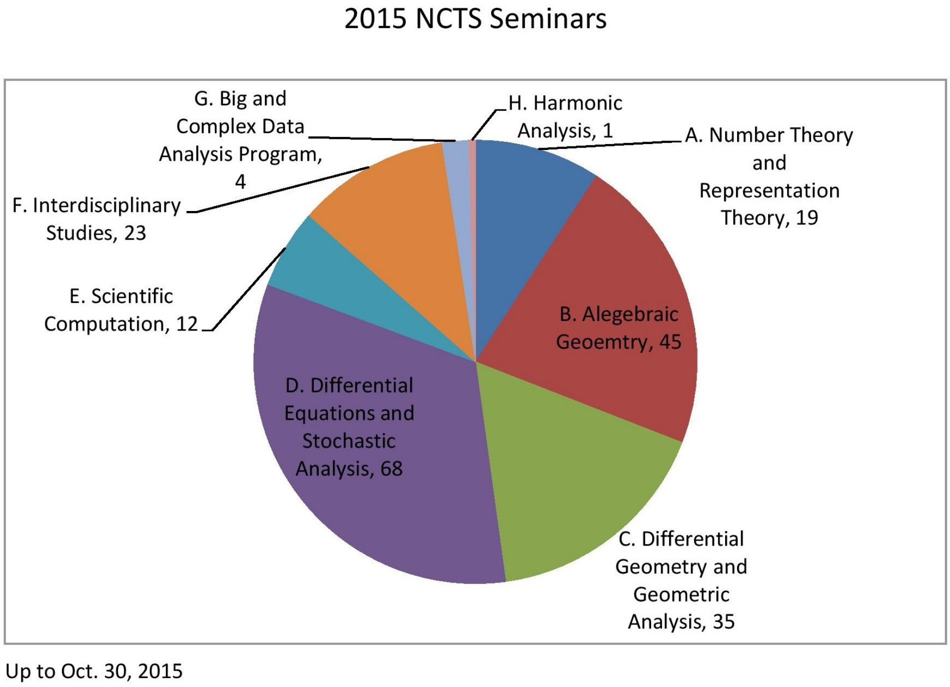

Figure 1. 2015 NCTS Seminars

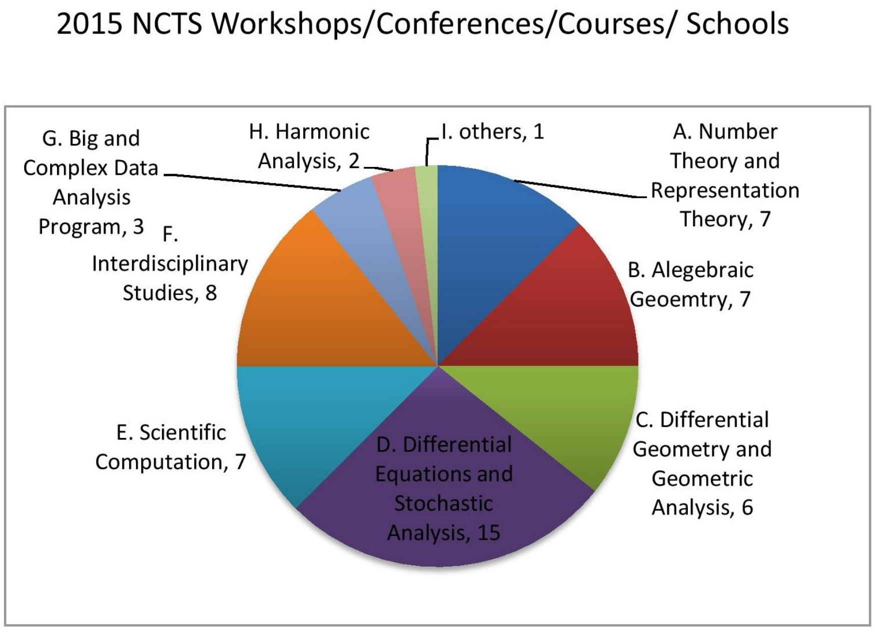

Figure 2. 2015 NCTS Workshops

2. Summary of visitors

USA	49
China	46
Japan	66
Korea	19
Europe	21
Others	10
Total	211

Up to Oct. 30, 2015

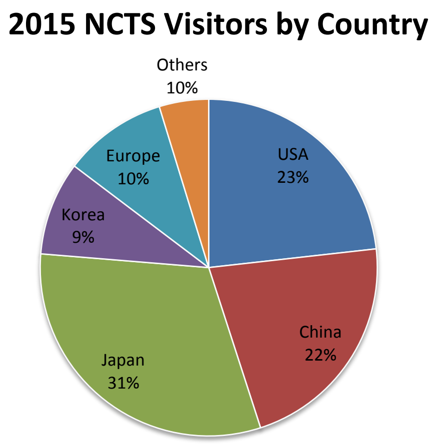

Figure 3. 2015 NCTS Visitors by Country

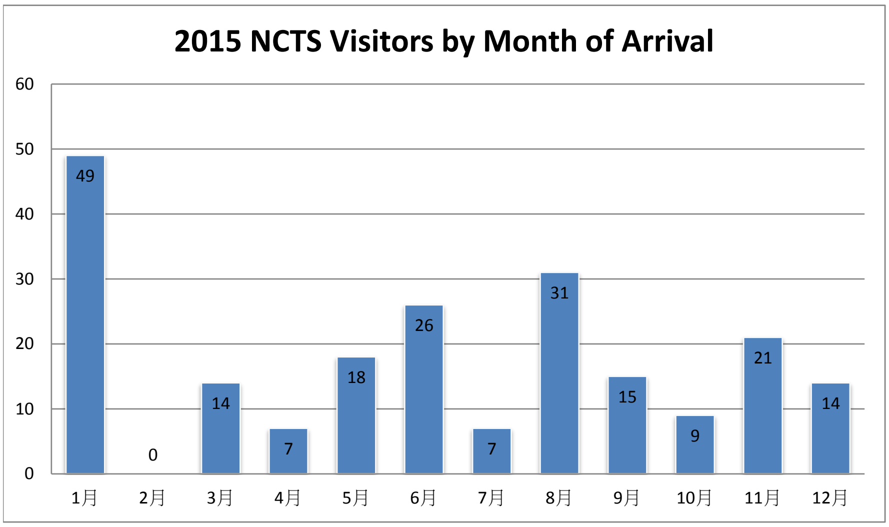

Figure 4. 2015 NCTS Visitors by Month of Arrival

II. Operation and Achievement of the Center

A. Visitor program

1. Short term and long term visitors

We encourage our members to invite visitors for longer stay, as this helps create more discussion and interaction between visitors and local researchers. We believe that this is an effective way to make research collaboration more solid. Up to the end of October, the Center hosted 211 foreign visitors. The total duration of their visits is 2843 days. The average duration is 13 days, which is the highest among 9.4 (2405/255) of the year 2012, 11.4 (3055/269) of the year 2013 and 10.3 (3162/307) of the year 2014.

2. NCTS scholars

The Program of NCTS Scholar is a program to recruit world leading experts to work in NCTS for 3 to 6 months during a span of three years. The scholars are free to choose their period of visit. We appointed 4 new NCTS Distinguished Scholars and Scholars this year, including: Fan Chung Graham (UC San Diego), Gunther Uhlmann (University of Washington), Paolo Cascini (Imperial College London), and Nikolaos Zygouras (University of Warwick). Together with existing Scholars, there are 4 NCTS Scholars visited the Center this year: a. NCTS Distinguished Scholar Yujiro Kawamata (University of Tokyo) visited the Center in August. During his visit, he co-organized an international workshop "Higher Dimensional Algebraic Geometry". b. NCTS Distinguished Scholar Horng-Tzer Yau (Harvard University) visited the Center from May 31 to July 31. During his visit, he gave a mini-course on random matrices. The course was video-taped and put on the web already. c. NCTS Distinguished Scholar Fan Chung Graham is visiting the Center from October to December. She is to give a course on spectral graph theory which will last from mid-October to late-December. d. NCTS Scholar Caucher Birkar visited the Center in May and August. He had given one mini-course each time. In his report, he said *"I believe NCTS is going in the right direction by appointing scholars who regularly visit the center and who attract other visitors. Scholars are asked to give lectures and organize conferences which is quite reasonable and necessary. A really positive thing about NCTS is the lack of bureaucracy for scholars."*

3. Research pairs

A center of excellence should be able to attract researchers from all over the world to conduct cutting edge research in the Center, no matter they are affiliated or associated with local people. Therefore, the program of Research in Pairs was launched this year. The basic rule is that a "Pair" consists of 2 - 4 members to meet at the Center and work on their research for 2 - 4 weeks. The very first Pair, consists of Korean researchers and a local researcher from NCKU will visit the Center in November 2015.

B. Postdoctoral program and cultivation program

1. **Postdoctoral program**

We received 40 applications this year. Since we have 4 existing postdocs with renewed appointments, the Executive Committee selected 16 postdocs fellows and put some in the backup list. At the end, we have 12 new postdocs and 4 renewed ones (Qi Gao, Rulin Kwan, Chi-Kwan Fok, Yunchang Seol). Among those 12 fresh postdocs, there are 4 Taiwanese and 8 international postdocs, which makes in total 5 Taiwanese and 11 international postdocs.

2015 NCTS Postdocs

Name	Group	Adviser	Nationality	Previous Affiliation
Yu-Yen Chien	A	Ching-Hung Lam	Taiwan	University of Southampton
Tsz On Mario Chan	B	Chen-Yu Chi	Hong Kong	KIAS
Hiep Dang	B	Jungkai Chen	Vietnam	Kaiserslautern
Chien-Hao Huang	D	Chii-Ruey Hwang	Taiwan	Academia Sinica
Gyeongha Hwang	D	Jenn-Nan Wang	Korea	Ulsan Nat'l Inst. of Sci. and Tech.
Yong-jie Wang	A	Shun-Jen Cheng	China	Univ. of Sci. and Tech. of China
Catalin Ion Carstea	D	Jenn-Nan Wang	Romania	Jiaxiang Foreign Lang. School
Jia-rui Fei	A	Shun-Jen Cheng	China	UC Riverside
Zheng-yu Hu	B	Jungkai Chen	China	University of Cambridge
Tien-Tsan Shieh	E	I-Liang Chern	Taiwan	NCU
Li-ren Lin	D	Jenn-Nan Wang	Taiwan	Academia Sinica
Kazuaki Miyatani	A	Jeng-Daw Yu	Japan	Hiroshima University
Ru-Lin Kuan*	D	Tai-Chia Lin	Taiwan	Academia Sinica
Qi Gao*	F	Tai-Chia Lin	China	University of Minnesota
Chi Kwong Fok*	C	Nan-Kuo Ho	Hong Kong	Cornell University
Yunchang Seol*	E	Ming-Chih Lai	Korea	Chung-Ang University

*: renewal

Table 1. 2015 NCTS Postdocs

2. **Graduate and undergraduate program**

The purpose of the Research Assistant Program of NCTS is to identify some young students who are very potential to be good theoretical scientists in the future. The offer is from one semester to one year. They are exposed to advanced research environment, which is very helpful for mathematical studies. Besides regular RA offers, we also provide several visiting RA offers for those students who are studying abroad during their visit to NCTS. Our previous program was quite successful. There are many active researchers in Taiwan and abroad who used to be RA of NCTS. For example, Chieh-Yu Chang of NTHU, Yu-Jong Tzeng of MinnesotaKetc. This year we selected 4 RA among

10 applicants.

Another aspect is the courses and schools for students. As always emphasized by NCTS, cultivation of youth is the major mission of NCTS. Therefore, each Topical Program provides considerable courses and schools for students. There were 3 courses and 20 mini-courses or schools designed for students this year.

2015 NCTS Research Assistants

Name	Group	Adviser	Previous Affiliation
陳延安	B	Jungkai Chen	NTU
郭柏均	E	Jenn-Nan Wang	NCTU
趙凱衛	C	Yng-Ing Lee	NTU
黃昱睿	C	Chin-Yu Hsiao	NTHU

Table 2. 2015 NCTS Research Assistants

C. International cooperation

1. Cooperation with international institutions

There are several on-going international exchange program based on existing MOUs. a. NCTS-POSTECH (Korea): Based on the MOU between NCTS and POSTECH, there have been joint meeting in the area of number theory and PDE for years. The joint meeting on number theory scheduled in August 2015 which was to be held in POSTECH has been postponed to January of 2016 due MERS in Korea. b. NCTS-RIMS (Japan): This MOU was made effective in the year 2014. The MOU between RIMS and NCTS focuses on the exchange of young researchers (Ph. D students, postdocs, and young faculties) for a longer period. We support the visit of Gi-Ren Liu to RIMS based on this MOU. c. NCTS-KIAS (Korea): The Directors of both institutes met and discussed the possible collaboration this October. It is a consensus that an exchange program similar to NCTS-RIMS will be carried out. The actual MOU shall be drafted and discussed. d. There was some informal discussion between with MPI Leipzig (Germany). We are also seeking possible cooperation with MSRI (USA) and MBI (USA). The realization of MOU with University of Bergen (Norway) was also discussed.

2. Cooperation with foreign research team

There are also several ongoing international cooperation with foreign research team. We list some of them. a. Reaction-Diffusion Network in Mathematics and Biomedecine (ReaDiNet): This network was initiated by French and Japanese team. It is now enlarged into a network between CNRS, INRIA, Paris Sud, Nice of France and Tokyo, Meiji of Japan, KIAS of Korea, and NCTS of Taiwan. NCTS will send two young researchers to attend the meeting in November 2015 in Meiji University. b. East Asian Core Doctorial Forum on Mathematics (EACDFM): This forum consists of 4 participating countries: Taiwan, Japan, Korea, and China. The forum is organized by students. Each country has a student representative to serve as organizing committee. And each country invites about 10 Ph.D. (or postdocs) for the forum. The 2015 forum was held in NCTS. The 2016 forum will be held in Fudan University.

III. Academic Programs

A. Number Theory and Representation Theory

Program A.	Number Theory and Representation Theory
Coordinator	Chiafu Yu, Yifan Yang, Chieh-Yu Chang, Ming-Lun Hsieh, Meng-Kiat Chuat
Core members	Shun-Jen Cheng, Ching-Hung Lam, Ngau Lam, Yung-Ning Peng

1. Overview of the Program

The purpose of this program at NCTS is to help Taiwanese algebraists create outstanding research, establish international cooperation and enhance international visibility of number theory and representation theory in Taiwan. The research activities in the program are currently organized by four PI's: Chiafu Yu(AS), Yifan Yang (NCTU), Chieh-Yu Chang (NTHU), Ming-Lun Hsieh (NTU) and Meng-Kiat Chuat (NTHU). In addition, our group includes eight postdocs Pin-Chi Hung, Kenichi Namikawa (assistant professor in Tokyo Denki University since April 2015), Tse-Chung Yang, Jia-Wei Guo, Mounir Hajli, Jia-Rei Fei, Yong-Jie Wang, and Fu-Tsun Wei and two Ph.D. students Yao Cheng and Shi-Yu Chen.

The research topics of this program cover a wide spectrum of number theory. This year we focus on the following four areas:

a. The geometry of Shimura varieties over finite fields and class number formulas of central simple algebras (Yu);
b. Explicit methods for classical modular forms and Shimura curves (Yang);
c. Transcendental number theory in positive characteristic, and functional field analogue of multiple zeta values (Chang);
d. Iwasawa theory and p-adic methods in algebraic number theory and automorphic forms (Hsieh). We have two number theory seminars in Taipei and Hsinchu every Wednesday with local and international speakers.

Ming-Lun Hsieh has participated in the JSPS program "Development of Concentrated Mathematical Center Linking to Wisdom of the Next Generation" led by professor Nobuo Tsuzuk in Tohoku University. In the future, we expect to have substantial cooperation with the number theory group including Kobayashi Shinichi and Masataka Chida in the mathematics department of Tohoku U. through this program or the "research pair" supported by NCTS.

Fostering the young generation of Taiwanese number theorists is also an important task in the program. To this end, Chiafu Yu gives two semester courses "Abelian Varieties and Related Topics" and "Representation Theory of Finite Groups of Lie

Type" during this fall. He will also organize "NCTS Winter School on Shimura varieties" during December 17-31, 2015.

Representation theory is the study of realizations of symmetries in mathematics and nature in general. Its main goal is to understand and give explicit realizations of in general abstract mathematical objects in terms of more concrete linear transformations on vector spaces. The focus group on representation theory investigates the representation theory of groups, Lie algebra, Lie superalgebras and vertex operator algebras. Research topics include the representation theory of finite groups and p-adic groups, the investigations of the relationship between the representation theories of classical Lie algebras and Lie superalgebras, the structure theory of vertex operator algebras and their relationship with certain sporadic simple groups and various quantum algebras such as Yangians and W-algebras. Although representation theory has been one of the focus programs at NCTS since its inception in 1997, the number of mathematicians working in related areas is still rather limited. Therefore, substantially more efforts were made in training students in this direction over the last decade. Due to this effort, we have sent many graduate students aboard. Most of them have just recently earned or are about to earn their Ph.D. degree. Some of them have already come back and have found tenure-track positions in national universities. We will continue this strategy. In 2015, we had regular representation theory seminar in Taipei every Friday. There were also seminar and colloquium talks from times to time. In March 2015, we organized an international workshop on finite groups and vertex operator algebras at Taitung. There were about 35 participants and about 20 of them were from US, Japan and China that included several students and postdocs from Taiwan, China and Japan. This was the fourth international workshop on finite groups and vertex operator algebras that we organized in the East Taiwan. We believe that it will be very important for establishing a long-term research relationship between the researchers in Taiwan and other countries in Asia. We plan to make this event an annual event. In January 2016, a conference in representation theory will be held in Academia Sinica, Taipei. This conference is dedicated to Geogre Luzstig on the ocassion of his 70th birthday. Many leading experts in representation theory will attend the conference.

2. Research Highlights and Reports

Recently our number theory group has obtained important breakthroughs in Iwasawa theory and multiple zeta values in positive characteristic.

a. Iwasawa theory

Iwasawa theory is a systematic p-adic method to study the connection between arithmetic and analytic objects associated to Galois representations: Selmer groups and L-functions. It was created by Kenichi Iwasawa (1959) in his study on the relationship between ideal class numbers and special values of zeta functions for the cyclotomic $\mathbb{Z}_p$-extension of the rational number field. Later this theory was further explored by Mazur (1972) to study rational points of abelian varieties in towers of number fields and eventually it was generalized by Greenberg (1994) to investigate Selmer groups and L-functions attached to p-adic families of Galois representations. With the development for more than five decades, Iwasawa theory becomes one of the most active areas in modern number theory, and its central problem "Iwasawa main conjectures" has proved to be the most powerful tool to attack various important problems in number theory such as the Bloch-Kato conjecture and the Birch and Swinnerton-Dyer conjecture. The latter is one of seven Millennium prize problems proposed by Clay Mathematical Institute in 2000.

Roughly speaking, Iwasawa main conjectures assert the equality between the size of a p-adic families of Selmer group and its related p-adic L-functions. Coates-Wiles (1977) and Rubin (1992) gave the first proved example of the BSD conjecture in the world by proving Iwasawa main conjecture for imaginary quadratic fields, and works of Kato (2002) and Skinner-Urban (2014) provide otherexamples of BSD conjecture via proving Iwasawa main conjecture for elliptic curves in many cases.

One focus in the number theory program at NCTS is Iwasawa theory, and one of the important results we obtained is a one-sided divisibility towards Iwasawa main conjecture for CM fields as well as p-adic Birch and Swinnerton-Dyer conjecture for CM elliptic curves over totally real fields. People had no idea how to study Iwasawa theory for general CM fields and CM abelian varieties since Rubin proved the Iwasawa main conjecture for imaginary quadratic fields and CM elliptic curves over the rational number field in 1991. The main difficulty has been that the method of Rubin crucially relies on special units in ray class fields of imaginary quadratic fields whose existence is one of the major open conjectures in algebraic number theory. Our new approach is an extensive study of congruence between Eisenstein series and cusp forms on the quasi-split unitary group $U(2,1)$ of degree three over totally real fields combined with the existence of Galois representations attached to automorphic representations of $U(2,1)$ due to Rogawski et al. This work was published in Journal of the AMS [1] and represents a significant advance in Iwasawa theory for CM fields.

Besides works on the man conjectures for CM fields, we successfully extend the work of Bertiloni-Darmon on the anticylotomic main conjecture for elliptic curves to elliptic modular forms. We construct anticyclotomic p-adic L-functions for modular forms and prove the vanishing of its mu-iinvariants [3]; we prove one-sided divisibility in the anticyclotomic main conjectures for modular forms [2]. In addition, in [4] we construct Euler system for generalized Heegner cycles and prove the corresponding Perrin-Riou's explicit reciprocity law, with which we obtain some new examples of Bloch-Kato conjectures under some p-ordinary hypothesis. These works were outcomes of our international cooperation with researchers from Japan and USA during their stay in NCTS.

b. Multiple zeta values in positive characteristic

Multiple zeta functions were first introduced by Euler, and the study on multiple zeta values (MVZ, special values of multiple zeta functions) recently becomes a very hot topic in number theory. A focus in NCTS is the functional analogue of MVZ, and we obtained leading progress along this direction. In [5], we establish a function field analogue of the Goncharov's conjecture for classical multiple zeta values, abbreviated as MZV's. Precisely, we show that the function field MZV's form a graded algebra (graded by weights) defined over the base rational function field. In [6], we consider the positive characteristic analog of the question for classical MZV's: is there any criterion to determine when a given MZV is Eulerian? This question in the classical setting is only known by Brown, who provides a sufficient condition. In the paper [6], we give an effective criterion for the analogous problem in the function field case. In the classical theory of MZV's, Zagier conjectured an explicit dimension formula for space of the double zeta values in terms of the dimension of elliptic cusp forms for the full modular group. Concerning Zagier's conjecture, the best known result due to Gangl, Kaneko and Zagier is that the conjectural dimension provides an upper bound for the space in question, and the dimension is unknown for weight greater than 4. In [7], we establish an effective criterion/algorithm to compute the dimension of the space of double zeta values in positive characteristic and shows that the na?ve analogue of Zagier's conjecture is not valid in the function field setting. The study of [8] is inspired by the conjectural correspondence between the Eulerian single zeta values at positive integers and the vanishing of the p-adic Kubota-Leopoldt p-adic zeta values at positive integers (note that one direction is well-known). In [8], we consider the Carlitz multiple polylogarithms, abbreviated as CMPLs, that were introduced in [5]. The major result in [8] is to show that the v-adic vanishing of CMPLs at algebraic points is equivalent to its ∞-adic counterpart being Eulerian. This establishes a nontrivial connection between v-adic and ∞-adic worlds.

c. Works on class numbers, abelian varieties and Shimura curves

Computation of class numbers explicit has been an important problem in Number Theory. The computation of class numbers of definite central simple algebras A over global fields F is of particular interesting because this also gives the dimension of the space of certain automorphic forms. In [9]. we completely solve the problem of computing class numbers for an arbitrary definite central simple algebra over an arbitrary function field.

The Honda-Tate theory gives a beautiful and explicit enumerate for isogeny classes of abelian varieties over finite fields. The integral theory was then investigated by Waterhouse and other authors. In [10], we continue to complete the explicit enumerate for the isogeny class corresponding to $\sqrt{p}$, the most unaccessible case over the prime field, which complements work of Waterhouse. We also develop all tools for calculating the number of abelian surfaces in this isogeny class-including the generalized Eichler class number formula and the optimal embedding formula.

We report briefly works on explicit methods for Shimura curves. By realizing modular forms on Shimura curves in two way, firstly in terms of solutions of Schwarzian differential equations and secondly in terms of Borcherds forms, we obtain special-value formulas for hypergeometric functions, where the values of certain hypergeometric functions are shown to be explicitly known algebraic multiples of the Chowla-Selberg periods. Then by constructing suitable Borcherds forms and using Schofer's formulas, we completely determine the equations of all hyperelliptic Shimura curves.

d. Vertex operator algebras

In the recent years, the research team lead by C. H. Lam has made important contributions towards the classification of holomorphic vertex operator algebras with $c = 24$. Schellekens originally conjectured that there are just 71 holomorphic VOAs of central charge 24, and gave extensive computational data to back his claims. He also asserted that this result has applications to the heterotic string. However, only 39 of the 71 proposed theories had been constructed in that time. In the last 20 years, Schellekens' conjecture has become one of the main conjectures in VOA theory, for at least two good reasons: (a) there are very few good construction techniques for rational VOAs, and Schellekens' problem is an interesting yardstick by which progress in this direction can be measured, and (b) the parallel result for self-dual positive-definite lattices has had an enormous influence on areas such as algebraic combinatorics, lattice-theory, and various moonshine theories. Therefore, one would expect that knowing the complete list of holomorphic $c = 24$ VOA's will - in the long run - have an even greater impact.

Now many VOAs in Schellekens' list have been constructed explicitly. In around 2010, a special class of holomorphic VOAs, called framed VOAs, of central charge 24 were studied by H. Shimakura, H. Yamauchi and C.H. Lam. A complete classification of holomorphic framed VOAs of $c = 24$ has also been achieved by H. Shimakura and C.H. Lam. In particular, they showed that there exist exactly 56 holomorphic framed VOAs of central charge 24 and they are uniquely determined by the Lie algebra structures of their weight one subspaces under the assumption that they are framed. On the other hand, a $\mathbb{Z}_3$-orbifold theory associated to lattice VOAs has been developed by Miyamoto in 2013. As an application, a holomorphic VOA whose weight one subspace has the Lie algebra structure $E_{6,3}G_{2,1}^3$ was constructed. By using the similar methods, two other holomorphic VOAs have been constructed by H. Shimakura and T. Sagaki. Recently, van Ekeren, Moller and Scheithauer announced that they have obtained a mathematically rigorous proof for Schellekens' list using modular invariant of trace functions and the theory of Jacobi forms. They also established the $\mathbb{Z}_n$-orbifold construction for lattice VOAs for general elements of arbitrary orders using an unpublished result of Miyamoto. As an application, they also claimed to have the constructions of several new VOAs in Schellenkens' list. In 2015, H. Shimakura and C.H. Lam developed a new construction method using Li's Delta operator and inner automorphisms. The technique itself was developed by Dong-Li-Mason nearly 20 years ago (using simple currents and inner automorphisms), but these ideas were never applied to substantial construction problems before. Using this technique, Shimakura and Lam are able to construct 6 new examples of holomorphic VOAs of $c = 24$. Combining all the known results, 70 of the 71 theories in Schellekens' list have been constructed now. The same technique using inner automorphisms and the Delta operator also provide some new insights for proving the uniqueness of the VOA structures. In fact, Shimakura and Lam are able to prove that the VOA structures of several holomorphic VOAs of $c = 24$ are uniquely determined by the Lie algebra structures of their weight one subspaces. We expect that these techniques would apply to more complicated situations.

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B. Algebraic Geometry

Program B.	Algebraic Geometry
Coordinator	Eugene Xia, Jiun-Cheng Chen, Wu-Yen Chuang
Core members	Jungkai Chen, Jeng-Daw Yu, Chen-Yu Chi, Ching-Jui Lai, Shin-Yao Jow, Wan Keng Cheong, Jia-Ming Liou

1. Overview of the Program

The research group of algebraic geometry in Taiwan Has obtained certain international reputation thanks to the consistent support of NCTS. Through the strategic planing of seminars courses and workshops, for undergraduate postdocs to reseachers, the size of the group grows notably in recent years. Therefore, the main goal of the Topical Program in Algebraic Geometry is to achieve world leading advances and to cultivate the young generation as well.

In the year of 2015, our focus is on, but on restricted to, the following three topics:

a. Minimal model program and related topics;

b. Derived categories and homological method in algebraic geometry;

c. Algebraic geometry in positive characteristics.

In fact, Taiwan has strong research group in higher dimensional birational geometry. Almost all renown world experts in the field of birational geoemtry visited NCTS for some period. Not only this, we also attracted world leading experts to visit NCTS for a longer period, for example, Yujiro Kawamata, Caucher Birkar, and Paolo Cascini. Together with our existing research groups, NCTS becomes an tempting places for younger people in algebraic geoemtry. We attracted three new international postdocs this academic year: Zhengyu Hu from Cambridge, Mario Chan from KIAS and Dang Hiep from Kaiserlautern together with another two existing Japanese postdocs: Furakawa and Miyatani. With these international young scholars, our weekly seminars run in English.

Due to the increasing popularity of researchers in NCKU and NTHU, we started to run our regular seminar rotationally in the pattern that twice in Taipei, once in

Hsinchu and once in Tainan during every 4 times. This formation and formulation of seminar brought great impact to our younger students.

2. Highlights

The highlights of the program consists of three parts: first, the important breakthrough on explicit birational geometry in dimension three; second, the studies on derived categories on special varieties; and third, the important activities of this year.

a. Explicit birational geometry in dimension three

Minimal model program is one of the main topic in algebraic geoemtry since 1980s. In order to study an algebaic variety, it is very tempting to find a good representative in a birational equivalent class and study the relative its geometry. The beautiful theory was established mainly due to Reid, Shokurov, Kawamata, Kollar, Mori and others. Moris proof of the existence of minimal model for threefolds in the 1990 was a milestone. The recent breakthrough, by Birkar-Cascini-Hacon-McKernan, shows that the minimal model exists under some mild conditions. In general higher dimensions, the main difficulty is to show the existence of flips, termination of flips and abundance conjecture.

However, even though minimal model program was proved by Mori for more than two decades. Not so much of the explicit understanding of 3-dimensional MMP was known. The elementary maps appeared in MMP are divisorial contractions, flips and flops. Divisorial contractions are higher dimensional analogue of blowdowns, while flips are considered to be more subtle maps.

The research team led by Jungkai Chen, together with Jheng-Jie Chen, Shin-Gu Chen, Ching-Jui Lai is one the leading team in the study of geometry of threefolds, especially in the problem related to three dimensional minimal model programs. The previous work of Jungkai Chen provides a decomposition of flips and divisorial contraction to curve into a set of explicit elementary maps, such as blowups and weighted blowups. Let $f\colon Y \to X$ be a divisorial contraction, or a flip in dimension three. There exists a sequence

$$Y = X_n \to X_{n-1} \to \ldots \to X_1 \to X_0 = X$$

such that each map $X_i \to X_{i-1}$ is one of the following explicit maps: a flop,a blowup along a smooth curve, a divisorial contraction to a point, or a weighted blowup over a point. By using the above decomposition, one can similarly obtain elimination of indeterminancies explicitly. As a consequence, it is proved by Chen and his collaboration Meng Chen of Fudan University that three dimensional version of Noether Inequality holds. In other words, let X be a minimal threefold of general type. Then the canonical volume $K^3 \geq 4/3p_g - 10/3$.

Together with the Bogomolov-Miyaoka-Yau inequality, This inequality also leads to the foundation of the study of geography of threefolds of general type. By geography, one means the distribution of invariants and relations of invariants. It is expected that explicit birational geometry of threefolds will be an indispensible part toward the geometry of threefolds.

The goal is to build up complete classification theory parallel to that of surfaces. A better understanding of threefolds explicitly will be helpful to general higher dimensional varieties

b. Derived categories on special varieties

Derived category draws lots of attention in recent years. It was first introduced as an algebraic foundations of algebraic geometry. The milestone was Mukai's invention of Fourier-Mukai transfor, which build up an fantastic equivalence between derived categories of abelian and its dual abelian varieties. This reveals the hidden symmetry for abelian varieties. Later it was extended to K3 surface and Calabi-Yau threefolds. Many more interesting results was discovered thanks to the work of Bridgeland. Not only he found some moduli interpretation of the Fourier-Mukai transform, but also he built up the theory of stability on the derived categories.

The work of Wu-Yen Chuang was around this circle. He considered elliptic fibrations with arbitrary base dimensions. In particular, some stability conditions on threefolds were carefully studied. More explicitly, he gave a criterion under which certain 2-term polynomial semistable complexes are mapped to torsion-free semistable sheaves under a Fourier-Mukai transform. As an application, one can construct an open immersion from a moduli of complexes to a moduli of Gieseker stable sheaves on higher dimensional elliptic fibrations.

Another important feature is to apply the derived categoty to the study to varieties related to abelian varieties. This was mainly the joint work of Jungkai Chen and Jiang, follow the previous joint work of Chen and Hacon.

A smooth projective variety X is said to be a variety of maximal Albanese dimension if there exists a generically finite morphism $f\colon X \to A$ to an abelian variety. First of all, the sheaf $f_*\omega_X$ is studied. It is known that the birational geometry of X is very much governed by the positivity of $f_*\omega_X$ and the sheaf $f_*\omega_X$ is known to be a GV-sheaf but is not necessary M-regular. The joint work of Chen and Jiang proved a general decomposition theorem for $f_*\omega_X$, which implies that $f_*\omega_X$ is not far from being M-regular.

In fact, Kollár proved another decomposition theorem for higher direct images in general in 1985. The new results shows that it is possible to characterize each components explicitly by cohomological properties for varieties with maximal Albanese dimension. One might expect that both of the decomposition theorems are related to Hodge theory. It will be an interesting topic for further investigation.

This decomposition theorem and its variant can be applied to prove globally generated properties for canonical or pluricanonical bundles. For instance, we have the following

Let $f : X \to A$ be a generically finite morphism to an abelian variety. Then ω_X^2 is globally generated away from the exceptional locus of f. Moreover, if X is of general type, then the bicanonical map of X is generically finite onto its image.

Recall that for a polarized abelian variety (A, H), $2H$ is globally generated and $3H$ is very ample. The above results are the birational analogues for canonical bundles of varieties of maximal Albanese dimension.

It is also applicable to the study the structure of smooth projective varieties X of general type and of maximal Albanese dimension with $\chi(X, \omega_X) = 0$. In recent years, these varieties have attracted considerable attention. Green and Lazarsfeld showed that a variety of maximal Albanese dimension satisfies $\chi(X, \omega_X) \geq 0$. It was conjectured by Kollár that a variety of general type and maximal Albanese dimension would satisfy $\chi(X, \omega_X) > 0$. A couple of years later, Ein-Lazarsfeld disproved the conjecture by providing an example of threefold of general type and maximal Albanese dimension with $\chi(X, \omega_X) = 0$. It is thus natural and important to characterize or classify varieties of general type and maximal Albanese dimension with $\chi(X, \omega_X) = 0$. The previous joint work of Chen, Jiang and Debarre was the starting point toward this direction. In which the case that the Albanese variety of X has at least three simple factors are handled. It is shown that the example of Ein and Lazarsfeld is the only possible variety in dimension three. Note that the characterizing properties of X are preserved under birational maps and finite étale maps, the classification of X would be up to birational maps and étale covers.

In fact, some birational criteria for morphisms between varieties of maximal Albanese dimension is essential. Then, these criteria surprisingly give strong restraints on the structure of X. In particular, when that A_X has only three simple factors, one can prove that the example of type of Ein-Lazarsfeld is essentially the only possible variety.

c. Most important activities

(1) Winter School in Algebraic Geometry (Feb. 9-11), organized by Jiun-Cheng Chen, Jungkai Chen, Chen-Yu Chi and Shin-Yao Jow

The purpose of the winter school is to introduce several important aspect in algebraic geometry for students and young researchers. The materials are chosen and designed for those who know the basic language (e.g. Ch. 1 of Hartshorne, divisors, ample line bundles and sheaf cohomology, etc) and are interested in knowing more toward the recent research topics in algebraic geometry.

It consists of the following three parts.

- Analytic method
  A survey of Hormander's L^2 estimates, Nadel's coherence theorem, and Nadel's vanishing theorem.
- Birational geometry
  A quick survey and overview of the MMP and prove some basic theorems, including: KawamataViehweg vanishing Theorem, nonvanishing Theorem, base point free theorem and the cone Theorem. Keel's base point free theorem in positive characteristic was also introduced.
- Moduli spaces
  A introduction of moduli spaces and some classical examples such as the Hilbert scheme and the moduli of curves. Effort was made to convey ideas rather than to present technicalities.

(2) Mini-conference on Algebraic Geometry (Mar. 6), organized by Jiun-Cheng Chen, Wu-Yen Chuang, Furukawa
The was organized in the occasion of a visit of a research group from Waseda University.

(3) Workshop Higher Dimenaional Algebraic Geometry (Aug. 19-23), organized by Jungkai Chen and Kawamata
This series of workshop was started in the year 2004, 2007, 2010, 2013. This is the fifth one belong to the series. Each time, we invited world leading experts and young prospects to talk about the most recent advances in higher dimensional algebraic geometry. There are 22 invited speakers.

(4) NCTS Mini-Course in Algebraic Geometry–Linear systems on algebraic varieties (May 8-19), lectured by Caucher Birkar
Many fundamental properties of a projective algebraic variety X are determined by linear systems $|D|$ of divisors D. Of particular interest are linear systems related to the canonical bundle K_X. The birationality of pluricanonical system and the so-called Iitaka fibration of X were discussed. Numerous tools and techniques of modern birational geometry naturally show up in the lectures.

C. Differential Geometry and Geometric Analysis

Program C.	Differential Geometry and Geometric Analysis
Coordinator	Mao-Pei Tsui, Nan-Kuo Ho, Chun-Chi Lin, Hong-Lin Chiu, River Chiang
Core members	Chung-Jun Tsai, Ziming Nikolas Ma, Suh-Cheng Chang, Yng-Ing Lee, Jih-Hsin Cheng, Siye Wu, Chiung-Jue Sung, Dong-Ho Tsai, Mei-Lin Yau, Rung-Tzung Huang, Kwok-Kun Kwong, Ching-Tung Wu

1. Overview of the Program

The main goal of the differential geometry and geometric analysis group at NCTS is to create an excellent research environment for Taiwanese differential geometers to work on world class research topics and provide necessary resources to conduct international research cooperation. We are hoping that our activities will enhance international visibility of differential geometers in Taiwan. The research activities in the program are currently organized by five PI's. In addition, our group includes several members: Chung-Jun Tsai, Ziming Nikolas Ma, Suh-Cheng Chang, Yng-Ing Lee (NTU), Jih-Hsin Cheng (Academia Sinica), Siye Wu, Chiung-Jue Sung, Dong-Ho Tsai (NTHU), Mei-Lin Yau, Rung-Tzung Huang (NCU), Kwok-Kun Kwong(NCKU), Ching-Tung Wu (NPU). We also have a group of very active postdoc: Kuo-Wei Lee, Chih-Wei Chen, Sung-Hong Min (NTU), Yang-Kai Lue (NTNU), Yen-Chang Huang, Sin-Hua Lai (NCU), Ting-Huei Chang, Chung-Yi Ho, Chi-Kwong Fok (NTHU), Ching-Hao Chang(AS), Chih-Chung Liu (NCKU). The research topics of this program cover a wide spectrum of in differentia geometry. This year we focus on the following four areas:

a. Geometric evolution equations (Tsui and Lin)
b. Mathematical general relativity (Tsui);
c. Symplectic geometry and contact topology (Chiang, Ho and C-J. Tsai);
d. Geometry of CR manifolds (Chiu and Cheng);

The highlights of this year's activities including:

a. Weekly differential geometry seminar at NTU, Sinica - NCTS Geometry Seminar at Academia Sinica and NCTS learning seminar on special holonomy
b. Two Taiwan Geometry Symposium (May 2, 2015 at NCKU and Oct 24-25, 2015 at NDHU)
c. Special Day on Symplectic Geometry and Geometric Evolution Equations (June 22, 2015) and Special Day on the mathematics of relativity (December 28, 2015)
d. NCTS Summer School on Kerr Geometry (August 17 - 21, 2015)
e. One Day Workshop for Young Differential Geometers (December 13, 2015)
f. 2015 NCTS Workshop on Subelliptic Operators and Singular Analysis (June 24-25, 2015)

Many differential geometers have visited NCTS including Mu-Tao Wang (Columbia University), Der-Chen Chang (Georgetown University), Ernst Kuwert(Albert-Ludwigs-Universität Freiburg, Germany), Stephan Luckhaus (University of Leipzig, Germany), Anna Dall'Acqua (Universität Ulm, Germany), Po-Ning Chen (Columbia University), Ye-Kai Wang (Michigan State University). We have also sponsored students and postdocs to attend international conferences and workshops.

Our activities and visitors have broaden and strengthen the research topics in our group. This will lead to a steady growth of the differential geometry and geometric analysis group.

2. Highlights

In the following, we describe some important breakthroughs in our group

a. Geometric Evolution Equations and Related Topics

(1) Generalized Lagrangian mean curvature flows in cotangent bundle

The study of Lagrangian submanifolds have been an important research topics. One particular example is the study of Generalized Lagrangian mean curvature flows in cotangent bundle. It is motivated to study the following conjecture.
Strong Arnold conjecture: Let Σ be a compact, exact, orientable embedded Lagrangian in T^*M, where M is also compact and orientable. Can Σ be deformed through exact Lagrangian to the zero-section?
in [**?**], Fukaya, Seidel and Smith proved that compact, exact, orientable embedded Lagrangian submanifolds are Floer-cohomologically indistinguishable from the zero-section in cotangent bundle under suitable conditions (triviality of the fundamental group of the cotangent bundle, and of the Maslov class and second StiefelVWhitney class of the Lagrangian submanifold). Another possible approach to investigate the Strong Arnold conjecture is to use the Lagrangian mean curvature flows in cotangent bundle to deform a exact Lagrangian submanifold to zero section in the cotangent bundle of a manifold. In fact, the Generalized Lagrangian mean curvature flows can be defined for an almost Kähler manifold with an Einstein connection. Recall that an almost Kähler structure on a symplectic manifold (N, ω) consists of a Riemannian metric g and an almost complex structure J such that the symplectic form ω satisfies $\omega(\cdot, \cdot) = g(J(\cdot), \cdot)$. Any symplectic manifold admits an almost Kähler structure and we refer to (N, ω, g, J) as an almost Kähler manifold. In [3], Smoczyk and Wang propose a natural evolution equation to investigate the deformation of Lagrangian submanifolds in almost Kähler manifolds. A metric and complex connection $\widehat{\nabla}$ on TN defines a generalized mean curvature vector field along any Lagrangian submanifold M of N. They study the evolution of M along this vector field, which turns out to be a Lagrangian deformation, as long as the connection $\widehat{\nabla}$ satisfies an Einstein condition. This can be viewed as a generalization of the classical Lagrangian mean curvature flow in Kähler-Einstein manifolds where the connection $\widehat{\nabla}$ is the Levi-Civita connection of g. Their result applies to the important case of Lagrangian submanifolds in a cotangent bundle equipped with the canonical almost Kähler structure.
In a joint project with Smoczyk and Wang [1], Tsui shows that the canon-

ical connection on the cotangent bundle of any Riemannian manifold is an Einstein connection (in fact, Ricci flat). The generalized mean curvature vector on any Lagrangian submanifold is related to the Lagrangian angle defined by the phase of a parallel n form, just like the Calabi-Yau case. They also show that the corresponding Lagrangian mean curvature flow in cotangent bundles preserves the exactness and the zero Maslov class conditions. The torsion of the Einstein connection has made the generalized Lagrangian mean curvature much more nonlinear than the Lagrangian mean curvature flow in a K*ä*hler-Einstein manifold. For example, the second fundamental form is no longer totally symmetric and the evolution equations of geometric quantities are also more nonlinear. Moreover, they utilize the technique from [4] to prove a long time existence and convergence result to demonstrate the stability of the zero section in the case of the cotangent bundle of the sphere. However, the general behavior of the generalized Lagrangian mean curvature flow is still remained unexplored. The most challenging aspect of this project is to understand the formation of singularities of generalized Lagrangian mean curvature flow. It is very important to understand the structure of singularities. Unfortunately, there are very few results in this direction.

(2) Higher-order variational problems and the related gradient flows
One of the simplest mathematical settings in elastic mechanics is the so-called Euler-Kirchhoff theory. The other example is the mechanics of ribbons and Möbius strips, which recently has attracted more and more attention. These simple mechanical models providing challenging mathematical problems, e.g., in the calculus of variations and related dynamical theory. One of the simpler dynamical theory is gradient flows motivated from geometric variational functionals. They are often called geometric flows and can also be viewed as over-damped dynamics of mechanical objects, when the geometric objects are related to mechanics.
Geometric flows for open curves associated with energy functional of higher-order derivatives have appeared in various research topics, for examples, higher-order variational problems in differential geometry and geometric control theory; interpolation problems of curves in computer-aided geometric design; mechanical modeling of polymers (e.g., mechanical modeling of DNA molecules [2] and filaments in biological cells). These functionals are often related to certain Sobolev norms of the first-order (e.g., stretching energy) and second-order derivatives (e.g., bending energy) of curves. The long-term existence of solutions of the so-called curve-straightening flow, using min-max method, has been established in the literature on curves in a Riemannian manifold). Their min-max method works for the case of either

open or closed curves. On the contrary, in the parabolic PDE approach, the long-time existence of smooth solutions has also been established for closed curves. There are however very few papers discussing the case of open curves in the PDE setting until recent years.

C.C. Lin's current research mainly focuses on higher-order variational problems, the related gradient flows, and their applications. There are many interesting problems motivating higher-order problems, e.g. Willmore-type functionals in differential geometry or biomechanics, thin films in applied sciences, and Hawking energy functional in general relativity. At this moment, he focus on the topic of gradient flows for one-dimensional elastic curves with various boundary conditions. With boundary conditions or obstacle (at interior points), new challenges in mathematical analysis have been found, even in the one-dimensional case. Besides, he also starts working on the higher-dimensional cases, in which singularities often appear and thus very different from the story of one-dimensional case. His recent achievement [**?**, **?**] is to derive the existence of smooth solutions for gradient flows of elastic curves with clamped boundary conditions and fixed total length. He also developed new tricks to tackle this case, which couldnt be resolved by the techniques that was used in previous papers. The main machinery is to apply energy estimates and interpolation inequalities, together with estimating the right terms which match well the boundary conditions and constraints from fixing total length. Without finding the proper terms to estimate, standard interpolation inequalities couldnt be directly applied, because there are many terms reaching the borderline cases. With the new result, it is very hopeful to extend these results to resolve more general boundary conditions in higher-order flows.

(3) Ricci flow, CR Yamabe flow and CR Yamabe soliton

There are several important breakthroughs in the study of the Ricci flow.

(a) O. Munteanu and J.P. Wang recenly have done a series of important works about Ricci solitons. In particular, they solved the conjecture "Gradient shrinking solitons with positive curvature are compact" this year. Their results provides rather flourish enumerates on the functional theory on Ricci solitons (especially for the shrinking case).

(b) The classification of solitons by their asymptotic behavior has been an important problem since S. Brendle solved Perelman's conjecture in 2012. In particular, B. Kotschwar and L. Wang solve the uniqueness problem for shrinking solitons by using a pde-theoretical method, which is different from Brendle's blow-down argument. Precisely, they showed that two shrinking solitons asymptotic to the same cone must be identically the same. On the other hand, although there had been a related result due to H.-D. Cao and X. Cui, the uniqueness problem is not yet solved for steady or shrinking Kähler-Ricci soliton because the convergence of complex structure is not easy to handle. We note that the expanding case is similar to Brendle's original situation and has been solved by O. Chodosh (real) and Chodosh-T.-H. Feng (Kähler).

One of the main theme in the study of the Ricci flow is to establish curvature estimates. It has been known that curvature operator Rm remains bounded along the flow whenever the Ricci curvature is bounded. However, we know very little about how the bound depends on the Ricci curvature. Recently, Chih-Wei Chen showed that this bound depends only on the bound of Ric and the lower bound of injectivity radius. Precisely, he derived uniform $C^{0,\alpha}$-bounds for ∇Ric and Rm for all Ricci flows with $|Ric_{g(t)}| \le K$ and $inj_{g(t)} \ge \delta > 0$. An immediate application of his result is the improvement of the regularity of classical compactness theorems. His result shows that, via the Ricci flow, $g(t)$ is $C^{2,\alpha}$-bounded whenever $|Ric_{g(t)}| \le K$ and $inj_{g(t)} \ge \delta > 0$.

In the joint work with H.-D. Cao and S.-C. Chang, C-W Chen classified the diffeomorphism types of three dimensional pseudo-gradient CR Yamabe solitons with zero torsion. Moreover, we can show that Reeb orbits are contained in each level set of the potential function φ. Our future project is to classify these solitons into more specific classes.

On the other hand, S.-C. Chang and C-W Chen have found a new entropy for CR Yamabe flow. They plan to study the convergence of the flow via the help of this new entropy.

b. Cohomology theory for symplectic manifolds

The main purpose of this project is to pursue a cohomology theory for symplectic manifolds which is analogous to the Dolbeault theory for complex manifolds. Primitive cohomology theory for symplectic manifolds was introduced by L.-S. Tseng and S.-T. Yau. Besides its own interest in symplectic geometry, they also discovered that the cohomology theory describes the deformation of certain supersymmetric equation.

One of the main goal is understand all the (Kähler) Calabi–Yau threefolds. However, it is known that they can have different homotopy type. In the mid 80s, Clemens and Friedman discovered the (complex) conifold transition which produces a Calabi–Yau threefold by contraction and deformation. Topologically, it replaces a bunch of rational curves by Lagrangian three-spheres. Therefore, the Betti number changes; the resulting Calabi–Yau threefold have less b_2, but more b_3. In 1987, Reid made an interesting proposal. He wanted to make sense of the vast collection of diverse Calabi–Yau threefolds. He speculated that *all* (Kähler) Calabi–Yau threefolds fit into a single universal moduli space in which families of smooth Calabi-Yau threefolds of different homotopy types are connected to one another precisely by the conifold transitions. On the other hand, it is possible that the resulting manifold have zero b_3. Hence, one ends up with a complex, non-Kähler Calabi–Yau threefolds. For instance, one can produce k-connected sum of $S^3 \times S^3$ by conifold transition, with $k \geq 2$.

Our main results in [5] aim to answer the following questions.

(1) The dimensions of primitive cohomologies can vary along the deformation of symplectic forms. Is there an *a priori* bound on their dimensions?

(2) The de Rham cohomologies with the wedge product form a graded ring. Is there a product operation for primitive cohomologies?

We will see that these two questions are related to each other, and there is a nice algebraic structure on the space of primitive forms.

It turns out that the first question is related to the cohomological property of ω, the so-called *Lefschetz map*. The Lefschetz maps are linear maps and so in particular we would like to know their kernels and cokernels. In [5, Theorem 4.2], we discovered that such information is exactly encoded in the filtered cohomologies. As a consequence, it gives *a priori* bounds on the dimensions of primitive cohomologies, and answers question (1). More importantly, it provides a clue about how to define the product structure on $PH^*(M)$.

We now briefly explain the algebraic structures on the space of primitive forms. Similar to the de Rham complex, there is a *graded algebra* $\mathfrak{A} = \bigoplus_{j\geq 0} \mathfrak{A}^j$ for primitive forms. The elements and gradings are suggested by the primitive elliptic complex. With the help of the exact triangle, we successfully define a product operator $\times$ on $\mathfrak{A}$. Due to the presence of the second order operator, the product involves *first order operators*. The product operator almost makes $\mathfrak{A}$ into a differential graded algebra, except that $\times$ is *not associative*. However, the non-associativity can be captured by higher order maps. And it turns out that $\mathfrak{A}$ forms an A_∞-algebra, which is some algebraic structure appears naturally in the study of homotopy theory and loop spaces. Moreover, the non-associativity disappear after taking cohomology, and thus $(PH^*(M), \times)$ is a graded ring. We also found some examples that the ring structure encodes much more information than the cohomology itself.

c. Geometry of CR manifolds

The application of the moving frame method to geometry of hypersurfaces and integral Geometry in Heisenberg group

In Euclidean space, the fundamental theorem of curves states that any unit-speed curve is completely determined by its curvature and torsion. More precisely, given two functions $k(s)$ and $\tau(s)$ with $k(s) > 0$, there exists a unit-speed curve whose curvature and torsion are the functions k and τ, respectively, uniquely up to a Euclidean rigid motion. The main theorems in the paper are to show the analogous theorems of curves and surfaces in the 3-dimensional Heisenberg group H_1.

Heisenberg group H_1 can be regarded as a pseudo-hermitian manifold by considering H_1 with the standard pseudo-hermitian structure (J, Θ). Recall that a pseudo-hermitian transformation on H_1 is a diffeomorphism on H_1 preserving the pseudo-hermitian structure (J, Θ). Denote $PSH(1)$ be the group of all pseudo-hermitian transformations on H_1, and we call the element in $PSH(1)$ a **symmetry**. A symmetry is the Heisenberg rigid motion in H_1 that can be characterized explicitly.

In the joint work with Dr. Sin-Hua Lai and Yen-Chang Huang, H-L Chiu [**?**] study the local equivalence problem of hypersurfaces in the Heisenberg groups via Cartans method of moving frames and Lie groups. For regular part, they find a complete set of invariant to distinguish different hypersurfaces. We also study the integrability condition for hypersurfaces and show some kind of embedding theorems for manifolds satisfying the integrability condition. The goal of this approach is to give some geometric enumerates for the contact topology.

They have also obtained some formulae to express some geometric measures as the expectations of some kind of numbers by moving frame methods. For examples, we have proved the Crofton-type formula and the Poincare-type formula in the integral geometry. Their goal in this approach is to give a complete proof for the isoperimetric inequality in the Heisenberg geometry.

d. CMC foliation conjecture and Constant mean curvature hypersurfaces in spacetimes

The related topics to constant mean curvature (CMC) hypersurfaces in spacetimes are very popular in recent decades. This is because CMC hypersurfaces have been considered as important objects in general relativity such as the analysis on Einstein constraint equations and gauge condition in the Cauchy problem of the Einstein equation. In addition, York suggested the concept of the CMC foliation and the CMC time function on relativistic cosmology, so many theoretical physicists and cosmologists try to find the CMC foliation in a variety of cosmological spacetimes.

There are many beautiful results of CMC hypersurfaces when the ambient space is Minkowski. However, the CMC hypersurfaces are not well-understood in the Schwarzschild spacetime or in its maximal analytic extension, called Kruskal extension. Up to now, we only have results of spacelike, spherically symmetric CMC hypersurfaces in the Schwarzschild spacetime. For nonsymmetric CMC hypersurfaces, there are many interesting and important problems to explore. Hence our future plan in general relativity area will focus on the nonsymmetric CMC hypersurfaces in the Schwarzschild spacetime such as the existence, uniqueness, smoothness, foliation property, etc.

The recent breakthrough in general relativity areas is that many results and properties of spherically symmetric CMC hypersurfaces in the Kruskal extension are proved. More precisely, K-W Lee and Y-I Lee [6, 7] consider the spacelike, spherically symmetric CMC equations with symmetric boundary data. This Dirichlet problem is a second order partial differential equation in the Kruskal coordinates, or a second order ordinary differential equation in the standard Schwarzschild coordinates. The constructions of spacelike, spherically symmetric CMC hypersurfaces in [6, 7] and previous results in [8] can help us prove the existence of the Dirichlet problem. However, the uniqueness part is much difficult.

One interesting thing is that the geometric analysis plays a very important role to overcome the uniqueness problem. Given (M^{n+1}, g) a Lorentzian manifold and a point $x \in M$, they can define the Lorentzian distance function $d_N(x)$ with respect to a spacelike hypersurface N. We can also restrict the distance function on another spacelike hypersurface Σ, called $d_N|_\Sigma$, and it is smooth if each point in Σ is not a focal point of N. Suppose that $q \in \Sigma$ is a maximum point of $d_N|_\Sigma$. They can get the estimates of $\Delta d_N|_\Sigma(q)$ in terms of the mean curvature of N and Σ, and this estimate can help them prove the uniqueness of the Dirichlet problem.

This is a very interesting result because general relativity, geometric analysis, and partial differential equation are closely connected in this topic. Moreover, as an application, the uniqueness and existence of the Dirichlet problem are equivalent to the existence of CMC foliaton in the Kruskal extension. Hence they have completely proved the CMC foliation conjecture by Malec and Ó Murchadha in [8].

Recently, Schinkel, Macedo, and Ansorg [9] considered the spacelike, axisymmetric CMC hypersurfaces in the Kerr spacetime. This numerical discussion would help us know more the non-symmetric CMC hypersurfaces in the Schwarzschild spacetime because Schwarzschild spacetime is a special case of Kerr spacetime.

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D. Differential Equations and Stochastic Analysis

Program D.	Differential Equations and Stochastic Analysis
Coordinator	Jenn-Nan Wang, Jung-Chao Ban, Chao-Nien Chen, Yuan-Chung Sheu
Core members	Chiun-Chuan Chen, Kuo-Chang Chen, Jong-Sheng Guo, Cheng-Hsiung Hsu, Sze-Bi Hsu, Ming-Chia Li, Ching-Lung Lin, Song-Sun Lin, Gi-Ren Liu, , Chih-Wen Shih, Je-Chiang Tsai, Kung-Chien Wu, Shuenn-Jyi Sheu, Chii-Ruey Hwang, Lung-Chi Chen, Shang-Yuan Shiu, Li-Hsien Sun, Guan-Yu Chen

1. Overview of the Program

In this program, we focus on the interplay between differential equations and probability. Besides of the interdisciplinary studies, we also put our efforts on improving research achievements in each field. In the subprogram of differential equations, we mainly concentrate on partial differential equations, mathematical biology, and dynamical systems. For the subprogram on PDE, we focus on the kinetic theory, fluid equations, nonlinear Schrodinger equations, inverse problems, etc.

The field of mathematical biology is closely related to PDE. There are many interesting problems arising in studying new phenomena observed in biology and ecology. The program of mathematical biology is to promote interdisciplinary research through the communication between mathematicians, biologists, statisticians, physicists and computational scientists.

Dynamical System is one of the major and classical research areas in mathematics and mathematical science. Various activities supported by NCTS are organized to widen the spectrum of research groups on dynamical systems in Taiwan. The aim of these programs is to promote international collaborations and to provide training for young scholars and students.

For the subprogram in the stochastic analysis, we plan to investigate Markov chain mixing: The Markov chain Monte Carlo (briefly, MCMC) method is a well-designed algorithm in sampling probability measures on discrete sets. When the MCMC method is simulated, it is important to select a (deterministic or random) time, say T, to stop the algorithm for sampling. Theoretically, the stopping time T can be the mixing time or the coupling time but none of them is easy to achieve. The cutoff phenomenon of Markov chains was introduced by Aldous and Diaconis in early 1980s in order to catch up the observation that the distribution of Markov chain is far from its stationarity before a time S and, after a relatively short period compared with S, the distribution turns out to be almost the limiting distribution. When a cutoff exists in a MCMC algorithm, the time S can be a good candidate for the stopping time of algorithm. The MCMC method arises in many disciplines including the statistic physics, computer science, molecular biology, mathematical finance and more. From the viewpoint of interdisciplinary research, the underlying machinery can be very complicated and a quick formula on the stopping time T and the cutoff time S will be very challenging but highly expected.

2. Highlights

Highlight of the most significant research results:

a.(1) Pointwise behavior of the linearized kinetic equation (Kung-Chien Wu). We study the pointwise behavior of the linearized Boltzmann equation on torus for non-smooth initial perturbations. The result reveals both the fluid and kinetic aspects of this model. The Mixture Lemma plays an important role in constructing the kinetic-like waves, and we supply a new proof of this lemma without using the explicit solution of the damped transport equations. We apply similar ideas to the linearized Fokker Planck Boltzmann model, the abstract way to proof this Mixture Lemma is necessary in this case.

(2) Nonlinear stability of the 3D Boltzmann equation (Kung-Chien Wu). We study the nonlinear stability of the Boltzmann equation in the 3-dimensional periodic box, which is based on the previous result. The initial perturbation is not necessary smooth. The convergence rate is algebraic for small time region and exponential for large time region. Moreover, the algebraic rate is optimal and the exponential rate depends on the size of the domain (Knudsen number).

b.(1) Higher dimensional and tree shifts of finite type (TSFTs). Shift space is a powerful tool to describe the solution structures in physical systems and lattice dynamical systems. While the dynamical behavior of one-dimensional shifts of finite type is explicitly characterized, the properties of multidimensional ones are barely known. J.-C Ban, W.-G Hu and S.-S Lin provided power methods and theory to characterize the mixing properties of higher-dimensional SFTs. J.-C. Ban and C.-H Chang provided the analogous results on TSFTs. The entropy theory on TSFTs is also established by J.-C Ban and C.-H Chang recently. Based on our knowledge, these results are new and useful.

(2) Entire solutions for delayed monostable epidemic models. S.-L Wu and C.-H Hsu studied the existence of entire solutions for delayed monostable epidemic models with and without the quasi-monotone condition. They established the comparison principle and construct appropriate sub-solutions and upper estimates in the quasi-monotone case. And some new types of entire solutions are constructed by using the traveling wave fronts and spatially independent solutions of two auxiliary quasi-monotone systems and a comparison theorem for the Cauchy problems of the three systems in the non-quasi-monotone case.

(3) Symbolic embeddings for difference equations. H.-J Chen and M.-C Li studied complexity of solutions of a high-dimensional difference equation f and provides a sufficient condition for any sufficiently small C1 perturbation of f to have symbolic embedding, that is, to possess a closed set of solutions that is invariant under the shift map, such that the restriction of the shift map to is topologically conjugate to a subshift of finite type.

c.(1) A joint work with Daniel Hernandez-Hernandez has been accepted for publication in a conference proceeding: (D. Hernandez and S.J. Sheu) Solution of the HJB equations involved in utility-based pricing, XI Symposium on Probability and Stochastic Processes, Mena, R.H., Pardo, J. C., Rivero, V., Uribe Bravo, G. (Eds.), (2015), Springer Verlag. We have joint work with Hideo Nagai (Kansai University, Japan) , Hiroaki Hata (Shizuoka University) on portfolio optimization problems. We continue to improve our result and is almost ready to submit for publication. (H. Hata, H. Nagai and S. J. Sheu) An optimal consumption problem for general factor model, preprint (40 pages).

(2) A joint work with Yu-Jui Huang (Dublin City University) on systemic risk has been presented in ICIAM 2015. We continue to improve our result and polish the paper in order to be ready to submit for publication: (Yu-Jui Huang and Li-Hsien Sun) Mitigating Systemic Risk via Central Banking, Preprint.

(3) For research results on Markov chain mixing, we would like to highlight the following progress in Markov chain mixing. (Cutoffs of birth and death chains.) In 2006, Diaconis and Saloff-Coste determined the cutoff in separation using the eigenvalues of transition matrices and, later in 2010, Ding, Lubetzky and

Peres determined the cutoff in the total variation using the mixing time and the spectral gap. In the year of 2014-2015, Chen and Saloff-Coste provide a probabilistic criterion on the existence of cutoffs and characterize the cutoff time and window using the expectation and variance of the first hitting time to large set. Those results are collected and published in the following article: Guan-Yu Chen and Laurent Saloff-Coste, Computing cutoff times for birth and death chains. Electronic Journal of Probability, 20 (2015), no. 76, 1-47. (Cutoffs of product chains.) This ongoing project is aroused by Chen and Kumagai in early 2015 and, recently, they obtained a fundamental inequality on the comparison of the total variation distance and the Hellinger distance, which says that both distances will be close to 0 or 1 simultaneously. This result identifies cutoffs in the total variation with cutoffs in the Hellinger distance, while the Hellinger distance of product chains can be easily represented by the Hellinger distance of the original chains. From the perspective of probability theory, this may lead to a new branch on the study of cutoffs in the total variation.

(4) The reconstruction of a nanoscale object from measurements of its diffraction patterns observed from the far field has attracted the attention of scientists and applied mathematicians over a century since it arises in many areas of optics such as X-ray crystallography, astronomy and coherent light microscopy. In the experiment, the nanoscale object is illuminated by high frequency coherent X-rays, so the optical sensors behind the sample cannot capture the phase information of diffraction waves. How reconstruct the nanoscale object through the phase-less diffraction measurements is the so-call phase retrieval problem. The alternating projection (AP) is the most popular idea for solving the phase retrieval problem. Both the Gerchberg and Saxton algorithm and the Error-Reduction algorithm belong to this type. Since the constraint generated by the collected data is not convex, the convergence of these algorithms cannot be implied by the classical convex optimization theory. Currently, we have proved some theoretical results about the local convergence of Gerchberg-Saxton algorithm and the Error-Reduction algorithm. The submission of these results to international journals is under preparation. These works are collaborated with Albert Fannjiang (University of California, Davis) and Pengwen Chen (National Chung Hsing University).

Highlight of the most significant activities:

a. Reaction-diffusion systems serve as relevant models for studying pattern formation in several fields of sciences. During June 4-5, there is a two-day workshop on Reaction-Diffusion Equations and Related Topics held at National Tsing Hua University. The aim of this workshop is to bring together active researchers to discuss recent and prospective advances on mathematical biology and related partial differential equations. In addition, Professor Xiaofeng Ren delivered a mini-course for giving more detailed information about new techniques used in the investigation of patterns in geometric inhibitory systems. Also, young mathematicians are invited to talk their recent research. Organizers Chiun-Chuan Chen (National Taiwan University) Chao-Nien Chen (National Tsing Hua University)

Invited talks Chueh-Hsin Chang (Taida Institute for Mathematical Sciences) Yung S. Choi (University of Connecticut) Wei-Ming Ni (East China Normal University & University of Minnesota) Chiun-Chang Lee (National Hsinchu University of Education) Takashi Teramoto (Asahikawa Medical University) Je-Chiang Tsai (National Chung-Cheng University)

Mini-Course on Geometric Inhibitory Systems Xiaofeng Ren (George Washington University) 1. A binary inhibitory system: one dimensional stationary points and two dimensional stationary disc assemblies 2. Half discs in stationary assemblies: the impact of domain geometry 3. A ternary inhibitory system and double bubbles in stationary assemblies

b. 2015 Workshop on Dynamical Systems (2015/05/21-2015/05/23); Organizers: J.-C Ban, K.-C Chen, C.-H Hsu, M.-C Li, S.-S Lin The 2015 Workshop on Dynamical Systems includes two lecture series (Y.-F Yi and J. Peyriere) and eight 50 minutes presentations. This workshop has been hosted each May since 2001 by NCTS. This year we have arranged lectures and talks centered on ergodic theory, fractal geometry, random and arithmetic dynamics.

c. 2015 Summer Course on Dynamical Systems (2015/07/21-2015/08/13): W.-G Hu (Sichuan University); Organizer: J.-C Ban The purpose of this summer course is to introduce the entropy theory and dynamical zeta functions for higher dimensional dynamical systems. This question is at present far from being solved. Some useful tools and open problems are presented in this course.

d. NCTS Summer Courses on Analysis and PDE (Sep. 7 11, 2015) Speaker: Chi Hin Chan (Department of Applied Mathematics, National Chiao Tung University) Title: Introduction to the classical De-Giorgi's method and its recent application in the regularity theory of incompressible Navier-Stokes equations. Speaker: Jenn-Nan Wang (iAMS, National Taiwan University) Title: The Dirichlet-to-Neumann map and electrical impedance tomography

e. NCTS Short Course on Financial Mathematics: Black-Scholes-Merton Theory, with financial and stochastic backgrounds, Oct 16, Oct 21, 2015. Speaker: Narn-Rueih Shieh (National Taiwan University)

f. NCTS Mini Course on Random Matrices, June 29 July 13, 2015. Speaker: Horng-Tzer Yau (Harvard University)

g. We have regular probability seminar at National Central University, National Chiao Tung University, National Chengchi University and Academia Sinica. A topic we focus is the SPDE (stochastic partial differential equations). We have been studying lecture notes by Prevot-Rochner and Koshinevisan and others, and also report some fresh research papers. Another topic we are interested in is the study of mean field game and its application in finance. We also invite visitors to visit us for discussion or joint work. We also ask them to present their research works. There are several stochastic calculus seminars supervised by our colleagues . The materials for the seminars may have different level and serve for different needs. There are two seminars in NCU campus, one is more advanced level and one is for beginner level. There is another on in NTU campus (supervised jointly by Yun-Shyong Chow and Shuenn-Jyi Sheu). We also run three short courses during the year: General theory of Markov processes, Introduction to SDE and Introduction to Random walks with applications

Reports of an important ongoing project:

a. Nonlinear stability of the 1D Boltzmann equation (Kung-Chien Wu) We study the nonlinear stability of the Boltzmann equation in the 1D periodic box, this is an interesting project. The 3D case was completely resolved. However, in 1D case, the effect of the nonlinear part is stronger and we cant apply standard iteration method, i.e., treat the nonlinear term as the tail term. In fact, we need to construct the main part of the solution, substitute it, and then do iteration as previous case. Then so called main part is not obvious in Boltzmann case, the challenge is to find it and supply an optimal estimate.
b. Exponential stability for the inhomogeneous Landau equation (Kung-Chien Wu) We want to deal with the inhomogeneous Landau equation on the torus in the cases of hard, Maxwellian and moderately soft potentials. We first investigate the linearized equation and we prove exponential decay estimates for the associated semigroup. We then turn to the nonlinear equation and we use the linearized semigroup decay in order to construct solutions in a close-to-equilibrium setting. Finally, our goal is to prove an exponential stability for such a solution, with a rate as close as we want to the optimal rate given by the semigroup decay.
c. Tree SFT (J.-C Ban and C.-H Chang) In recent works, J.-C Ban and C.-H Chang established the entropy formula for a TSFT. What is still lacking is an explicit enumerate of the measure-theoretic entropy and variational principle. J.-C Ban and C.-H Chang will restrict their attention to those topics as an ongoing project.
d. Spatial-Temporal Delay Equations Hsu-Wu S.-L Wu and C.-H Hsu work on the topics of stability of traveling wave fronts for delayed monostable lattice differential equations and multilayer cellular neural networks. Such topics are interesting and attract considerable attention in recent years.
e. The ongoing project with W. H. Fleming (Brown University) and Hideo Nagai

(Kansai University) continue to make progress. A manuscript is under preparation: (W.H. Fleming and Hideo Nagai and S. J. Sheu) Portfolio optimization in incomplete markets. In this project, we study the use of dynamic programming approach and the martingale method to study the portfolio optimization problem. We observe the introduction of the parameter which describes the market completions plays important role. To understand the approach, a recent study with Jun Sekine shows that the value functions for the market completions is convex. This convexity has not been observed before and has close connection with our study with Fleming and Nagai. We believe this will have important consequence. The ongoing project with a graduate student Weida Chen is to study Martin boundary for diffusion processes using stochastic calculus. This is different from traditional approach by introducing Martin topology. This previous approach is abstract, therefore its connection to diffusion theory is vague. The approach using stochastic analysis may suggest a new way to understand the theory of Martin boundary. The study of Weida Chen has been making good progresses in the last one year.

f. The ongoing project with J.-P. Fouque (University of Califronia, Santa Barbara) is proceeding. A manuscript is under preparation, (J.-P. Fouque and Li-Hsien Sun) Systemic Risk and Stochastic Games with Delay. In this project, we propose a model of lending and borrowing with the corresponding refund. In order to describe this behavior, we introduce a non-cooperative stochastic differential game with delay. Nash equilibrium (open-loop) in this game can be obtained using forward and backward stochastic differential equations. In addition, we still consider the Hamilton-Jacobi-Bellman approach to solve the closed-loop equilibrium.

E. Scientific Computing

Program E.	Scientific Computation
Coordinator	Wen-Wei Lin
Core members	I-Liang Chern, Tsung-Ming Huang, Ming-Chih Lai, Weichung Wang, Suh-Yuh Yang

1. Overview of the Program

Due to the enormous progress in computer technology and numerical software that have been achieved in recent years, the use of numerical simulations in exploring new sciences and engineering gains more and more importance. Scientific computation in many cases offers a cost effective technique, such as numerical linear algebra, computational fluid dynamics and computational electromagnetism, to investigate the real-life sciences which has been regarded equally ubiquitous along with the experiment and theory. Besides the need for developers of corresponding hardware, there is a strong demand to cultivate qualified applied mathematicians who are able to efficiently apply numerical methods to solve complex problems arising from natural sciences and engineering. Thus, it is very important that the next phase of National Center for Theoretical Sciences (NCTS) has the Scientific Computation focused program to enforce such research trend and to cultivate our young generations. Meanwhile, scientific computation nowadays is seen as a strongly interdisciplinary field in which several aspects of numerical mathematics, natural sciences, computer science and the corresponding industrial applications are simultaneously important.

A challenging topic in scientific computing is how to design efficient algorithms for numerical PDE, numerical linear algebra, computational electromagnetism, computational fluid dynamics, fluid-structure interaction problems etc., on modern computer systems (CPU/GPU parallelization). The numerical difficulties include the requirement of large computational resources, especially for 3D real-world applications, the characteristics of multi-scale behavior in both temporal and spatial domains, and the strong local nonlinearity of the problem.

Compressive sensing is a hot interdisciplinary research topic in the fields of statistical science, computer science and mathematical science. It is an optimization technique to efficiently acquire or reconstruct sparse information from few measurements. Compressive sensing has been applied successfully in signal/image processing, data analysis, medical imaging, machine learning. Recently, it is also used to design efficient algorithms for high dimensional partial differential equations and integral equations. This is what we are pursuing and it is indeed related to our first topic, the multi-scale problems. In addition, we are also interested in the applications of compressive sensing in massive data analysis.

The multi-scale problems appear in virtually all areas of modern science and engineering. These problems are very important but usually difficult to solve due to the high degree of variability and the multi-scale nature of formation properties. Thus, it is a challenge to develop efficient numerical methods for multi-scale problems.

Based on the existing research man power in Taiwan and the current frontier research directions internationally, the following areas are of particular necessity to explore in the next phase of NCTS.

a. Large scale linear system computations and high performance computing

b. Numerical methods for PDEs and computational fluid mechanics

c. Numerical methods for multi-scale problems

d. Uncertainty quantification and applications

e. Compressive sensing and big data analysis

f. Real world scientific computing applications in industry

2. Highlights

In recent years, the scientific computing groups in NCTS have yielded significant advances in the topics of multi-scale finite element methods, fluid-structure interaction problems, compressive sensing and imaging processing. The most significant contributions include:

a. A new framework of multi-scale finite element method (MsFEM) for solving the second-order partial differential equations exhibiting multi-scale behavior. The key ingredient of the MsFEMs is a set of multi-scale basis functions, which is constructed by solving locally the PDE problem with some proper boundary conditions. The selection of boundary conditions plays an important role on the overall performance of MsFEM. Finding an appropriate boundary condition setting for some particular application is the current topic in the MsFEM research. Either using purely local information or purely global information are two popular classes of MsFEMs in the current available literature. In the proposed framework, which we call the iteratively adaptive MsFEM (i-ApMsFEM), the local-global information exchanges through iteratively updating local boundary condition for these multi-scale basis functions. Once the multi-scale solution is recovered from the solution of global numerical formulation on coarse grids, which couples these multi-scale basis functions, it provides a feedback for updating the local boundary conditions on each coarse element. The key successful step of i-ApMsFEM is to perform a few steps of smoothing iteration for the multi-scale solution before it is used for setting the boundary condition for the multi-scale basis functions in order to eliminate the high frequency error introduced by the inaccurate coarse solution. As the number of iteration increases, the quality of the MsFEM solution gets improved, since these adaptive basis functions are expected to more accurately capture the multi-scale feature of the approximate solution. We demonstrate the advantage of the proposed method through a number of numerical examples, including the high-contrast elliptic interface problems, the convection-dominated diffusion problems, and the multi-scale elliptic problems. The work has been submitted to an international journal.

b. Penalty immersed boundary methods and improved coupling interface method. Penalty immersed boundary method: a successful penalty immersed boundary method is developed for simulating the dynamics of inextensible interfaces interacting with solid particles. The main idea of this approach relies on the penalty techniques by modifying the constitutive equation of Stokes flow to weaken the incompressibility condition, relating the surface divergence to the elastic tension to relax the interface's inextensibility, and connecting the particle surface-velocity with the particle surface force to regularize the particles rigid motion. The advantage of these regularized governing equations is that when they are discretized by the standard centered difference scheme on a staggered grid, the resulting linear system can easily be reduced by eliminating the other unknowns than velocity directly, so that we just need to solve a smaller linear system of the velocity approximation. This advantage is preserved and even enhanced when such approach is applied to the transient Stokes flow with multiple compound vesicles. Moreover, this smaller linear system is symmetric and negative-definite, which enables us to use efficient linear solvers. Another important feature of the proposed method is that the discretization scheme is unconditionally stable in the sense that an appropriately defined energy functional associated with the discrete system is decreasing and hence bounded in time. Several numerical examples for the accuracy and stability of the immersed boundary discretization scheme have been tested. The tank-treading and tumbling motions of inextensible interface with a suspended solid particle in the simple shear flow have studied extensively. The simulation of the motion of multiple compound vesicles has performed as well. Numerical results illustrate the superior performance of the proposed penalty method. The findings of this research were published in a recent issue of Journal of Scientific Computing (2015). Improved coupling interface method (CIM): This is a continuation of our earlier work on this subject. Two papers were published recently. One is a joint work with the front tracking team from Stony Brook. We have merged our Coupling Interface Method 3D code with their FronTier Code and have studied fabric dynamics (CCP 2013). The second work is an improvement of the CIM. Our original CIM was proposed in 2007 and has attracted much attention since then (60 citations so far). In our earlier CIM, the second order method cannot be applied at some exceptional grid points when the underlying interface is very complex, and a hybrid method was proposed. Although the overall error is still second order through least squares fitting, the absolute errors fluctuate at different mesh sizes. In our new work (JCP 2014), we proposed two recipes to avoid these annoying errors. It is second order accurate everywhere, and it can handle quite complex interfaces in three dimensions without error fluctuation.

c. Compressive sensing and image processing. A registration method for solving point set matching problems has been proposed. The method is a combination of a global affine transform and a local curl-free transform. The curl-free transform is estimated by optimizing some kernel correlation function weighted by a square root of a pair of correspondence matrices, which can be regarded as an approximation of the mass transport problem. By applying this method to match two sets of lung branch points whose displacement is caused by lung volume changes. Nearly perfect match performances verdict the effectiveness of this model. This paper was published on SIAM Journal on Image Sciences. In a joint project with engineers, we study how to mitigate B_1^+ inhomogeneity in a high-field magnetic resonance imaging (MRI). Such inhomogeneity causes spatially dependent contrast and makes clinical diagnosis difficult. The proposed method is a two-step design procedure in which (a) a combination of linear and quadratic spatial encoding magnetic field is used to remap the B_1^+ map in order to reduce the inhomogeneity problem to one dimension, (b) the locations, the amplitudes and the phases of spokes are estimated in one dimension. It is shown both numerically and experimentally that this design can mitigate the B_1^+ inhomogeneity at 7T efficiently. This work was published on Magnetic Resonance in Medicine (2013). In another joint work, we study the Expectation-Maximization (EM) algorithm for Maximum Likelihood Estimation (MLE) in Positron Emission Tomography (PET) and propose a new algorithmic structure called the String-Averaging Expectation- Maximization (SA-EM). In our simulation study, high-contrast and less noisy images with clear object boundaries are reconstructed with the proposed SA-EM algorithm in less computation time. Together with the new scheme, we propose a stopping criterion for this and other fast algorithms in tomography based on the curvature of the likelihood, as well as an L-curve to analyze iterations quality. Also, we present new convergence results for this family of algorithms.

d. A hybrid immersed boundary and immersed interface method for electrohydrodynamic simulations A hybrid immersed boundary (IB) and an immersed interface method (IIM) are developed to simulate the dynamics of a drop under an electric field in Navier-Stokes flows. Within the leaky dielectric framework with piecewise constant electric properties in each fluid, the electric stress can be treated as an interfacial force on the drop interface. Thus, both the electric and capillary forces can be formulated in a unified immersed boundary framework. The electric potential satisfies a Laplace equation which is solved numerically by an augmented immersed interface method which incorporates the jump conditions naturally along the normal direction. The incompressible Navier-Stokes equations for the fluids are solved using a projection method on a staggered MAC grid and the potential is solved at the cell center. The interface is tracked in a Lagrangian manner with mesh control by adding an artificial tangential velocity to transport the Lagrangian markers to ensure that the spacing between markers is uniform throughout the computations. A series of numerical tests for our scheme have been conducted to illustrate the accuracy and applicability of the method. We run a series of simulations with different permittivity and conductivity ratios and compare with the results obtained by the small deformation theory and other numerical results in literature. In addition, we also study the electric effect for a drop under shear flow.

e. Vesicle electrohydrodynamic simulations by coupling immersed boundary and immersed interface method A coupling immersed boundary (IB) and an immersed interface method (IIM) are developed to simulate the electrodeformation and electrohydrodynamics of a vesicle in Navier-Stokes leaky dielectric fluids under a DC electric field. The vesicle membrane is modeled as an inextensible elastic interface with an electric capacitance and an electric conductance. Within the leaky dielectric framework and the piecewise constant electric properties in each fluid, the electric stress can be treated as an interfacial force so that both the membrane electric and mechanical forces can be formulated in a unified immersed boundary method. Unlike the droplet interface behavior, when applying the electric field, opposite charges accumulate on either side of the vesicle membrane and thus form a transmembrane potential. The transmembrane potential can be calculated from the conservation law of current density across the membrane. The electric potential and transmembrane potential are solved simultaneously via an efficient immersed interface method. The fluid variables in Navier-Stokes equations are solved using a projection method on a staggered MAC grid while the electric potential is solved at the cell center. A series of numerical tests have been carefully conducted to illustrate the accuracy and applicability of the present method to simulate vesicle electrohydrodynamics. In particular, we investigate the prolate-oblate-prolate (POP) transition and the effect of electric field and shear flow on vesicle electrohydrodynamics. Our numerical results are in good agreement with those obtained in previous work using different numerical algorithms. Furthermore, the previous immersed boundary (IB) method for 3D axisymmetric inextensible vesicle in Navier-Stokes flows can be extended to general three dimensions. Despite a similar spirit in numerical algorithms to the axisymmetric case, the fully 3D numerical implementation is much more complicated and is far from straightforward. A vesicle membrane surface is known to be inextensible and exhibits bending resistance. In all of numerical methods for vesicle, how to impose the membrane inextensibility constraint is an important issue. The surface tension in vesicle problems, which has a different physical meaning from that in general two-phase flow problems, is unknown a priori and in fact acts like Lagrange multiplier to enforce the local inextensibility along the surface. This is exactly the same role played by the pressure to enforce the fluid incompressibility in Navier-Stokes equations. As in 3D axisymmetric case, instead of keeping the vesicle locally inextensible, we adopt a modified elastic tension energy to make the vesicle surface patch nearly inextensible so that solving the unknown tension (Lagrange multiplier for the inextensible constraint) can be avoided. This approach significantly simplifies the numerical algorithm. Nevertheless, the new elastic force derived from the modified tension energy has exactly the same mathematical form as the original one except the different definitions of tension. The vesicle surface is discretized on a triangular mesh where the elastic tension and bending force are calculated on each vertex (Lagrangian marker in the IB method) of the triangulation. A series of numerical tests on the present scheme are conducted to illustrate the robustness and applicability of the method. We perform the convergence study for the immersed boundary forces and the fluid velocity field. We then study the vesicle dynamics in various flows such as quiescent, simple shear, and gravitational flows. Our numerical results show good agreements with those obtained in previous studies.

F. Interdisciplinary Studies

Program F.	Interdisciplinary research
Coordinator	Tai-Chia Lin
Core members	Tzyy-Leng Horng, Chih-hao Hsieh, Sze-Bi Hsu, Hsisheng Teng, Je-Chiang Tsai, Hung-Chi Kuo

1. Overview of the Program

We have four projects which focus on the following topics.

I. Modeling, simulation and analysis of electric double layers

II. Life history traits and exploitation affect the spatial mean-variance relationship in fish abundance

III. Models of harmful algae with toxin degradation

IV. Mathematical models of cancer stem cell

Activities of Project I:

a. Summer school (8/18-9/2, 2015) entitled "Modeling, Simulation and Analysis of Electrolytes"

b. Winter school (12/18, 22, 25-30, 2015) entitled "Modeling, Simulation and Analysis of Biology and Physiology".

There are about 35 registered students from different universities of Taiwan to attend the summer school and learn magneto-hydrodynamics equations, charge conserving Poisson-Boltzmann equations, physics and Application of electrolyte in a charged nanopore, numerical methods for simulating ion flows in electrolytes, and molecular dynamics simulation and its application.

Activities of Project II:

a. International workshop (9/16-18, 2015) entitled "International workshop on development and application of empirical dynamic modeling and forecasting for nonlinear systems"

b. Summer school for weather prediction (8/12-13, 2015) which cover the topics of the predictability and the uncertainty of the weathers and numerical weather prediction.

All activities of Project I and II are located at National Taiwan University (Taipei).

Activities of Project III:

a. Summer school (9/3-10, 2015) for Mathematical Biology which covers the topics of wave theory for monotone semi-flows and reaction-diffusion equations
b. International workshop (10/29-30, 2015) at National Tsing-Hua University (Hsinchu), which joints with Project IV.

Activities of Project IV:

- Summer school (7/27-8/6, 2015) at National Chung-Cheng University (Chia-Yi) entitled "Analysis and Modeling of High Throughput DNA Methylation Data".

The summer school covers the topics of Kaplan-Meier estimator, Log-rank test, cox proportional hazard model, and analysis of DNA methylation data. Besides, each project has its own regular seminars and group meeting every month.

2. Highlights

Modeling, simulation and analysis of electric double layers (by T.L. Horng, T.C. Lin and Hsisheng Teng et al) The electric double layer (EDL) consists of the Stern layer (a compact layer of charge at the electrode-solution interface) and the diffuse layer (a diffusion layer extending into the solution). When voltage is imposed, the EDL may appear with the thickness of nanoscales and form a capacitor to store energy which has many applications on biology, physics and chemical engineering. For instance, how to increase the efficiency of energy storage device is an important energy research subject nowadays. Nano-structured supercapacitor shows the potential of high-efficiency energy storing and output, and has become an intensive research subject in the area of energy storage device these years. Supercapacitors use electrostatic force to separate the cations and anions in electrolyte, immersed inside nanopores of electrodes, to store electric charges. As the nanopores become smaller, steric effect becomes important in the distribution of ions. Here we study the steric effect of ions inside the nanopores of supercapacitor theoretically and then verify the developed mathematical model by experiments. To describe the diffuse layer, T.C. Lin et al (cf. [1]) justify the standard Poisson-Nernst-Planck (PNP) system (a macroscopic continuum model) by the diffusion limit of Vlasov-Poisson-Fokker-Planck system in a bounded domain with reflection boundary conditions for charge distributions. To see the stability of diffuse layer, T.C. Lin et al (cf. [2]) develop mathematical arguments to prove exponential decay estimates of the solution of the perturbed problem with global electroneutrality for the stability of 1D boundary layer solutions to PNP systems. Near the Stern layer, charged particles (ions) become crowded so the finite size effect should be considered and the standard PNP system has to be modified. Using techniques of Fourier analysis, T.C. Lin et al (cf. [3]) find approximate Lennard-Jones potentials and derive a new PNP type model called the steric PNP (PNP-steric) system, and T.L. Horng et al (cf. [4]) numerically simulate the selectivity of ion channels. Ion channels are protein molecules that conduct ions through a narrow pore of fixed charge formed by the amino acids of the channel protein. Membranes are otherwise quite impermeable to natural substances, so channels are gatekeepers for cells. Channels are natural nanovalves that control a wide range of biological function. Experiments measuring currents through single protein channels show unstable currents. Channels switch between 'open' or 'closed' states in a spontaneous stochastic process called gating. Currents are either (nearly) zero or at a definite level, characteristic of each type of protein, independent of time, once the channel is open. The steady state Poisson-Nernst-Planck equations with steric effects (PNP-steric equations) describe steady current through the open channel quite well, in a wide variety of conditions. Here we study the existence of multiple solutions of steady state PNP-steric equations to see if they themselves, without modification or augmentation, can describe two levels of current. T.C. Lin et al (cf. [5]) prove that there are two steady state solutions of PNP-steric equations for (a) three types of ion species (two types of cations and one type of anion) with a positive constant permanent charge, and (b) four types of ion species (two types of cations and their counter-ions) with a constant permanent charge but no sign condition. The excess currents (due to steric effects) associated with these two steady state solutions are derived and expressed as two distinct formulas. Our results indicate that PNP-steric equations may become a useful model to study spontaneous gating of ion channels. Spontaneous gating is thought to involve small structural changes in the channel protein that perhaps produce large changes in the profiles of free energy that determine ion flow. Almost all biological activities involve transport in ionic solutions, which involves various couplings and interactions of multiple species of ions. Many complicated types of electrolytes involved in biological processes, such as those in ion channel proteins, certain amino acids (movable side chain) are crucial to the functions of these ion channels. The electrostatic properties involving multispecies (at least three species) ions can be fundamentally different to those with only one or two species. To see such difference, T.C. Lin et al (cf. [6]) study charge conserving Poisson-Boltzmann equation for multispecies ions which is derived from steady state PNP systems with charge conservation law, and is the surface potential model for the generation of a surface charge density layer related to electric double layers. The asymptotic limits of the solution values (electric potentials) at interior and boundary points with a potential gap (related to zeta potential) are uniquely determined by explicit nonlinear formulas which can be verified by experiments. For electric double-layer capacitors, T.L. Horng and H. Teng et al (cf. [7]) study facile simulation of carbon with wide pore size distribution for electric double-layer capacitance based on Helmholtz models. Their simulation results regarding the capacitance values of each carbon are in excellent agreement with experimental data, thereby verifying the reliability of the proposed model. Furthermore, the finite-size effects also should be considered and the steric PNP system may be applicable for carbon with narrow pore size. Such an interdisciplinary research is going on now.

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Life history traits and exploitation affect the spatial mean-variance relationship in fish abundance (by C.H. Hsieh et al) Nonlinear dynamics, with hallmarks of multiple stable states, regime shifts, hysteresis, and nonlinear amplifications of external forcing, are ubiquitous in nature. These phenomena pose challenges in understanding and forecasting system behavior. Most of existing methods are not effective in studying these kinds of systems. For example, most existing statistical methods assume that the system is linear and separable, and thus, these methods are ill posed for a nonlinear-coupled dynamic system. While it is possible to develop a set of nonlinear equations to model such systems, we typically do not know the equations. Thus, alternative approaches are needed. Here we apply empirical dynamic modeling approaches that can effectively deal with nonlinear systems (cf. [1-4]). These methods can help provide mechanistic understanding of nonlinear systems. Moreover, they provide capacity to forecasting. These novel methods can be applied in ecosystem managements, public health, medical sciences, neural sciences, and many others. Overfishing is predicted to undermine the spatial heterogeneity of fishes. As an effective index to quantify spatial heterogeneity, the exponent b in Taylor's power law ($V = aMb$) measures how spatial variance (V) varies with changes in mean abundance (M) of a population, with larger b indicating higher spatial aggregation (i.e. less heterogeneity). Theory predicts b is related with life history traits, but empirical evidence is lacking. Using 50-year spatiotemporal data from the California Current Ecosystem, we examined fishing and life history effects on Taylor's exponent by comparing exploited and unexploited fishes living in the same environment. We found that unexploited species with smaller size and generation time exhibit larger b, supporting theoretical prediction. In contrast, this relationship in exploited species is much weaker, as the exponents of large exploited species were higher than unexploited species with similar traits. Our results suggest that fishing may change the degree of spatial aggregation of a species, likely through degrading their size/age structure. In addition, results of moving-window cross-correlation analyses on b versus age structure indices (mean age and age evenness) for some exploited species corroborate our findings. Furthermore, through linking our findings to other fundamental ecological patterns (occupancy-abundance and size-abundance relationships), we provide theoretical arguments for the usefulness of monitoring the exponent b for management purposes. We propose that age/size-truncated species might have lower recovery rate in spatial occupancy, and the spatial variance-mass relationship of a species might be non-linear. This study is important in exploring the unification of relationships among spatial distribution, abundance, and body size in ecology; moreover, it provides profound implications in management.

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Models of harmful algae with toxin degradation (S.B. Hsu et al) Resource competition model mentioned previously assumed constant yield for population dynamics. This implies no energy conservation from one stage to the next. However, Grover suggested that storage of excess resource could lead to variable yield for converting nutrient into organism. The dynamics of a reaction-diffusion system for two species of microorganism in an unstirred chemostat with internal storage is studied [8]. It is shown that the diffusion coefficient is a key parameter of determining the asymptotic dynamics, and there exists a threshold diffusion coefficient above which both species become extinct. On the other hand, for diffusion coefficient below the threshold, either one species or both species persist, and in the asymptotic limit, a steady state showing competition exclusion or coexistence is reached. It has been known that nutrients and light are essential resources for growth of phytoplankton. We investigate a competition model of two phytoplankton species for a single nutrient with internal storage and light in a well-mixed aquatic environment [7]. By appealing to the theory of monotone dynamical system, we are able to determine the outcomes of competition: extinction of two species, competitive exclusion, stable coexistence and bistability of two species. We also present the graphical presentation to classify the competition outcomes and to compare outcome of models with and without internal storage. Phytoplankton species in a water column compete for mineral nutrients and light, and the existing models usually neglect differences in the nutrient content and the amount of light absorbed of individuals. In [3], we examine a size-structured and nonlocal reaction-diffusion-advection system which models the dynamics of a single phytoplankton species in a water column where the species depends simply on light for its growth. Our model is based on the assumption that the amount of light absorbed by individuals is proportional to cell size, which varies for populations that reproduce by simple division into two equally-sized daughters. We first establish the existence of a critical death rate and our analysis indicates that the phytoplankton survives if and only if its death rate is less than the critical death rate. The critical death rate depends on a general reproductive rate, the characteristics of the water column, cell growth, cell division, and cell size. Photoinhibition is characterized by a decreasing rate of photosynthesis with increasing light, which occurs in many phytoplankton species that are sensitive to strong light. We study the effect of photoinhibition in a nonlocal reaction-diffusion-advection equation, which models the dynamics of a single phytoplankton species in a water column where the growth of the species depends solely on light [1]. Our results show that, in contrast to the case of no photoinhibition, where at most one positive steady state can exist, the model with photoinhibition possesses at least two positive steady states in certain parameter ranges. Our approach involves bifurcation theory and perturbation-reduction arguments. Algae that produce harmful blooms are notable for their production of toxins, which can have inhibitory, or allelopathic effects on their algal competitors, on grazers of algae, and on organisms further removed in the food web, such as fish. Some researchers have suggested that allelopathy contributes to the formation of harmful blooms. Theory addressing competition for nutrient resources has a long history in ecology and is a mature area of research. However, theory for allelopathic interactions is currently in a stage of development. In [9], we extended a classical model of competition for two essential resources that can be stored within individuals by introducing additional competition through allelopathy. Our study also suggests that when both allelopathy and resource competition occur in phytoplankton communities, population dynamics will be less predictable in in eutrophic systems with high nutrient supply. In [6], we study a mathematical model arising from crop raiding of large-bodied mammals living in the biodiversity-rich tropics. The topic is important because it involves highly threaten species that can cause significant economic damage and be killed in retribution. Our mathematical model consists of a non-local spatially heterogeneous parabolic problem of logistic type. Our main analytical result characterizes the existence of positive solutions of the model and it provides us with some multiplicity results. Blooms of the harmful algae have increased the intensity worldwide in coastal as well as inland waters. The blooms have direct impacts for human health, and food webs in aquatic ecosystems. To understand longitudinal patterns arising along the axis of flow, the research team of Grover proposed reaction-diffusion-advection system modeling the dynamics of one nutrient, one single population of algae, and algal toxin with spatial variations in an idealized riverine reservoir where a main channel was coupled to a hydraulic storage zone. Their systems are not only interesting biologically but also extremely challenging from the mathematical point of view, as some of its equations have no diffusion terms, and hence, the associated solution flows cannot be compact, which makes mathematics harder. Our first task is to study the linearized stability of the trivial and semi-trivial steady states of the model. The linearized system at all these states is cooperative, but the "compactness" of the flow is as well needed for applying the classical Krein-Rutman theory in order to infer the existence of a principal eigenvalue whose sign can provide with the local attractive, or repulsive, character of these steady states. In [2], we established the existence of the principal eigenvalue problem of a linear system under suitable conditions, and analyzed the dependence, or sensitivity, of the principal eigenvalue with respect to significant parameters of the model from the ecological point of view. In [4], we introduce the basic reproduction ratio R_0 for algae and show that R_0 serves as a threshold value for persistence and extinction of the algae. With an additional assumption, we obtain the uniqueness and global attractivity of the positive steady state in the case where $R_0 > 1$. In [5], we investigated three species food web models of predator-prey type with an omnivorous top predator which is defined as feeding on more than one trophic level. Analytically, we completely classify the parameter space into three categories containing eight cases, show the extinction results for five cases, and verify uniform persistence for the other three cases. Moreover, in the region of the parameter space where the system is uniformly persistent we prove the existence of periodic solutions via Hopf bifurcation and present the chaotic dynamics numerically. Biologically, the omnivory module blends the attributes of several well-studied community modules, such as food chains (food chain models), exploitative competition (two predators-one prey models), and apparent competition (one predator-two preys models). Malaria and typhoid are among the most endemic diseases, and thus, of major public health concerns in tropical developing countries. In [10], we develop novel mathematical models describing the co-infection dynamics of malaria and typhoid. Through mathematical analyses of our models, we identify distinct features of typhoid and malaria infection dynamics as well as relationships associated to their co-infection. Using our model, we present illustrative numerical results with a case study in the Eastern Province of Kenya to quantify the possible false diagnosis resulting from this co-infection. In Kenya, despite having higher prevalence of typhoid, malaria is more problematic in terms of new infections and disease deaths. We find that false diagnosis - with higher possible cases for typhoid than malaria - causes significant devastating impacts on Kenyan societies. Our results demonstrate that both diseases need to be simultaneously managed for successful control of co-epidemics.

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Mathematical models of cancer stem cell (J.C. Tsai et al) In [1], we have constructed a simple model to study the B-cell activation in the immune system. The analysis of stationary waves of this model suggests that for the parameter σ less than $1/2$, we have to initially activate a very small amount of receptors close to the north pole of the sphere to induce the total activation of the cell, whereas for the parameter σ larger than $1/2$, we have to initially activate the receptors from almost the whole sphere to induce the total activation of the cell. Our results identify the critical role of stationary B-cell activation waves in the immune system. In [2], we have addressed wave propagation problems in a class of predator-prey systems which include Holling type I, type II, type III, and Ivlev type functional responses. As one of the referees commented that the result of the paper addresses a long-standing issue on spreading speeds in ecological invasion problems. We develop mathematical theorems for ecological invasion problems in a wide class of predator-prey systems.

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G. Big and Complex Data Analysis (Open Call Program)

Program G.	Big and Complex Data Analysis Program
Coordinator	Meihui Guo and Ray-Bing Chen
Core members	Mong-Na Lo Huang, Yun-Chan Chi, Miin-Jye Wen, Ching-Kang Ing, Yu-Fen Huang, Shih-Feng Huang

1. Overview of the Program

The Big and Complex Data Analysis program was started from April, 2015. The programs goals and achievement of this year are stated below.

a. Organize Big Data workshops and related conferences. Big data workshops provide opportunities for researchers to exchange and discuss important ideas and learn new technology. This year we organized three big data workshops at National Sun Yat-sen University (NSYSU). The first one "Toward Big Data Analysis Workshop" was held on June 5-June 6, 2015. The second one "Big Data One Day Workshop" will soon be held on November 12, 2015, and the third one "Workshop on Complex and High-dimensional Data Analysis" will be held on December 9-10, 2015.

b. Encourage domestic young researchers and PhD students to attend international conferences. It is important for researchers to establish international cooperation. We encourage young faculties, Post-doctors and PhD students to attend international conferences or to have short-term academic visit to establish their international connections and cooperation. Up to now, there are two applications approved by the NCTS (Huang, Chieh-Sen and Lee, Tsung-Lin). Lee, Tsung-Lin was supported to attend "Workshop on Scientific Computation with Applications" at Michigan State University. With intensive discussion in the workshop, a paper related to the big data computation is accomplished and submitted. There are three other applications are under reviewed which includes a PhD student (Ping Yang Chen, submitted his application for attending the 9th Conference of the Asian Regional Section of the IASC (IASC-ARS 2015) in Singapore).

c. Promote interdisciplinary cooperation especially for Applied Mathematics and Statistics. The Multidisciplinary Data Science Research Center (MDSRC) of NSYSU Involves researchers in Statistics, Applied Mathematics and other science area. To promote interdisciplinary cooperation, we will hold two big data workshops in November and December, together with the MDSRC. Besides, the program coordinators and core members have several ongoing interdisciplinary co-projects. For example, Meihui Guo served as a co-PI of a Communication Engineering project with Wong, KainamThomas (Hong Kong Polytechnic University). Ching-Kang Ing leads a group of young faculty to work on an industry-academia collaboration project. Ray-Bing Chen has a long term cooperation with applied mathematician Weichung Wang. Chieh-Sen Huang works on an industry-academia collaboration project with China Steel Corporation. Mong-Na Lo Huang joined an engineering project with Nan-Lu Chan in Department of Electrical Engineering.

d. Invite international scholars to promote international cooperation. With support from NCTS, Professor Kerby Shedden from Department of Statistics, The University of Michigan, Ann Arbor, visited the Department of Applied Math., NSYSU from July 15 to August 15. During this period of time, Prof. Shedden attended our regular group meetings and participated in the discussions actively with our faculties and graduate students. As the director of the Consulting Center of the University of Michigan, he offered us many useful comments and suggestions on how to proceed on analysis with real world problem. He has also given two talks with the titles: 1. Matrix-variate Models; 2. Data Science and Statistics in an American Research University. The first one introduced a new technique to handle high dimensional and highly correlated genomic data, and the second one introduced newly established Ph.D. or master programs about Data Science and their connections with Statistics in the US research universities. Moreover, he introduced several research project directions related to big data analysis including one about self-driving cars. It has made us come to understand that while trying to promote big data analysis on high-tech industry problems etc., special interests should be focused on the unique features of research topics with strong local strengths and heritage. Prof. Sheddens visit has been very fruitful and support from the NCTS is an important source for achieving all these things.

2. Highlights

Research highlight and report

a. Highlight of the most significant research result. The concern with high-dimensional statistical inference has been growing for the last decade and received increasing attention. Consider the linear regression model

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i x_t i + \epsilon_t, t = 1, 2, \ldots, n,$$

where y_t denotes the response, $x_{t1}, x_{t2}, \ldots, x_{tp}$ are p predictor variables, and ϵ_t is the random error component. When p is larger than n, there are statistical and computational difficulties in estimating the regression coefficients. Moreover, standard regression methods may not work well due to the curse of dimensionality. In the last decades, major advances to tackle these dilemmas have been made with the introduction of L_2-boosting (Bhlmann and Yu (2003)), LARS (Efron et al. (2004)), and Lasso (Tibshirani (1996)). Other approaches to this issue are available, including adaptive Lasso (see Zou (2006)), sure independence screening (SIS) method (see Fan and Lv (2008)), iterative sure independence screening (ISIS) method (see Fan and Lv (2010)) and so on. A method that is widely used in applied regression analysis to handle a large number of input variables is stepwise least squares regression. Temlyakov (2000), Tropp (2004), and Tropp and Gilbert (2007) use orthogonal greedy algorithm (OGA) (also called orthogonal matching pursuit in information theory) and focus on approximating the noiseless model (i.e. $\epsilon_t = 0$). Moreover, Ing and Lai (2011) proposed a fast stepwise regression method and obtained the variable selection consistency via OGA+HDIC+Trim where HDIC is an abbreviation for high-dimensional information criterion. The approach of OGA+HDIC+Trim consists of (i) forward selection of input variables, (ii) a stopping rule to terminate forward inclusion of variables, and (iii) stepwise backward elimination of variables. In this study, we consider the following location-dispersion model

$$\begin{aligned} y_t &= \beta_0 + \textstyle\sum_{i=1}^{p} \beta_i x_{ti} + \sigma_t \epsilon_t, \quad t = 1, 2, \ldots, n, \\ \sigma_t^2 &= \exp\{\alpha_0 + \textstyle\sum_{i=1}^{p} \alpha_i x_{ti}\}, \end{aligned}$$

where disturbance term of location model, $\{\epsilon_t\}$, admits an $MA(\infty)$ representation, and dispersion model of σ_t^2 is depended on those predictor variables $x_{t1}, x_{t2}, \ldots, x_{tp}$. It is obvious that the response vector $\{y_t\}$ can have a time series structure. We plan to use OGA+HDIC+Trim to do model selection for both location and dispersion model and expect that we achieve variable selection consistency. In other words, include all relevant variables and exclude irrelevant variables when sample size is large enough. Moreover, we estimate the coefficients α_i's and β_i's based on the model selection result. We will establish the consistency of the coefficients estimates. Furthermore, the proposed approach will be applied to improve high technique production line quality.

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b. Highlight of the most significant activities.

(1) Toward Big Data Analysis Workshop was held on June 5-June 6, 2015 at National Sun Yat-sen University. The workshop has ten speakers sharing with the audiences about how to deal with high dimensional and complex data. The talks cover Data Visualization; Big Data Analytics; Biostatistics Applications with Large Data Set; Numerical Optimization; Computational Issues related to Big Data, as well as Machine Learning and Data Mining Techniques. The speakers have also provided problems and software for the participants to practice with real and big data analysis. The support of the National Center for Theoretical Sciences (NCTS) is indispensable for gathering the experts in the related fields, and researchers and graduate students in Southern Taiwan to learn the most updated methodologies toward big data analysis. There have been very active discussions during the workshop, and some Ph.D. students have also had chance to present some of their project results based on the techniques learned from this workshop.

The new ideas and methodologies presented in this workshop have also inspired a lot of interests with some new research directions among different researchers in certain research topics. To follow up with the results of this workshop, it is expected to form some focus groups to work on some specific research problems in the future.

(2) Big Data One Day Workshop will be held on November 12, 2015 at National Sun Yat-sen University. There are six invited domestic speakers and some of them are directors of data science center in Taiwan.

(3) We will organize Workshop on Complex and High-dimensional Data Analysis on December 9-10, 2015. There are four internationally renowned keynote speakers and 12 invited speakers including two from US, one from Singapore, one from Hong Kong and eight from Taiwan.

c. Report of an important ongoing project. The followings are two important ongoing projects:

(1) Principal Huber-type Expectile Components Analysis.
In many applications, practitioners may be more interested in the tail variations of the data rather than the variations around the mean. For instance, investors are interested in the principal factors that cause big jump in stock prices during financial crisis. A tail indicator named expectiles proposed by Newey and Powell (1987) is an analogue of the mean for quantiles. They defined an asymmetric L_2-norm in order to capture the tail behavior. Taylor (2008) shows quantile and expected shortfall can be assessed via expectiles. Tran et al. (2014) develops an analogue of principal component analysis (PCA) for expectiles namely principal expectile components (PEC). In financial application, we occasionally confront heavy-tailed observations or outliers. We propose to use the Huber norm (Huber, 1973) which is a hybrid of squared error for relatively small errors and absolute error for relatively large ones. The Huber norm is defined as follows.

$$L_\delta(a) = \begin{cases} a^2, & |a| \le \delta \\ 2\delta|a| - \delta^2, & |a| > \delta \end{cases}$$

The Huber criterion becomes closer to the L_2-norm as is larger. By controlling the value of , the Huber criterion is more robust against outliers but less efficient for normally distributed data. The value of is usually evaluated by 1.345 times the median of the absolute data divided by 0.6746. In this project, we define an asymmetric Huber norm to capture the tail behavior and prevent the heavy tailed observations and outliers at the same time. The Huber-type expectile can be obtained analogically. Then, we find the principal expectile components based on the asymmetric Huber norm, namely principal Huber-type expectile component (PHEC). The PSO or other numerical optimization methods are used to find the principal components. The PHEC can be used to convert a set of multivariate observations into a set of linearly uncorrelated variables for heavy-tailed observations or data with outliers.

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d. New development or new breakthrough and its relation to the program Optimal design Generator via Metaheuristic Optimization Approaches: In optimal design, the statistical model is assumed to be

$$y = g(x, \theta) + \epsilon,$$

where y is an univariate response; x is in the pre-specified design space, $\chi \in \mathbb{R}^d$; θ is the unknown parameter vector, and the error ϵ is assumed be from normal samples with zero mean and constant variance σ^2. Here the mean function $g(x, \theta)$ can be a linear or nonlinear function of θ.
Following the notations in optimal design, a design is a probability measure and is represented as

$$\xi = \left\{ \begin{array}{ccc} x_1 & \dots & x_k \\ p_1 & \dots & p_k \end{array} \right\},$$

where $x_1, \dots, x_k$ are the design points in χ, and p_is are the corresponding weights with $p_1 + \dots + p_k = 1$. Then given a pre-specified mean function $g(x, \theta)$, the corresponding Fisher information matrix, $I(\theta, \xi)$, can be computed as the expectation of the negative of the matrix of second derivatives of the log-likelihood function. Since this Fisher information matrix, $I(\theta, \xi)$, is related to the parameter estimation and prediction, thus the optimal design criterion is a convex function of $I(\theta, \xi)$. For example, the D-optimal criterion is $\log |(\theta, \xi)^{-1}|$, and the D-optimal design is to minimize this D-criterion w.r.t. all possible designs, ξ, in the design space χ. Thus given an optimal criterion, $\Phi(\theta, \xi)$, the corresponding optimal design is found by solving the following global optimization problem,

$$\xi^* = \text{argmin}_\xi \Phi(\theta, \xi).$$

How to numerically generate the design for a given criterion is an important issue, because the theoretical results might not be obtained, and have their own limitation. A commonly used design search algorithm can be found in Fedorov (1972). This Fedorov's algorithm is a sequential algorithm by iteratively adding design points or exchanging some design points to improve the criterion values. Until now Fedorov type algorithm is still popularly used and studied, for example, the cocktail algorithm proposed by Yu (2011). Our research team has worked on the related design search problems for more than 5 years. Instead of using the Fedorov exchange type methods, we apply a metaheuristic approach, particle swarm optimization, into the design search problems.
Particle swarm optimization (PSO) proposed by Eberhart and Kennedy (1995). Basically PSO is an iterative method and is to simulate the behaviors of bird flocking in search for food. Consider a scenario where there is only one piece of food in the area being searched and all the birds do not know exactly where the food is. However, they increasingly know how far the food is with each iteration. The effective strategy is to share information constantly among the flock and follow the bird which is nearest to the food. PSO first generates a group of initial particles randomly. At every iteration, it searches for the optimal point by updating each particle its two"best" values. The first one is the best solution it has achieved so far. This personal best value is called p_{best} and is stored. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called g_{best}. After finding the two best values, the particle updates its velocity and positions using the equations,

$$v_i^{t+1} = \omega_i v_i^t + c_1\beta_1(p_i - x_i^t) + c_2\beta_2(p_g - x_i^t) \text{ and } x_i^{t+1} = x_i^t + v_i^{t+1}.$$

Here, v_i^t is the particle velocity, ω_i is the inertia weight that modulates the influence of the former velocity and can be a constant or a decreasing function (Fan and Chang, 2007), x_i^t is the current position of the i-th particle. The vectors p_i and p_g respectively denote the personal best position for the i-th particle and the global best position for all particles. That is $p_{\text{best}_i} = f(p_i)$ and $g_{\text{best}} = f(p_g)$. The constant c_1 is the cognitive learning factor, c_2 is the social learning factor. The variables β_1 and β_2 are random vectors with the same dimension as x_i and multiplication of two random vectors are taken to be componentwise. The two constants c_1 and c_2 respectively control how each particle moves toward its own

local best position and overall global best position. Usually $c_1 = c_2 = 2$ seems to work well (Kennedy, 1997).
Our research team has used this PSO to generate the locally optimal designs for the different nonlinear models and different design criteria. This paper (Qiu et al., 2014), was published in Swarm and Evolutionary Computation. We also worked on the minimax (or maximin) design generators. This is a challenge problems because it is a nested optimization problems and the optimality criterion is non-differentiable. To deal with this minimax design problem, we proposed a nested PSO and we have demonstrated the efficiency of this nested PSO with respect to the different types of models and design criteria.
In this year, we focus on the "mixture experiment". In the mixture experiments, instead of the simple box constraints for the experimental region, the design region is a simplex, i.e., $\{(x_1, \ldots, x_q) \in [0,1]^q, \quad \sum_i x_i = 1\}$, and might also include some other constraints. Thus the experimental region is very irregular! Thus this irregular region would cause some troubles for finding the best design. To deal with this complex constraint optimization problem, a modified PSO approach was proposed in Wong et al. (2015). The basic idea is as follows. For our mixture experiments design problems, we find that a more effective way to first optimize over the regular hypercube, and then use a projection function to identify our target optimal design in the search space. Thus we define a proper project approach from the regular hypercube space onto the regular (or irregular) simplex. Therefore, this modified PSO is called ProjPSO. In Wong et al. (2015), we have demonstrated this ProjPSO worked well for the design problems with regular or irregular simplex spaces. We compare the performances of this ProjPSO with some commonly used methods to illustrate the advantages of the proposed method. We also demonstrate the ProjPSO can be used for large dimensional problem, for example, we did consider an optimization problem with more than 400 dimensionalities. This paper was published in PloS ONE on this June.
The key researchers in Taiwan of these series of projects are Prof. Weichung Wang (NTU) and Prof. Ray-Bing Chen (NCKU). We focused on the modifications of the PSO type algorithms for the different optimal design search problems, and also worked on the efficiency of the program implementation due to the complex optimization problems with huge dimensionalities. Thus our Taiwan research team plays the key role in this series of the researches.

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H. Harmonic Analysis (Open Call Program)

Program	Harmonic Analysis
Coordinator	Chin-Cheng Lin
Core members	Hua-Long Gau, Xiang Fang, Ming-Yi Lee, D.-M. Nhieu, Chun-Yen Shen

1. Overview of the Program

Harmonic Analysis has its root dating back to the 1800's, a time when Fourier made his contributions. That is to say, classical Harmonic Analysis treats problems concerning the Fourier transform and Fourier series. Overtime, it has broadened its investigation and influenced the development of mathematics in a substantial way. The Harmonic Analysis group in Taiwan, although small, focuses on a wide variety of topics which include the classical two weight problems, theory of function spaces, operator theory and numerical range. The details of our achievement in 2015 can be found in the publication list and described below. Our upcoming goal is to promote knowledge and development of this field in Taiwan. We outline our forth coming plan and report on activities during the year 2015. In May of 2015, a symposium aimed at young researcher in Analysis was organized at NCU. From September till the end of the year, a weekly seminar on "Continuous martingales and Brownian motions" is being held on every Thursday. Near the end of 2015, prominent scholars in the field such as Alexander Volberg, Yongsheng Han, Oliver Newton, Xuan Thinh Duong will visit NCU and will be given lectures and workshops. Their visits will certainly benefit the mathematical community here at home. This is only the beginning with more to come in the year 2016.

2. Highlights

a. Carleson measure on function spaces

In [2], Carleson characterized the positive measure μ on the unit disk such that

$$\|f\|_{L^p(d\mu)} \leq C_p \|f\|_{H^p} \tag{1}$$

for $p \geq 1$, where H^p denotes the analytic Hardy space over the unit disk. This type of measures are later called Carleson measures on the Hardy spaces and the inequality (2) is called the Carleson embedding. Carleson measures are crucial to Carleson's celebrated solution of the corona problem. Several analogues of Carleson's theorem have subsequently been obtained. It is extended to the Hardy space of unit ball on $\mathbb{C}^n$ by Hormander [5]. Later on, Hastings [4] proved a Carleson type theorem on the Bergman space. Clearly Carleson measurse started at function spaces of holomorphic functions, however it has repercussions far beyond such a scope. It have been applied in real Hardy space defined by Fefferman and Stein in [3]. Carleson measures are also important in the elliptic operators [6, 7].

b. Reverse Carleson measures on the function spaces

It is natural to expect, and it is indeed important for applictions in many situations to consider the converse inequality of (2). Precisely, to characterize the positive measure μ such that there is a positive number $\delta > 0$

$$\|f\|_{\mathcal{H}(\mu)} \geq \delta \|f\|_{\mathcal{H}} \tag{2}$$

where $\mathcal{H}$ is a function space over $\mathbb{D}$. This measure is called the $\mathcal{H}$-reverse Carleson measure. This turns out to be a much harder question for most of the situations of interests. Very recently, this question is solved when $\mathcal{H}$ is Hardy space and Model space on the unit disk. This is open for other spaces. Furthermore, Luecking [8] posted this question when $\mathcal{H} = L^2_a(\mathbb{D})$ which is still a core open question in operator theory except for some special cases.

One of the most interesting questions in function theory is to determine the sample sequence in a function space. Let $\{\gamma_n\}$ be a sequence of distinct points in the disk. It is called a sample sequence for the Bergman space $L^2_a(\mathbb{D})$ if there exist constant c_1 and c_2 such that

$$c_1\|f\|^2_{L^2(\mathbb{D})} \leq \sum_{n=1}^{\infty}(1-|\gamma_n|^2)^2|f(\gamma_n)|^2 \leq c_2\|f\|^2_{L^2(\mathbb{D})} \tag{3}$$

for $f \in L^2_a(\mathbb{D})$.
To see its connection with Carleson and reverse Carleson measure, we define a mass measure μ supported on $\{\gamma_n\}$ as

$$\mu_n = (1-|\gamma_n|^2)^2\delta_{z_n} \tag{4}$$

where

$$\delta_{z_n}(z) = \begin{cases} 0, & z \neq z_n; \\ 1, & z = z_n. \end{cases} \tag{5}$$

Then (3) can be rewritten by

$$c_1\|f\|^2_{L^2(\mathbb{D})} \leq \|f\|_{L^2(\mu)} \leq c_2\|f\|^2_{L^2(\mathbb{D})}. \tag{6}$$

It follows that $\{\gamma_n\}$ is a sample sequence for $L^2_a(\mathbb{D})$ if and only if μ is Carleson and reverse Carleson.

Remark: It is a good exerciseto show that there does NOT exist sample sequences for the complex Hardy space over the unit disk. However, there does exist sample sequences for the Bergman space. Such sequences are first characterized by Seip in his seminal work [11]. See also [9].
Another type of questions which are usually studied in conjunction with sample sequences is on the so called interpolation sequences. A sequence is called an interpolation sequence for $L^2_a(\mathbb{D})$ if it only satisfies the second equality of (3). Carleson [1] himself solved the interpolation sequence problem for the Hardy space and in that paper he introduced Carlson measure for Hardy space. [10] solved this question for the Bergman space via certain extremal functions.
Reverse Carleson measure has many other applications in function spaces in operator theory.
Let $\varphi \in L^\infty$. Then we recall that a Toeplitz operator T_φ on the Bergman space $L^2_a(\mathbb{D})$ is defined by

$$T_\varphi f = PM_\varphi f. \tag{7}$$

Here P is the orthogonal projection from $L^2(\mathbb{D})$ onto $L^2_a(\mathbb{D})$.
Obviously T_φ is bounded. Then a longstanding open problem is to characterize its invertibility. This question is still far away to present a solution. The latest result is due to Zhao and Zheng [12]. When $\varphi \geq 0$, they can answer this question by Berezin transform. The tool of Zhao and Zheng's proof is the characterization

of reverse Carleson measure for a special measure $\chi_G(w)dA(w)$ where $G \subset \mathbb{D}$ is a measurable set.

To illustrate the role of reverse Carleson measures in the problem of invertibility, suppose that $\varphi \in H^\infty$. Observe that if T_φ is invertible, then T_φ is bounded below. Therefore, there is a constant $\delta_0 > 0$ such that

$$\|f\|_{L^2(\mathbb{D})} \leq \delta_0 \|T_\varphi f\|_{L^2(\mathbb{D})}. \tag{8}$$

Since $\varphi \in H^\infty$, we have

$$T_\varphi f = P(\varphi f) = \varphi f. \tag{9}$$

It follows that

$$\int_{\mathbb{D}} |f(z)|^2 dA(z) \leq \delta_0^2 \int_{\mathbb{D}} |f(z)|^2 |\varphi(z)|^2 dA(z). \tag{10}$$

Thus if $\varphi \in H^\infty$ and T_φ is invertible then

$$|\varphi(z)|^2 dA(z) \text{ is a reverse Carleson measure on } L_a^2(\mathbb{D}). \tag{11}$$

The major problem in non-homogeneous Harmonic analysis is to deal with the boundedness property of the so-called Calderón-Zygmund operators. Broadly speaking, a singular integral operator is defined as following with a kernel function $K(x,y)$ that satisfies some size and smoothness conditions.

$$Tf(x) = \int K(x,y) f(y) dy, \qquad \text{a.e. } x \notin \text{supp}(f),$$

In the celebrated paper of David and Journé (Ann of Math 1984), they gave the famous boundedness criterion on $L^2(\mathbb{R}^n)$ space. That is T is L^2 bounded if and only if $T1$ and $T^*1 \in BMO$ and T satisfies the weak boundedness property. It is a very beautiful (and useful) result that one only needs to test the simplest function 1 in the operator. Note that via the theory of Calderón-Zygmund T is L^p bounded for all $1 < p < \infty$ as long as T is L^2 bounded. Shortly after the successful one weight A_2 condition was established, the two weight problem had been raised, and it was realized and considered an extremely important and difficult problem. Roughly speaking, the two weight problem was to find a necessary and sufficient condition with two arbitrary Borel measures σ, ω such that

$$\int |T_\sigma f(x)|^2 d\omega \lesssim \int |f(x)|^2 d\sigma,$$

where $T_\sigma f(x) = \int K(x,y) f(y) d\sigma(y)$. It has been found that the solution will have many important applications in various areas. In a beautiful series of papers of Nazarov, Treil and Volberg that they have developed the modern nonhomogeneous

Harmonic analysis. In their celebrated papers (J of Amer. Math. Soc 1999 and Acta Math 2003), they have proved a boundedness criterion for nonclassical one weight Cauchy transform, namely

$$\int |C_\sigma f(z)|^2 \sigma(z) \lesssim \int |f(z)|^2 \sigma(z),$$

where $C_\sigma f(z) = \int_{\mathbb{C}} \frac{f(w)}{z-w} \sigma(w)$ if and only if $\int |C_\sigma \chi_Q)|^2 \sigma(z) \lesssim \int_Q \sigma(z)$ for any cube Q. In addition, as an application of this result they also settled a long standing conjecture of Vitushkin in complex analysis. We point out that for the classical one weight case if the weight ω satisfies the A_2 condition, then ω is doubling, that is $\omega(2Q) \leq C\omega(Q)$. In the nonclassical one weight case, or even the two weight case, the measures σ, ω are arbitrary (i.e. may not be doubling and can be a singular measure). Hence the two weight problem becomes extremely difficult.

The first two weight result was proved in 1980 by Eric Sawyer for maximal operators and Poisson integral operators. However it is important to mention that these operators are positive, therefore Sawyer was able to obtain the two weight $T1$ theorem for these operators. The singular operators are much harder, and the two weight $T1$ theorem for singular operators remained open since then, even for one of the most important singular operators, the Hilbert transform. Finally the two weight $T1$ theorem for the Hilbert transform was settled in a series of papers (Duke Math Journal 2014, one of the authors is Chun-Yen Shen at National Central University, NCTS Harmonic Analysis Program 2015). The main theorem is:

In fact the two weight inequality for the Hilbert transform was studied as early as 1976 by Muckenhoupt and Wheeden. But, it received much wider recognition as an important problem with the 1988 work of Sarason. The work of Sarason was part of important sequence of investigations that identified de Branges spaces as an essential tool in operator theory. The question was concerned with the boundedness property of composition of Toeplitz operators. This question finally was realized to be highly related to the two weight problem for the Hilbert transform. In particular, Sarason conjectured that the full Poisson A_2 condition would be sufficient for the two weight inequality. In an important development, F. Nazarov showed that this was not the case. It immediately reveals that the two weight problem is really very subtle. In addition, the two weight problem was also important to study the model spaces, namely some certain embedding questions for model spaces can be realized as a two weight inequality for the Cauchy transform, $Cf(z) = \int \frac{f(w)}{z-w} dA(w)$. This will play an important role in our future investigation.

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IV. Goals and Planning of Next Year

A. Overview

There are several plans for the coming year in order to achieve the goals and missions of the Center.

First of all, our program for postdocs and students is definitely one of core of the Center. To design and develop suitable activities for postdocs and students is of fundamental importance. On the other hand, our young members are encouraged to travel abroad or even to do research abroad for a longer period. We hope that our programs can help them to broaden their view dramatically.

Secondly, we will work on the organization of Topical Program more systematically. Since there are only limited members in Taiwan here. It is essentially to design a system with many research activities in an effective way. In some sense, we need to design the system to save the time of our researchers. Therefore, we will work harder to promote some joint courses between nearby university. Or we will try to cooperate with various departments to reduce the regular load of outstanding researchers.

Also, we will encourage more long-term visitors. By having more long-term visitors, we expect that closer academic relation could be consolidated. In fact, instead of hosting many conference, we would prefer to invite researchers to stay longer for collaboration.

Last but not the least, we are going to organize Special Programs. For the first semester, we are organizing a Special Program focus on interdisciplinary research. For the second semester, we are organizing a Special Program on special varieties.

B. Academic programs

1. Number Theory of Representation Theory

For the future research plan, we continue to broaden and deepen our investigation on the four topics.

a. Geometry of Shimura varieties: We hope to complete the two current projects concerning non-emptiness of strata in Shimura varieties and the ordinary locus of quaternionic Shimura varieties. More specifically,

(1) About the construction of admissible integral models for Shimura varieties of abelian type, and prove the non-emptiness of KR strata for the Pappas-Zhu integral model, the non-emptiness of NP strata as general as possible and the non-emptiness of EO strata for good reduction of Hodge type.

(2) Investigate the density and existence of the ordinary locus for type C Shimura varieties. We like to understand the sufficient and necessary condition for the density of the ordinary locus. On the other hand, joint with Jiangwei Xue, Tse-Chung Yang, Mounir Hajli, we hope to build as many as possible

the cases in which we can compute the number of abelian varieties in a given isogeny class.

b. Explicit methods in classical modular forms: Currently, we are working on a thorough enumerate of quaternionic loci in Siegel's upper-half space. We are able to determine the number of Shimura curves of discriminant D in Siegel's upper-half space in terms of class numbers of imaginary quadratic orders. In addition, we are working on explicit modular parametrizations of these curves. After this work is completed, we plan to obtain a formula for the values of derivatives of Borcherds forms at CM-points. Such a formula will have many applications in the study of Shimura curves.

c. Multiple zeta values in positive characteristic: Our goal is to understand and determine the linear relations among the same weight MZVs in positive characteristic. Based on [7], we wish to generalize it to higher depths. That is, we wish to establish an effective algorithm to compute the dimension of the space of the same weight MZVs over function fields, whose characteristic zero counterpart is a very difficult problem, addressed as Zagiers dimension conjecture for MZVs.

d. p-adic methods in algebraic number theory and automorphic forms: Along this direction, our short-term goals are to:

   (1) Prove the exceptional zero conjecture for Katz p-adic L-functions by a closer investigation on Euler system of elliptic units (with M. Chida);

   (2) Establish congruences between Yoshida lifts and stable forms on $\mathrm{GS}_p(4)$ with application to Bloch-Kato conjecture for Rankin-Selberg convolution (with Kenichi Namikawa);

   (3) Extend works with F. Castella to supersingular case. The long-term project is to understand dynamics of special subvarieties of Shimura varieties under the action of Hecke operators with application to the non-vanishing of period integrals and special L-values.

   To this end, we plan to study recent works of Hida on transcendence of Hecke operators ad papers on the explicit computation of period integrals with our post-docs and Ph.D. students. We plan to organize international conferences and workshops next year. Chiafu Yu, Xuehua He and Kai-Wen Lan will organize 2016 Summer School of Shimura varieties and Related Topics during May 23-27 and Taipei Workshop of Shimura varieties and Related Topics during May 30-June 3, 2016. Ming-Lun Hsieh will organize the third Japan-Taiwan number theory conference at Taipei. This is the follow-up of the conferences initiated by Yifan Yang and Yamazaki Takao, and the aim is to provide opportunities for young number theorists between Japan and Taiwan to exchange ideas and stimulate the future potential collaboration.

In the next year, we will continue our program the classification of holomorphic vertex operator algebras (VOA) of central charge 24. It is one of fundamental problem in vertex operator algebra theory and seems to have many applications in mathematical physics and string theory. Due to our effort, 70 out of the 71 theory proposed by Schellekens have been constructed explicitly. We also developed a new method, called reverse orbifolding, and showed that many holomorphic vertex operators are uniquely uniquely determined by the Lie algebra structure of the weight one subspaces using this method. Our goal is to construct all 71 cases in Schellekens list and showed that the VOA structures are uniquely determined by the Lie algebra structure of the weight one subspace for all cases. In addition, we will continue our investigation on the two so-called strange series of simple Lie superalgebras, called the queer and the preiplectic Lie superalgebras. They provide some of the most interesting examples of Lie superalgebras and from a point of view, they are true super-analogues of the general linear and the orthogonal (or symplectic) Lie algebras. These two series have no classical counterparts in the theory of semisimple Lie algebras, as here one requires the dimensions of the even and the odd space to coincide. It is well-known that the finite-dimensional representation theory of the queer Lie superalgebra has intruiging connections with various areas of classical mathematics, e.g., Lie algebra of type B, symmetric function theory etc. It would be very interesting to study the representation theories of these superalgebras in the general BGG categories, even in the small rank cases. We believe that any insights gained here, even in small rank cases, could be quite significant for further development of Lie superalgebras. We will also study further connections between Lie algebras and Lie superalgebras in the spirit of super duality. An important class of Lie superalgebras so far not covered by super duality is the class of affine Lie superalgebras associated with finite-dimensional classical Lie superalgebras. For these Lie superalgebras representation theory is known to be very difficult. However, as representation theory of affine Lie superalgebras have many applications to other areas of mathematics and physics, we think that their study is a very important future direction. The remarkable connection between Yangians and finite W-algebras are by now well understood. It is therefore expected that there should be a super analogue as well. We would like to establish a realization of the finite W-superalgebras in terms of certain quotients of the so-called shifted super Yangians (both of type A). As an application, this provides a useful tool to the study of the representation theory of the finite W-superalgebras, which we expect to be very rich.

In the coming year, our stress will still be on student training and nurturing young researchers. In particular, we hope to lead students to directions which we feel will be substantial in the future. We will continue to organize weekly reading seminars on recent developments. We also plan to organize several conferences and workshops. Two international conferences Taipei conference in representation theory (Jan 4-8, 2016) and Conference on finite groups and vertex algebras (Aug 22-26, 2016)- have been organized. In addition, we also plan to organize a small workshop on finite groups and VOA in the east Taiwan in March 2016.

2. Algebraic Geometry

In the coming year of 2016, we are two NCTS Scholar Kawamata who will visit in March and July and Paolo Cascini who will visit in March. Also we are going to organize NCTS Special Program on Special Varieties starting from Fall of 2016.

i. In March, we plan to organize a mini-workshop on minimal model program when both Kawamata and Cascini are in NCTS.

ii. In Summer, we plan to have a summer school on derived category in preparing the Special Program.

iii. Special Program on Special Varieties By Special Varieties, we are mostly considering Calabi-Yau varieties and Fano varieties.

Calabi-Yau varieties are compact complex Kahler manifolds with trivial first Chern classes. Research on Calabi-Yau manifolds plays a central role in mathematical physics and mathematics, including differential geometry, complex geometry, algebraic geometry, number theory, representation theory. The algebra-geometric aspects of this rich topic focus on the study of moduli and arithmetic of Calabi-Yau manifolds. The further studies of moduli of K3 surfaces lead to the newly development of derived categories, Fourier-Mukai transform and Bridgeleand stabilities.

The research of Calabi-Yau threefolds is highly related to string theory and mirror symmetry in mathematical physics. It is speculated that all Calabi-Yau threefolds can be connected by a sequence of elementary process so that one can easily compare various mathematical invariants with physical meaning.

On the other hand, the modularity of Calabi-Yau manifolds over Q is very interesting from the perspective of arithmetic geometry. Jeng-Daw Yu'ss research was around this topic. His formulation of irregular Hodge fibration attracts notable attentions.

Fano varieties is another important type of varieties. There are ample rational curves on Fano varieties. The boundedness problem of Fano varieties have very strong application in the whole theory of minimal model program. Also the studies of rational curves have various impact and application in a lot of disciplinarians, including differential geometry, mathematical physics.

Therefore, during the semester, we are going to organizing many mini-courses and a few more workshops. We hope that our younger generation could benefit a lot from it.

3. Differential Geometry and Geometric Analysis

For the future research plan, we continue to broaden and deepen our investigation on the existing topics. We will list only some of the newer directions here.

a. Geometric Analysis in Physical Sciences (Chun-Chi Lin)

   We are interested in geometric analysis motivated from physical sciences (in particular, material sciences, general relativity, and quantum mechanics). Note that differential geometries provide various formulations of configuration states of physical objects. It has been observed from experimental data or theoretical analysis that many configuration states contain singularities, which are responsible to many phenomena. For example, dislocations in solid mechanics are closely related to material properties, e.g. elasticity and plasticity. The goal is to find the proper geometry and to develop the mathematical analysis for describing physical objects including defects or singularities Below are possible future research issues related to physical sciences:

   (1) From elasticity to plasticity
   (2) Static, deformation, or dynamic theory for describing configuration states of materials with defects (using differential geometries, e.g. Weyl-Cartan-Riemann geometries, and mathematical analysis)
   (3) A passage from discrete theory to continuum theory (using PDEs, geometric measure theory, calculus of variations)
   (4) Defects in elastic sheets (e.g. Foppl-von Karman equations)
   (5) Biomembranes and diblock polymers (e.g. Ohta-Kawasaki functional)

b. Cohomology theory in Symplectic manifolds (Chung-Jun Tsai)

   We are currently establishing tools to compute primitive cohomologies. We also apply it to understand how the primitive cohomology changes under important symplectic construction, for instance, the symplectic blow-up, the fiber sum of Gompf, the symplectic Lefschetz fibration of Donaldson. Donaldson proved that almost every symplectic manifold admits a symplectic Lefschetz fibration. It follows that one only need to study the vanishing cycles and monodromy action to understand the symplectic manifold. The vanishing cycles are Lagrangian spheres of the fiber, and Lagrangian cycles canonically correspond to middle dimensional primitive forms. Hence, the primitive cohomology theory has a nice interplay with the story of Lefschetz fibration of Donaldson.

   Then, we aim to focus on symplectic Calabi–Yau threefolds. During the last decade, many interesting examples are constructed, in particular, the symplectic conifold transition and the twistor space construction. The symplectic conifold transition involves the Lagrangian three-spheres; the twistor space construction involves a fibration structure. We hope to learn more about the non-Kähler Calabi–Yau equations by studying these constructions.

   Another direction is about the geometric interpretation of the A_∞-algebra structure. Sullivan showed that the differential graded algebra structure of the de

Rham complex encodes rational homotopy groups under certain mild topological assumptions. It is interesting to see whether the A_∞-algebra here also encodes the information of rational homotopy information of symplectic manifold.

c. Contact topology (River Chiang)
My current research interests fall in the following two categories: (1) contact structures in higher dimensions, (2) groups of symplectomorphisms. In particular I am interested in understanding various notions of fillability of contact manifolds, and the geometry and dynamics of the group of Hamiltonian diffeomorphisms.

We plan to organize the following activities next year.

a. The Third Taiwan International Conference on Differential Geometry (January, 2016)
b. Summer school in geometric evolution equations and symplectic geometry (Summer 2016)
c. Special day on several topics (2-4 topics)
d. International conference on Geometry of Several Complex Variables, probably (December, 2016)
e. Workshop on the moduli space of connections (Winter, 2016)
f. Workshop on Geometric Analysis in Physical Sciences
g. Intensive lectures series: Kazushi Ueda (Univ. of Tokyo) Homological mirror symmetry and Yuichi Nohara (Kagawa Univ.) Geometry of completely integrable systems on flag manifolds

We also plan to invite the following visitors: Simon Brendle (Stanford University), Mu-Tao Wang (Columbia University), Jian Song (Rutgers University), Xiao-Dong Cao (Cornell University), Knut Smoczyk (Leibniz Universitat Hannover), Spiro Karigiannis (University of Waterloo), Oliver Schnurer (Universitat Konstanz), Martin Guest (Waseda University), Philp Boalch (University of Orsay), Anton Alekseev (University of Geneva), Paul Biran (ETH), Viktor Ginzburg (UCSC), Yoshihiro Ohnita (OCU), Felix Schlenk (Neuchatel), Otto van Koert (SNU), , Jeffrey Case (Penn State University), Andrea Malchiodi (SNS, Pisa), Paul Yang (Princeton University), Yuguang Shi (Peking University), Luen-Fai Tam (The Chinese University of Hong Kong), Pengzi Miao (University of Miami), A. Goriely (Oxford) R. Hardt (Rice Uni.), S. Luckhaus (Uni. Leipzig + MIS.MPI) , Tonegawa (Uni. Hokaido) ,Li-Sheng Tseng (University of California, Irvine), Kwok Wai Chan (the Chinese University of Hong Kong), Siu-Cheong Lau (Boston University) , Hisashi Kasuya (Tokyo Institute of Technology).

4. Differential Equation and Stochastic Analysis

a. As usual, the 2016 Workshop on Dynamical Systems and regular seminars on dynamical systems will be organized next year. Besides, we plan to organize a workshop on arithmetic dynamics in 2016. The aim of this conference is to bring together active researchers on arithmetic dynamics, e.g. beta transformation, Gauss map and continued fraction dynamics, to discuss recent and prospective advanced. Finally, the important event of next year is the visit of Prof. Jiang. He is a famous mathematician on dynamical systems, and he will provide a 6-months lecture of dynamical zeta functions and quasi-conformal geometry in Teichmueller spaces. We will organize a regular seminar for the preliminaries of his work before July 2016. The tentative plans of next year are as follows.

(1) 2016 Workshop on arithmetic dynamics (March, 2016)

(2) 2016 Workshop on Dynamical Systems (May, 2016)

(3) The visit of Yun-Ping Jiang (2016/07/01-2017/01/31)

(4) Regular Seminar (Dynamical Zeta Functions, 2016/02-2016/07)

b. Pattern formation became an important research field, in which the traditional disciplines of biology, chemistry, physics and mathematics interact and certain progress has been made through the exchange of ideas. In the next few years, the goal of research activities will further enhance the interaction between different areas; in particular to develop new mathematical tools for better understanding various complicate patterns in biological systems and related wave signals in dynamics.

c. For the subprogram on PDE, we will continue regular seminars (2016/01-2016/06, 2016/09-2016/12) on PDE and Analysis at NTU. We have organized two workshops for young scholars this year. We plan to organize such activity once a year and the next one is scheduled in June, 2016. Also, in next year, we schedule another summer school (before summer break) on subjects related to PDE and probability. For international participation, the 8th International Conference on Inverse Problems and Related Topics will take place at Ewha w. University, Seoul, Korea, June 27 - July 1, 2016. We will organize a mini-symposium at the conference with speakers from Taiwan.

d.(1) On the study of SPDE, we hope and expect more people in the group to conduct research on this topic. We will invite experts to present lectures and also look for chance to have joint research. On the study of portfolio optimization problems, there is a possibility that our approach can be applied to different problems (including models with transaction costs) and can also be applied to general utility functions. The theory of mean field game has applications in finance for the study of systemic risk. For the models with common noisea class of SPDEs arises from the study. Therefore, we will be interested to learn the possible applications of the SPDE theory in the study of mean field game and its applications in finance.

(2) For the mixing time research, we will keep on exploring cutoffs of product chains and start a new subject on the spectral analysis of graph connection Laplacian. As there are very few domestic scholars working on these two topics, we will focus on the communication with international specialists and, thus, we are planning to organize a workshop in the Spring of 2016, say March, and invite several academic collaborators, including Takashi Kumagai, L. Saloff-Coste and Hau-Tieng Wu and more.

(3) Lung-Chi Chen would like to complete the project with professor Akira. They have taken a lot of time to do it. So I visited Akira to discuss this project face to face from September 8 to 13, 2015 and professor Sakai will visit me from November 21 to 28, 2015. We both hope that we can complete this project in next year. The second goal of next year, I would like to complete the project with professor Shu-Chiuan Chang and Dr. Chien-Hao Huang. It is almost done and we should make a finial check carefully. We hope that we can submit the manuscript to suitable journal in 2015. Finial, I will like to do the project with professor Akira Sakai at Hokkaido University and Markus Heydenreic at the University of Munich in Germany. In this project, we want to show the mean-field behavior for nearest-neighbor percolation when dimensions larger than 7 on the body centered cubic lattices.

5. Scientific Computing

a. Focuses Based on the current interests of the scientific computing groups in NTU and NCU, in 2016, we will focus on the following topics:

(1) Design and analysis of numerical methods for multi-scale partial differential equations: developing efficient numerical methods for high contrast interface problems, convective-dominate convective-diffusive problems, and the wave-number Helmholtz equation etc. The parametric studies of the proposed methods and their convergence analysis for these particular applications will be investigated. Possible to extend the proposed methods to the case of the system of multi-scale PDE problems. Target applications include the least-squares finite element formulation for the convective-diffusive problems, high-Reynolds number incompressible fluid flows, and the drift-diffusion model for the semiconductor device simulation will be considered.

(2) Developing efficient numerical methods for complex fluid problems: the most challenging issue is how to develop efficient algorithms for solving 3-D problems. It should also be related to the topic of some computational geometry problems such as how to accurately compute geometric quantities on surfaces. On the other hand, from the algorithmic viewpoint, the fluid-structure interaction solution algorithms can be classified as monolithic and partitioned approaches after the discretization by using, e.g., the arbitrary

Lagrangian-Eulerian method or the immersed boundary method. There are still some challenging issues that need to be resolved in future studies, such as the development of nonlinear preconditioning algorithms for monolithic approaches and efficient algorithms for solving large sparse linear systems of equations arising from the partitioned approaches.

(3) We look for possible applications of compressive sensing in PDEs in high dimensions that standard numerical methods encounter difficulties. We shall also look for possible applications in medical imaging.

b. The potential visitors for year 2016

(1) Eric C.-W. Chu (Monash University, Australia), Specialty: Numerical Algebra, Optimal Control

(2) Huoyuan Duan (Wuhan University, China), Specialty: stabilized finite element methods

(3) Albert Fannjiang (UC Davis, USA), Specialty: compressive sensing, stochastic PDEs

(4) Ming Jiang (Peking University, China), Specialty: medical imaging

(5) Eunjung Lee (Yonsei University, Korea), Specialty: least-squares finite element methods

(6) Ren-Cang Li (University of Texas at Arlington, USA), Specialty: high Performance Computing, numerical linear algebra

(7) Daniel Szyld (Temple University, USA), Specialty: numerical analysis, scientific computing

(8) Haijian Yang (Hunan University, China), Specialty: domain decomposition methods

(9) Yuan Yao (Peking University, China), Specialty: data science

c. The possible workshops for year 2016

(1) Workshop on Complex Fluids, Jan 18-19, 2016.

(2) One-Day Workshop on Numerical Partial Differential Equations

(3) One-Day Workshop on Numerical Methods for Fluid-Structure Interaction Problems

(4) One-Day Workshop on Multi-scale Computational Methods and Compressive Sensing

(5) Interdisciplinary Workshop on Computational Biomechanics and Engineering

d. NCTS/NTU/NCU/NTUST Joint seminar on compressive sensing and its applications Organizers: I-Liang Chern (NTU), Yu-Jie Li (NTUST), Suh-Yuh Yang (NCU) Time: The seminar will be held bi-weekly on Friday, 4:30-5:30, 5:30-6:00 for discussion Venue: Room 440, Astromath Building, NTU Theme:

Compressive sensing is a hot interdisciplinary research topic in the fields of statistical science, computer science and mathematical science. It is an optimization technique to efficiently acquire or reconstruct sparse information from few measurements. It has been applied successfully in signal/image processing, data analysis, medical imaging, machine learning, etc. We organize this regular seminar to pursuit possible applications of compressive sensing in the following, but not limited to, areas:

(1) numerical partial differential equations and integral equations in high dimensions,

(2) machine learning, and

(3) signal/image processing.

Speakers:

(1) October 16, 2015, 4:30-5:30, Mr. Y. C. Zhang (NCU, Taiwan) Title: Algorithms for overcoming the curse of dimensionality for certain Hamilton-Jacobi equations arising in control theory and elsewhere (a recent work by Darbon and Osher)

(2) November 27, 2015, Prof. Ming Jiang (Peking University, China) Title: TBA

6. Interdisciplinary Studies

a. Researchers of Project I (Modeling, simulation and analysis of electric double layers) propose to derive various PNP type models and study them theoretically and numerically. It is expected to develop several mathematical theorems and numerical schemes of PNP type models with applications on supercapacitors.

b. Researchers of Project II (Life history traits and exploitation affect the spatial mean-variance relationship in fish abundance) plan to further develop this framework for understanding dynamics systems, such as forecasting and developing early warning signals.

c. Researchers of Project III (Models of harmful algae with toxin degradation) want to extend their study of models of harmful algae with toxin degradation in the following ways:

(1) To study the competition system,

(2) To include two essential nutrients in our model,

(3) To study the interactions of algae and the grazers of algae.

d. Researchers of Project IV (Mathematical models of cancer stem cell) shall use stationary waves in B-cell activation system to study the propagation of kinases in the immune system. Furthermore, they will study how binding of E2F6 and the polycomb repressor complex to the promoter of miR-193a epigenetically down-regulate the transcription of miR-193a in ovarian cancer stem cell.

7. Big and Complex Data Analysis

a. Enhance international cooperation:
   In this year, we only had one international visitor. In the next year, we hope to invite more international scholars. The visiting scholars do not only work on the joint research projects but also can help us on the short course or workshop to introduce their own novel research results. Currently Prof. Weng Kee Wong at the Department of Biostatistics, UCLA, is plan to visit us in next December. In addition to continue the optimal design related projects, we would like to have a workshop or short course on experimental design.

b. Encourage young researchers and PhD students to attend international conferences and short-term academic visiting:
   We will continue encouraging young faculties, post-doctors and Ph.D. students to attend international conferences or to have short-term academic visiting. From our experiences, it is really important to build their international connection and also to explore their global view. We also believe that it would also inspire them to work on some advance researches and catch up the current academic trend and innovation.

c. Short course and workshops: Based on specific topics, we would like to organize related short courses to introduce the details of the topics, for example, biostatistics, clinical trial, optimal design and so on. Then we will organize a one day workshop together with several universities in South of Taiwan. In this workshop, we will encourage the young researchers (include the Post-doctor and Ph.D. students) to present their on-going researches and might have panel discussion sessions for the possibilities of the future cooperation. To follow up with the workshop, it is expected to form some focus groups to work on some specific research problems in the future. In addition, we will continues to organize other Big and Complex Data analysis related workshops.

8. Harmonic Analysis

During 2016, we consider to invite the following mathematicians from abroad to contribute to our program and to offer either short courses or lectures.

a. Ji Li (Australia): January 17 - Feburary 6
b. Yongsheng Han (USA): March 1-31
c. Denny Leung (Singapore): April 1 - May 31
d. Qingbo Huang (USA): June 1-14
e. Guozheng Cheng (China) and Jiayang Yu (China): July 1-31
f. Pengfei Guan (McGill University, Canada): inviting
g. Tuomas Hytonen (University of Helsinki, Finland): inviting
h. Eric Sawyer (McMaster University, Canada): inviting

Planned seminars, workshops, and Symposiums:

a. Weekly seminar: every Thursday afternoon

b. Workshop on Harmonic Analysis: during the period of Yongsheng Hans visiting

c. 2016 Symposium for Young Analysts: in May, 2016

d. Workshop on Analysis: during the period of Pengfei Guans visiting

For the research subjects, we will focus on

a. Two weight problem for singular integral operators

b. Bergman-type singular integral operators and applications to operator theory

c. Besov spaces and BMO spaces associated to a differential operator/equation

V. Budget and Expenditure

A. Budget and expenditure of 2015 Below please find the summary of the expenditure of the year 2015 (including the budget of NTU's commitment). We consider most parts of the budget were carried out as in our plan.

1. The major difference comes from the budget of postdoc fellows. We allocated 20 positions of 20M for hiring postdocs. Since 12 fresh postdocs started their appointment from August. So the total budget used is about
   4 (renewal) x 1 yr $+$ 12 (fresh) x $5/12$ yr $+$ 3 (terminated) x $7/12$ yr,
   which is equivalent to about 10 positions. This is the main reason for the difference of about 10M.
2. Another notable difference comes from the budget of domestic long-term visitors. We allocated 1M for this purpose. The idea is to attract domestic researcher to conduct research by visiting NCTS for a longer term. However, only about 0.2M were used. Perhaps the program is not well-known to researchers or perhaps people tend to go abroad when they have a sabbatical.

B. Budget request of 2016

For the coming year of 2016, we request a budget increase of 5M so that the total budget supported by Ministry of Science and Technology is 50M. There are three main reasons.

a. The first one is that we are expecting more long-term visitors for the coming year. We are going to have Prof. Aoki of Kyoto University to visit NCTS for one year starting from April 2016, Prof. Yun-Ping Jiang of CUNY to visit for half an year starting from July. We are contacting Prof. Nakamura of Hokaido to visit for 6 months. These shows that our program of long-term visitors will be even more active next year.

   Other than these, the newly established Research Pair Program receiving more applicants. We expect to have large program next year.

   Taking these into consideration, we would like to request an increase of 3M for the visitors program in total.

b. The second reason is about the scientific program. After comparing with the world leading research centers, it shows that hosting semester-long Special Programs is a quite successful model. One one hand, it will attracts more leading experts and on the other hand, it will increase the visibility of the Center notably. We would like to request an extra 1M for this purpose.

c. The increasing importance of the research in data sciences is overwhelming. The research of data sciences requires further fundamental theoretical studies together with interdisciplinary cooperations. We plan to invest more budget on this aspect. Therefore, we would like to request an extra 1M for this purpose.

VI. Host institution's commitment

The commitment of National Taiwan University was completed fulfilled this year. It mainly consists of the following three parts.

A. Budget

In the proposal, the NTU committed to provide the budget of 20M for the operation of NCTS. This part was completed realized as shown in the Budget section. The host institution NTU will continue to keep this firm commitment.

B. Space

In the proposal, the NTU committed to provide part of the space of Astro-Math Building and Mathematical Research Center for NCTS. This part was completed realized. Those spaces are operated and maintained by NCTS with the assistance of the Department of Mathematics.

C. Administrative Issues

In order to run NCTS more smoothly in the campus of NTU, NCTS was registered as an Research Center at the University level. This means that NCTS has her identity in the administrative system of NTU and entitle to exercise relative privileges and rights. For example, our visitors are entitled to apply for university guest house and access the facilities of NTU.

VII. Appendix

1. Data of Seminars

Title	Dates	Gp.	Place	#
NCTS Number Theory Seminar	3/25, 4/15, 5/13, 6/1, 9/16, 9/23, 9/30	A	NTHU	20-30
NCTS Seminar on Arithmetic Geometry and Representation Theory	8/3, 8/5, 8/5, 8/12, 8/19, 8/19, 8/26, 9/17, 9/24, 10/1, 10/8, 10/15	A	NCTS	25-30
NCTS Reading Seminar in Algebraic Geometry	1/15 ,1/22, 1/29, 4/30, 5/28, 6/9, 7/9, 9/3	B	NCTS	25-40
NCTS Seminar in Algebraic Geometry	4/10, 4/24, 6/5, 6/12, 6/26, 7/3, 7/10, 8/7, 9/25, 10/2, 10/16, 10/23, 10/30	B	NCTS	30-40
NCTS Algebraic Geometry Seminar at NCKU	3/26, 4/9, 5/14, 5/28, 6/4, 6/18, 7/23, 7/30, 8/6, 9/15, 9/22, 9/29, 10/6, 10/20	B	NCKU	35
Learning Seminar in Algebraic Geometry	6/9	B	NCTS	50
Topics in Algebraic Geometry	3/30, 4/13, 4/20, 4/27, 5/4, 5/11, 5/25, 6/1, 6/15	B	NTHU	35
NCTS Differential Geometry Seminar	1/7, 1/15, 3/5, 3/17, 3/18, 3/19, 3/19, 4/9, 4/16, 4/23, 4/30, 5/21, 5/28, 6/11, 9/20, 9/25, 10/1, 10/8, 10/16	C	NCTS	15-40
Sinica-NCTS Geometry Seminar	4/27, 5/18, 5/29, 7/22, 10/16	C	NCTS	18-35
NCTS & NTHU Joint Geometry and Topology Seminar	3/25, 4/1, 4/15, 4/22, 10/14	C	NTHU	25-39
Sinica-NCTS Reading Seminar on Geometry	4/27	C	NCTS	20
NCTS Seminar on Stochastic PDEs	7/6	C	NCTS	41
NCTS Differential Geometry Seminar-Series of Talks on Willmore-type functional	3/17, 3/18, 3/19	C	NCTS	28
NCTS Learning Seminar on Special Holonomy	10/1	C	NCTS	35
IAMS/NCTS Applied Math Seminar	1/9, 3/4, 3/9, 3/18, 3/27, 5/27	D	NCTS	23-44
NCTS Probability Seminar at	3/20, 4/17, 5/1, 5/15, 5/22	D	NCCU	18-36

NCCU				
NCTS Symbolic Dynamics Seminar	3/27, 4/10, 4/24, 5/15, 6/5, 6/12, 6/26	D	NCTS	30-40
IAMS/NCTS/NCU Fluid PDE Seminar	1/9, 3/6, 3/6, 3/13, 3/13, 3/20, 3/20, 4/10, 4/10, 4/24, 4/24, 5/1, 5/1, 5/29, 5/29	D	NCTS	35
NCTS Arithmetic Dynamics Seminar	4/10, 4/24, 5/15, 5/29, 6/5, 6/12, 6/26	D	NCTS	27-37
NCTS Seminar in Partial Differential Equations	4/24, 4/28	D	NCTS	21-40
NCTS PDE & Analysis Seminar	4/20, 5/7, 5/14, 6/11, 6/25, 7/16, 8/20, 9/24, 10/1, 10/15, 10/22	D	NCTS & NTHU	25-40
NCTS & NCU PDE Seminar	7/2, 7/9, 7/22, 8/12, 9/9, 9/16,	D	NCTS & NCU	20-51
NCTS/CMMSC Serminar on Probability Theory and Related Topics	5/27	D	NCTS	34
NCTS Applied Mathematics Seminar	10/15	D	NCTS	30
NCTS/CMMSC Seminar on Probability and Statistics with Applications	5/14, 6/1, 6/18, 6/18, 6/29, 6/30, 7/6	D	NCTU	35
NCTS/CMMS Seminar on Scientific Computing	3/18, 3/27, 4/17, 4/17, 4/24, 4/30, 5/29, 6/17	E	NCTU	22-38
NCTS/CMMSC Seminar on Scientific Computing with Applications	3/18	E	NCTU	25
NCTS/NTU/NCU/NTUST Joint Seminar on Compressive Sensing and Its Applications	10/16, 10/23	E	NCTS	40
NCTS Seminar on Scientific Computing for Data Science	10/23	E	NCTS	50
NCTS Weekly Seminar in Mathematical Modeling	4/25, 4/30, 5/1, 5/1, 5/18, 5/23, 5/21, 6/11, 7/15, 7/21, 9/29, 10/6	F	NCTS & NCHU & CCU	18-45
NCTS Mathematical Biology Seminar	4/24, 5/8, 5/15	F	NTHU	22-33

NCTS Seminar on Signals of ion channels	8/20	F	NCTS	24
NCTS Interdisciplinary Talks	9/4, 9/4, 9/8, 9/8, 9/24	F	NCKU	40
NCTS Interdisciplinary Talks in Mathematical Biology	10/16, 10/23	F	NTHU	40
NCTS /IAMS/ NTUST Data Science Seminar	5/1, 5/8, 5/19	G	NCTS	32
Big Data Seminar	4/8	G	NCTS	50
NCTS Optimization Seminar	1/5	H	NTNU	30

Table 5. 2015 NCTS Seminars

Data of workshops/conferences/courses

Dates	Title	Gp	Place	#	Countries
1/14	One Day Workshop on Nonlinear Analysis, Combinatorial Analysis, and Matrix Analysis	H	NCTS	80	Taiwan, Japan, China, Poland
1/19-1/22	2015 East Asian Core Doctorial Forum on Mathematics		NCTS	60	Taiwan, China, Korea, Vietnam Japan, Korea
2/5	2015 Analysis and PDE Young Scholars Symposium	D	NCTS	50	Taiwan
2/9-2/11	2015 NCTS Winter School in Algebraic Geometry	B	NCTS	31	Taiwan
3/5	2015 NCTS Workshop on Applied M athematics	D	TKU	37	Taiwan, Janpn, USA
3/6-5/13 weekly	NCTS 2015 年 MPI 平行計算春季翻轉教室短期課程	E	NCTS, NSYSU	40	Taiwan
3/6	Mini-Conference on Algebraic Geometry	B	NCTS	51	Taiwan, Japan
3/23-3/27	NCTS Nini-Course on "Periodic Homogenization of Elliptic Problems	D	NCTS	22	Taiwan, U.S.A.
3/4, 3/11, 3/25	NCTS Nano-Course on Scientific Computing	E	NCTS	30	Taiwan
3/23, 3/30, 4/13, 4/20, 4/27	2015 NCTS/CMMSC Short Course in Probability	D	NCTU	50	Taiwan
5/2	The 10th Taiwan Geometry Symposium	C	NCTS	40	Taiwan
5/5, 5/8, 5/12, 5/15, 5/19	NCTS Mini Course in Algebraic Geometry-Linear systems on algebraic varieties	B	NCTS	40	Taiwan, UK
5/5-5/7	NCTS/NCU Short Course on Data Science	G	NCTS	60	Taiwan, China
5/15-5/16	2015 NCTS 計算數學薪傳及新苗研討會	E	NCTU	90	Taiwan
5/15, 5/25, 6/1	2015 NCTS/CMMSC Short Course in Probability	D	NCTU	31	Taiwan
5/19, 5/21, 5/27	NCTS/NCU Short Course on Data Science	E	NCTS & NCU	40	Taiwan, Canada
5/21-5/23	2015 NCTS Workshop on Dynamical Systems	D	NCTS	60	Taiwan, USA, Japan, Korea, China, Hong Kong, France

5/27	NCTS Mini-workshop on Special Values in Positive Characteristic	A	NTHU	30	Taiwan, USA
6/1	NCTS Nano-Course on Scientific Computing	E	NTU	43	Taiwan
6/4-6/5	NCTS Workshop on Reaction-Diffusion Equations and Related Topics	D	NTHU	40	Taiwan, USA, Japan
6/5-6/6	Toward Big Data Analysis Workshop	G	NSYSU	48	Taiwan
6/12-6/13	NCTS 2015 Spring Workshop on Fluid Dynamics	D	NCTS	40	Taiwan, USA, Netherlands
6/15	2015 NCTS Workshop on Applied Mathematics at Tainan	D	Tainan Univ.	50	Taiwan, USA, Japan
6/22	NCTS 2015 Special Day on Symplectic Geometry and Geometric Evolution Equations	C	NCTS	35	Taiwan, UK
6/23	NCTS 2015 Analysis and PDE Young Scholars Summer Symposium	D	NUK	50	Taiwan
6/24-6/25	2015 NCTS Workshop on Subelliptic Operators and Singular Analysis	C	NCTS	40	Taiwan, USA, Japan, China, Hong Kong, Germany
6/25	NCTS/NCU One-Day Workshop on Multiscaling Computational Methods and Compressive Sensing	E	NCU	40	Taiwan, USA
6/29,7/6, 7/13	NCTS Mini Course on Random Matrices	D	NCTS	10 0	Taiwan, USA
2015/6/30 & 7/1	NCTS 2015 暑期資料分析與統計短期課程	E	NCTS	52	Taiwan
7/13, 7/15, 7/17, 8/20, 8/22, 8/24	NCTS Summer Short Course on Drinfeld modules	A	NCTS	28	Taiwan
7/14, 7/16, 7/21, 7/23, 7/28	NCTS Short Course on Random Walks	D	NCTS	36	Taiwan
7/20, 7/22, 7/24	NCTS Summer Short Course on Drinfeld modules and t-motives	A	NTHU	30	Taiwan
7/21, 7/22, 7/23	Mini-course on Cloaking and Invisibility	D	NCTS	40	Taiwan

7/21, 7/23, 7/28, 7/30, 8/4, 8/6, 8/8, 8/11, 8/13	2015 Summer course on Dynamical Systems-- Pattern Generation Problem and Spatial Entropy inHigher Dimensiomal Lattice	D	NCTS	31	Taiwan
7/27, 7/29, 7/ 31, 8/4, 8/5, 8/6	NCTS Summer School: Analysis and Modeling of High Theoughput DNA Methylation Data	F	CCU	36	Taiwan
8/4-8/8	NCTS Summer School: Modeling, Simulation and Analysis of Nonlinear Optics	F	NCKU & NUK	29	Taiwan
8/11,8/18	NCTS Lecture series in Algebraic Geometry-Birational geometry of 3-folds in char p	B	NCTS	40	Taiwan
8/12, 8/13	NCTS 2015 暑期數值天氣預報簡介課程	F	NCTS	31	Taiwan, U.S.A.
8/17-21	NCTS Sunmmer School on Kerr Geometry	C	MCTS	58	Taiwan
8/18	NCTS/NTU/NCU Special Talks on Applied Mathematice	E	NCTS	45	Taiwan, U.S.A.
8/18	NCTS Special Talk on Algebraic Geometry	B	NCTS	24	Taiwan, Janpan
8/18-9/2	NCTS Summer School: Modeling, Simulation and Analysis of Electrolytes	F	NCTS	20	Taiwan
8/19-23	Higher Dimensional Algebraic Geometry	B	NCTS	60	Taiwan, USA, Japan, UK, Germany, Korea, China
9/3,4,7	2015 NCTS Summer Course in Mathematical Biology	F	NTHU	41	Taiwan, USA
9/16-9/18	NCTS 2015 International Workshop on Development and Application of Empirical Dynamic Modeling and Forecasting for Nonlinear Systems	F	NCTS	55	Taiwan, U.S.A., Germany, Japan, France, Australia, Netherlands
9/22-	NCTS Fall Course: Representation Theory of Finite Groups of Lie Type	A	NCTS	13	Taiwan
9/25-	NCTS Fall Course: Abelian Varieties and Related Topics	A	NCTS	35	Taiwan
10/2-10/4	NCTS Distinguished Lecture Series, by Weiming Ni	D	NCTS	60	Taiwan,

10/14	One Day Workshop on Optimization	H	NCTS	50	Taiwan, China
10/12-12/21	NCTS Distinguished Lecture Series, by Fan Chung Graham	A	NCTS	50	Taiwan, USA.
10/24-10/25	NCTS Eleventh Taiwan Geometry Symposium at NDHU	C	NDHU	50	Taiwan
10/29-10/30	2015 NCTS: Mathematical Modeling on Epigenetic Regulation of Gene Expresion	F	NTHU		Taiwan, USA, Italy, Denmark
11/12	One Day Workshop on Big Data	D	NSYSU		Taiwan
12/17-12/31	NCTS Winter School on Shimura Varieties and Related Topics	A	NCTS		Taiwan
12/18-12/31	NCTS Winter School Modeling, Simulation and Analysis of Biology and Physiology	F	NCTS		Taiwan
2016/1/18-2016/1/22	2016 The Third Taiwan International Conference on Differential Geometry	C	NCTS		Taiwan, USA, Japan, China, Canada, France

Table 6. 2015 NCTS Workshops/Conferences/Courses

2. Visitor data

Arr.	Dep.	Name	Title	Nationality	Affiliation
1/9	1/16	Hua, Zheng	Professor	Hong Kong	The University of Hong Kong
1/13	1/15	Goebel, Kazimierz	Professor	Poland	Maria Curie-Sklodowska University in Lublin,
1/13	1/15	Takahashi, Wataru	Professor	Japan	Tokyo Institute of Technology, Japan
1/15	1/19	Shimojo, Masahiko	Professor	Japan	Okayama University, Japan
1/18	1/22	Lin, Wei	Professor	China	Fudan University, China
1/18	1/23	Ogawa, Takayoshi	Professor	Japan	Tohoku University, Japan
1/18	1/23	He, Lingbing	Professor	China	Tsinghua University, China
1/18	1/24	Kim, Panki	Professor	Korea	Seoul National University, South Korea
1/18	1/21	Motoko,Kotani	Professor	Japan	Tohoku University, Japan
1/18	1/21	Yoshio,Tsutsumi	Professor	Japan	Kyoto University, Japan
1/18	1/23	呉泉水	Professor	China	Fudan University, China
1/18	1/23	王志強	Professor	China	Fudan University, China
1/18	1/23	于品	Professor	China	Fudan University, China
1/18	1/23	鍾學秀	Student	China	Tsinghua University, China
1/18	1/23	熊昌偉	Student	China	Tsinghua University, China
1/18	1/23	陽恩林	Student	China	Tsinghua University, China
1/18	1/23	關國卉	Student	China	Tsinghua University, China
1/18	1/23	王躍循	Student	China	Tsinghua University, China
1/18	1/23	Ozawa, Tmomi	Student	Japan	Tohoku University, Japan
1/18	1/23	Sato, Ryuichi	Student	Japan	Tohoku University, Japan
1/18	1/23	Kato, Tsuyoshi	Student	Japan	Tohoku University, Japan
1/18	1/23	Hasegawa, Shoichi	Student	Japan	Tohoku University, Japan
1/18	1/23	Kunikawa, Keita	Student	Japan	Tohoku University, Japan
1/18	1/23	王子鵬	Student	China	Fudan University, China
1/18	1/23	呉歆	Student	China	Fudan University, China
1/18	1/23	匡杰	Student	China	Fudan University, China
1/18	1/23	張理評	Student	China	Fudan University, China
1/18	1/23	任益斌	Student	China	Fudan University, China
1/18	1/23	易超	Student	China	Fudan University, China
1/18	1/23	Yokota, Maho	Student	Japan	Kyoto University, Japan
1/18	1/23	肖吉福	Student	Japan	Kyoto University, Japan
1/18	1/23	Minamide, Arata	Student	Japan	Kyoto University, Japan
1/18	1/23	陽煜	Student	China	Kyoto University, Japan
1/18	1/23	Hasui, Sho	Student	China	Kyoto University, Japan
1/18	1/23	Nishiguchi, Junya	Student	China	Kyoto University, Japan
1/18	1/23	Gotoda, Takeshi	Student	China	Kyoto University, Japan
1/18	1/23	Fukumoto, Yoshiyasu	Student	China	Kyoto University, Japan
1/18	1/23	Komatsu, Takashi	Student	China	Tohoku University, Japan

1/18	1/23	Yobuko, Fuetaro	Student	China	Tohoku University, Japan
1/18	1/23	Kee, Minku	Student	Korea	Ewha Womans University, Korea
1/18	1/23	Kim, Taekyung	Student	Korea	Seoul National University, South Korea
1/18	1/23	Song, Min Ho	Student	Korea	Sungkyunkwan University, Korea
1/18	1/23	Mojallal, Seyedahmad	Student	Korea	Sungkyunkwan University, Korea
1/18	1/23	Oh, Se Jin	Student	Korea	Seoul National University, South Korea
1/18	1/23	Yoon, Jihun	Student	Korea	Seoul National University, South Korea
1/18	1/23	Kim, Kyung Youn	Student	Korea	Seoul National University, South Korea
1/18	1/23	Oh, Jehan	Student	Korea	Seoul National University, South Korea
1/18	1/23	Choi, Woocheol	Student	Korea	Seoul National University, South Korea
1/20	1/23	Kim, Hyun Jin	Student	Korea	Ewha Womans University, Korea
3/2	3/6	Nonomura, Taku	Professor	Japan	JAXA
3/2	3/8	Kaji, Hajime	Professor	Japan	Waseda University, Japan
3/3	3/8	Luckhaus, Stephan	Professor	Germany	University of Leipzig, Germany
3/4	3/7	Suzuki, Taku	Professor	Japan	Waseda University, Japan
3/4	3/7	Watanabe, Kiwamu	Professor	Japan	Saitama University, Japan
3/5	3/7	Ishikawa, Daizo	Student	Japan	Waseda University, Japan
3/5	3/7	Nagai, Yasunari	Professor	Japan	Waseda University, Japan
3/8	3/19	Wang, Zhi-Qiang	Professor	USA	University of Utah, USA
3/12	3/16	Lai, King Fai	Professor	China	Capital Normal University, Beijing, China
3/12	3/24	Dall'Acqua, Anna	Professor	Germany	University of Ulm, Germany
3/13	3/20	Trihan, Fabien	Professor	Japan	Sophia University, Japan
3/14	3/19	Kuwert, Ernst	Professor	Germany	Albert-Ludwigs-University Freiburg, Germany
3/22	4/4	Shen, Zhongwei	Professor	USA	University of Kentucky, USA
3/24	3/26	Tamagawa, Akio	Professor	Japan	RIMS
4/1	5/31	Li, Tie-Xiang	Professor	China	Southeast University, China
4/1	5/31	李鐵香	Professor	China	Southeast University, China
4/7	4/12	Gavrilyuk, Sergey	Professor	France	Aix-Marseille University, France
4/12	4/18	Jinbo, Yoshinori	Post Doc	Japan	Hokkaido University, Japan
4/23	4/30	Ducrot, Arnaud	Professor	France	University of Bordeaux, France
4/29	5/12	Yao, Yuan	Professor	China	Peking University, China
4/30	5/23	Birkar , Caucher	Professor	UK	University of Cambridge, UK
5/1	6/30	Brownawell, W. Dale	Professor	USA	Pennsylvania State University, USA
5/3	5/5	Zhu, Xuding	Professor	China	Zhejiang Normal University, China
5/12	6/4	Wu, Hau-Tieng	Professor	Canada	University of Toronto, Canada
5/15	5/30	Chebotar, Mikhail	Professor	USA	Kent State University , USA
5/20	5/27	Zhu, Yu-Jun	Professor	China	Hebei Normal University, China
5/20	5/24	Dong Han Kim	Professor	Korea	Dongguk University
5/20	5/24	Hiroki Takahasi	Professor	Japan	Keio University, Japan

5/20	5/24	Hiroshi Kokubu	Professor	Japan	Kyoto University, Japan
5/20	5/24	Kazuyuki Yagasaki	Professor	Japan	Kyoto University, Japan
5/20	5/24	Shin Kiriki	Professor	Japan	Tokai University, Japan
5/20	5/24	Yang Wang	Professor	USA	Hong Kong University
5/20	5/24	Yinfei Yi	Professor	USA	Georgia Institute of Technology, USA
5/20	5/24	Jacques Peyriere	Professor	France	Tsinghua University, China
5/21	6/4	Papanikolas, Matthew	Professor	USA	Texas A&M University, USA
5/22	6/18	Choi, Yung-Sze	Professor	USA	University of Connecticut, USA
5/24	7/11	Lo, Chieh Cheng	Post Doc	USA	University of Illinois at Urbana-Champaign,
5/31	6/13	Ren, Xiaofeng	Professor	USA	George Washington University, USA
5/31	7/31	Yau, Horng-Tzer	Professor	USA	Harvard University, USA
6/1	7/15	Tsai, Yen-Hsi Richard	Professor	USA	UT Austin
6/1	3/31	Jinbo, Yoshinori	Post Doc	Japan	Hokkaido University, Japan
6/3	6/6	Takashi Teramoto	Professor	Japan	Asahikawa Medical University
6/3	6/11	Ni, Wei-Ming	Professor	USA	University of Minnesota, USA
6/4	6/17	Wang, Shouhong	Professor	USA	Indiana University, USA
6/7	6/19	Toschi, Federico	Professor	Netherlands	Eindhoven Univ. of Technology, Netherlands
6/7	6/19	Lee, Chung-Min	Professor	USA	California State University, Long Beach, USA
6/8	6/15	Segata, Jun-ichi	Professor	Japan	Tohoku University, Japan
6/11	6/16	Mimura, Masayasu	Professor	Japan	Meiji University, Japan
6/14	6/20	Hu, Bei	Professor	USA	University of Notre Dame, USA
6/14	7/10	Chen, Yu-Ting	Post Doc	USA	Harvard University, USA
6/15	7/4	Pei, Yuchen	Post Doc	UK	University of Warwick, UK
6/16	6/26	Furutani, Kenro	Professor	Japan	Tokyo University of Science, Japan
6/19	6/27	Bauer, Wolfram	Professor	Germany	Leibniz University at Hannover, Germany
6/20	6/27	Lotay, Jason D.	Professor	UK	University College London, UK
6/22	7/8	Xiang, Qing	Professor	USA	University of Delaware, USA
6/22	7/17	Che, Ziliang	Student	USA	Harvard University, USA
6/23	7/2	Chang, Der-Chen	Professor	USA	Georgetown University, USA
6/23	6/26	Iwasaki Chisato	Professor	Japan	Univ. Osaka and Univ. of Hyogo, Japan
6/23	6/26	Li, Yutain	Professor	HongKong	Hong Kong Baptist University
6/23	6/26	Wang, Wei	Professor	China	Zhejiang University, China
6/23	6/26	Xiaojing Lyu	Post Doc	China	Univ. Potsdam, and Tianjin University, China
6/23	7/3	Schulze, Bert-Wolfgang	Professor	Germany	University of Potsdam, Germany
6/24	7/1	Yau, Stephen S-T.	Professor	China	Tsinghua University, China
6/24	7/31	Liu, Chun-Hung	Post Doc	USA	Princeton University, USA
6/25	7/20	Liu, Daphne Der-Fen	Professor	USA	California State University, Los Angeles, USA
7/1	9/3	Wang, Haining	Student	USA	Pennsylvania State University, USA
7/4	7/31	Xue, Jiangwei	Professor	China	Wuhan University, China

7/15	8/15	Shedden, Kerby	Professor	USA	Dept Statistics, University of Michigan,
7/20	7/24	Liu, Hongyu	Professor	Hong Kong	Hong Kong Baptist University
7/22	8/21	Lin, Chin-Hung	Student	USA	Iowa State University, USA
7/26	7/31	Jin, Shi	Professor	USA	Univ. of Wisconsin at Madison
7/29	7/30	Ma, Man Shun John	Student	Canada	University of British Columbia, Canada
8/1	8/23	Birkar , Caucher	Professor	UK	University of Cambridge, UK
8/2	8/30	Kawamata, Yujiro	Professor	Japan	University of Tokyo, Japan
8/3	8/21	Chen, Po-Ning	Professor	USA	Columbia University, USA
8/3	8/21	Wang, Ye-Kai	Post Doc	USA	Michigan State University
8/11	8/23	Wang, Mu-Tao	Professor	USA	Columbia University, USA
8/12	8/24	Okawa, Shinnosuke	Lecturer	Japan	Osaka University, Japan
8/12	8/24	Okawa, Shinnosuke	Professor	Japan	Osaka University, Japan
8/14	8/21	Ito, Atsushi	Professor	Japan	Kyoto University, Japan
8/14	8/22	Hung, Pei-Ken	Student	USA	Columbia University, USA
8/14	8/31	Ito, Atsushi	Post Doc	Japan	Kyoto University, Japan
8/15	9/15	Yang, Ting-Hui	Professor	Taiwan	Tamkang University
8/16	8/30	Hu, Xianpeng	Professor	Hong Kong	The Chinese University of Hong Kong
8/17	8/27	Sosna Pawel	Professor	Germany	University of Hamburg, Germany
8/18	8/27	Min, Misun	Professor	USA	Argonne National Laboratory, USA
8/18	8/23	Patakfalvi, Zsolt	Post Doc	USA	Princeton University, USA
8/18	8/23	Choi, Sung Rak	Post Doc	Korea	Institute for Basic Science, Korea
8/18	8/23	Lee, Yongnam	Professor	Korea	KAIST, Korea
8/18	8/24	Chen, Jiang	Post Doc	Japan	University of Tokyo, Japan
8/18	8/24	Sannai, Akiyoshi	Professor	Japan	University of Tokyo, Japan
8/18	8/24	Ejiri, Sho	Professor	Japan	University of Tokyo, Japan
8/18	8/24	Gongyo, Yoshinori	Professor	Japan	University of Tokyo, Japan
8/18	8/24	Sano, Taro	Post Doc	Japan	Kyoto University, Japan
8/18	8/24	Nakamura, Yusuke	Professor	Japan	University of Tokyo, Japan
8/18	8/26	Chen, Meng	Professor	China	Fudan University, China
8/18	8/27	Fu, Bao-Hua	Professor	China	Chinese Academy of Sciences, China
8/19	8/21	Lin, Jessica	Student	USA	University of Wisconsin at Madison, USA
8/20	8/23	Takagi, Shunsuke	Professor	Japan	University of Tokyo, Japan
8/25	9/1	Horng, Tzyy-Leng	Professor	Taiwan	Feng Chia University
8/30	9/24	Hoshi, Akinari	Professor	Japan	Niigata University, Japan
8/30	9/24	Yamasaki, Aiichi	Professor	Japan	Kyoto University, Japan
8/31	9/6	Ma, Yu-Mei	Professor	China	Dalian Nationalities University
9/13	9/19	Ethan, Deyle	Professor	USA	University of California, San Diego, USA
9/14	9/19	Sugihara, George	Professor	USA	University of California, San Diego, USA
9/14	9/19	Telschow, Arndt	Professor	Germany	University of Munster, Germany

9/15	9/18	Boettiger, Carl	Professor	USA	University of California-Berkeley, USA
9/15	9/19	Munch, Stephan	Professor	USA	NOAA, USA
9/15	9/19	Ushio, Masayuki	Professor	Japan	Ryukoky University, Japan
9/15	9/20	Van Ness, Egbert	Professor	Netherlands	Wageningen University, Netherlands
9/15	9/20	Tsai, Cheng-Han	Professor	Australia	James Cook University, Australia
9/15	9/28	Tasaka, Koji	Professor	Japan	Nagoya University, Japan
9/16	9/18	Yang, Albert	Doctor	Taiwan	Taipei Veterans General Hospital, Taiwan
9/16	9/18	Yen, Chien-Chang	Professor	Taiwan	FuJen Catholic University, Taiwan
9/17	9/21	Guest, Martin A.	Professor	Japan	Waseda University, Japan
9/21	11/20	Fang, Xiaoli	Professor	China	Shaoxing University, China
9/27	10/10	Ni, Wei-Ming	Professor	USA	University of Minnesota, USA
9/27	10/7	Suzuki, Masahiro	Professor	Japan	Tokyo Institue of Technology, Japan
10/1	12/23	Graham, Fan Chung	Professor	USA	University of California, San Diego, USA
10/1	10/6	Kwon, Bongsuk	Professor	Korea	Ulsan National Institute of Science and Technology,
10/2	12/15	Aksoy, Sinan G.	Post Doc	USA	University of California, San Diego, USA
10/16	10/23	Roche-Newton, Oliver	Professor	China	Wuhan University, China
10/25	11/15	Aguda, Baltazar D.	Professor	USA	Disease Pathways LLC, USA
10/25	11/1	Sneppen, Kim	Professor	Denmark	University of Copenhagen, Denmark
10/25	11/1	Bosia, Carla	Post Doc	Italian	Human Genetics Foundation Torino, Italy
10/28	10/30	Fried, Eliot	Professor	Japan	Okinawa Institute of Science and Technology Graduate Univ.
10/28	11/4	Chen, I-Kun	S.Lecturer	Japan	Kyoto University, Japan
11/1	2/1	Ha Seung-Yeal	Professor	Korea	Seoul National University, South Korea
11/5	11/10	Pyo, Juncheol	Professor	Korea	Pusan National University, Korea
11/5	11/10	Lee, Hojoo	Professor	Korea	Korea Institute for Advanced Study
11/5	11/10	Kwong, Kwok-Kun	Professor	Taiwan	National Cheng Kung University
11/8	11/11	Kang, Hyeonbae	Professor	Korea	Inha University, Korea
11/10	11/19	Sere, Eric	Professor	France	University Paris-Dauphine, France
11/15	12/13	Wang, Jiaping	Professor	USA	University of Minnesota, USA
11/18	11/26	Heijster, Petrus van	S.Lecturer	Australia	Queensland University of Technology,
11/19	11/24	Iso, Yuusuke	Professor	Japan	Kyoto University, Japan
11/20	12/2	Jiang, Ming	Professor	China	Peiking University
11/20	11/23	Nishimura, Naoshi	Professor	Japan	Kyoto University, Japan
11/20	11/23	Kato, Hiroshi	Professor	Japan	Japan Aerospace Exploration Agency
11/20	11/23	Okawa, Shinpei	Professor	Japan	National defense medical College
11/20	11/24	Kawagoe, Daisuke	Professor	Japan	Kyoto University, Japan
11/20	11/24	Imai, Hitoshi	Professor	Japan	Doshisha University, Japan
11/20	11/24	Fujiwara, Hiroshi	Professor	Japan	Kyoto University, Japan
11/20	11/25	Nishida, Takaaki	Professor	Japan	Kyoto University, Japan
11/20	11/23	Hashimoto, Koh	Professor	Japan	Keio University, Japan

11/21	11/23	Endo, Ryuji	Professor	Japan	Polytechnic University of Japan
11/21	11/23	Higuchi, Tomoyuki	Professor	Japan	The institute of Statistical Mathematics
11/21	11/23	Yamamoto, Masahiro	Professor	Japan	University of Tokyo, Japan
12/12	12/18	Wang, Mu-Tao	Professor	USA	Columbia University, USA
12/14	12/21	Totaro, Burt	Professor	USA	UCLA
12/15	1/7	Chen, Miaofen	Professor	China	East China Normal University, China
12/15	1/2	Shen Xu	Professor	China	Chinese Academy of Sciences, China
12/16	12/28	Nie, Sian	Professor	China	Chinese Academy of Sciences, China
12/16	1/6	Jiang, Zhi	Professor	France	Paris Sud, France
12/16	12/23	Li, Bo	Professor	USA	University of California, San Diego, USA
12/19	12/30	Yasuda, Seidai	Professor	Japan	Osaka University, Japan
12/20	1/1	Liu, Weishi	Professor	USA	University of Kansas, USA
12/20	12/26	Duong, Xuan Thinh	Professor	Australia	Macquarie University
12/20	12/31	Cheng, Xiao-Lin	Professor	USA	University of Tennessee, USA
12/21	1/12	Geisser, Thomas	Professor	Japan	Nagoya University, Japan
12/21	12/30	Xu, Zhiliang	Professor	USA	Norte Dame University, USA
12/22	12/31	Liu, Chun	Professor	USA	Pennsylvania State University, USA

Table 7. 2015 NCTS Foreign Visitors

3. Summary of publications

Publications of Key Members

Journal Title	2014	2015	to appear
ADV APP MATH MECH	1		
ADV MATH	1	2	
ADV MATH. PHYS.	1		
ALGEBR COLLOQ			1
ALGEBR REPRESENT TH		1	
AM J MATH		1	
ANAL APPL	1		
ANN FUNCT ANAL	1		
ANN HENRI POINCARE			1
ANN INST STAT MATH		1	
ANN SC NORM SUPER PISA CL	1		1
ANN PROBAB		1	
APPL STOCH MODELS BUS IND	1		
APPL MATH COMPUT		1	
APPL MATH MODEL		1	
ARCH RATIONAL MECH ANAL		1	2
ASIAN J MATH	1		2
Bernoulli Journal.	1		
BIT NUMER MATH		1	
BULL INST MATH ACAD SIN	2		1
CALC VAR PARTIAL DIF	1	1	
CAN MATH BULL		1	
COMM MATH PHYS	1		
COMM MATH SCI	1		1
COMMUM PDE		1	
COMMUN ANAL GEOM		1	
COMMUN COMPUT PHYS	2	2	1
COMMUN INF SYSTEM	2		
COMMUN MATH PHYS		1	
COMMUN NONLINEAR SCI NUM SIMUL	1		
COMMUN PUR APPL ANAL	1		
COMPOS MATH	2	2	
COMPUT MATH APPL	1		

Comput. and Num. Simul.	1		
COMPUT. METHODS APPL. MECH. ENG	1		
Computational Statistics	1		
Computational Statistics and Data Analysis	3		1
CONTEMP MATH	1		
DISCRET CONTIN DYN S -A	3	1	
DISCRET CONTIN DYN S -B	1		
DISCRET CONTIN DYN S -S	1		
DUKE MATH J	1		
FORUM MATH		2	
IMA J NUMER ANAL	1	1	
IMAG COMPUT	1		
INT MATH RES NOTICES	1		2
INTEGR EQUAT OPER TH	2		
INVERSE PROBL	2		
INVERSE PROBL IMAG	1		
J ALGEBRA	1	4	1
J AM MATH SOC	1		
J APPL STAT	1		
J COMP APPL MATH	1		
J COMPUT APPL MATH	1		
J COMPUT GRAPH STAT		1	
J COMPUT MATH	1		
J COMPUT PHYS	6	2	
J DIFFER EQUATIONS	3	5	1
J DYN DIFFER EQU		1	
J ELASTICITY	1	1	
J FOURIER ANAL APPL	2		
J FUNCT ANAL	4	1	
J GEOM ANAL	1		1
J GLOB OPTIM	1		
J MATH ANAL APPL	1	1	
J MATH PHYS	2	2	
J MATH SOC JPN		1	
J OPERAT THEOR	1		
J PURE APPL ALGEBRA	1	1	
J REINE ANGEW MATH	2		4

J SCI COMPUT	1	5	
J STAT PHYS		2	
J SYMB COMPUT			1
J THEOR PROBAB	1		
KODAI MATH J	1		1
LINEAR ALGEBRA APPL	3		
LINEAR MULTILINEAR A	2	1	
MANUSCRIPTA MATH	1		
MATH RES LETT		1	
MATH Z	1		
NAGOYA MATH J	1		
Neural Processing Letters	1		
NEURAL NETW		1	
NEW YORK J MATH		1	
NONLINEARITY		3	
NUM METHOD FOR PDE	1		
NUMER ALGORITHMS	1		
NUMER MATH		1	
P AM MATH SOC	2	1	
P ROY SOC EDINB A		1	
PAC J MATH	2		
Parallel Computing	1		
PHYS LETT SECT A	1	1	
PHYSICA A		1	
PLOS ONE	1	2	
PURE APPL MATH Q	1	1	
Quantitative Finance	1		
REV MAT IBEROAM			1
REV MATH PHYS	1		
SCI CHINA MATH			1
SIADS		1	
SIAM J MATH ANAL	1		1
SIAM J SCI COMP		1	
SIAM MATRIX ANAL APPL		1	
SIAM SCI COMP		2	
SIAM J APPL MATH	1	1	
SIAM J MATH ANAL	1	1	

SIAM J MATRIX ANAL APPL		1	
SIGMA	1		
STAT COMPUT		3	
Statistics and Computing	1		
T AM MATH SOC	2	3	1
TAIWAN J MATH		1	3
Technometrics		1	1
TOHOKU MATH J			1
Total	**106**	**81**	**31**

Table 8. Summary of Publication of NCTS Key Members

4. Publications of key members

Publication List			
Author	Title	Journal	Year
Ban, Jung-Chao	Realization problem of multi-layer cellular neural networks	NEURAL NETW	2015
	Solution Structure of Multi-layer Neural Networks with Initial Condition	J DYN DIFFER EQU	2015
	The spatial complexity of inhomogeneous multi-layer neural networks	Neural Processing Letters	2014
	The multifractal spectra for the recurrence rates of beta-transformations	J MATH ANAL APPL	2014
	Neural networks equations and symbolic dynamics	INT. J. MACH. LEARN. CYBER	2014
	Dimension Spectrum for Sofic Systems	ADV MATH. PHYS.	2014
	Tracking mean field dynamics by synchronous computations of recurrent multilayer perceptrons	Comput. and Num. Simul.	2014
Chern, I-Liang	Stability of non-monotone critical traveling waves for reaction-diffusion equation with time-delay	J DIFFER EQUATIONS	2015
	A Complete Study of the Ground State Phase Diagrams of Spin-1 Bose-Einstein Condensates in a Magnetic Field via Continuation Methods	J. SCI. COMPUT	2015
	Accurate Gradient Approximation for Complex Interface Problems in 3D by an Improved Coupling Interface Method	J COMPUT MATH	2014
	A kinetic energy reduction technique and characterizations of the ground states of spin-1 Bose-Einstein condensates	DISCRETE CONT DYN-A	2014
	String-averaging expectation maximization for maximum likelihood estimation in emission tomography	Inverse Problems	2014
	Spectral collocation and a two-level continuation scheme for dipolar Bose-Einstein condensates	J. COMPUT. PHYS	2014
	Mitigate B11 Inhomogeneity Using Spatially Selective Radio-frequency Excitation with Generalized Spatial Encoding Magnetic Fields	Magnetic Resonance in Medicine	2014
Chen, Ray-Bing	Bayesian Variable Selection for Finite Mixture Model of Linear Regressions.	Computational Statistics and Data Analysis	to appear
	BSGS: Bayesian Sparse Group Selection	R J	to appear
	Optimizing Two-level Supersaturated Designs using Swarm Intelligence Techniques	Technometrics	to appear
	Bayesian Variable Selection for Probit Model with Componentwise Gibbs Sampler.	Comm. in Stat. –Simul. and Comput.	to appear
	Minimax Optimal Designs via Particle Swarm Optimization Methods	Statistics and Computing	2015
	A Modified Particle Swarm Optimization Technique for Finding Optimal Designs for Mixture Models	PLOS ONE	2015
	Bayesian Sparse Group Selection	J. COMPUT. GRAPH. STAT	2015

	COPICA - Independent Component Analysis via Copula Techniques.	STAT. COMPUT	2015
	A Note on Conditionally Optimal Star Points in Central Composite Designs for Response Surface Methodology.	Journal of the Chinese Statistical Association	2015
	Using Animal Instincts to Design Efficient Biomedical Studies via Particle Swarm Optimization.	Swarm and Evolutionary Computation	2014
	Adaptive Block Size for Dense QR Factorization in Hybrid CPU-GPU Systems via Statistical Modeling.	Parallel Computing	2014
	Time Varying Independent Component Analysis and Its Application to Financial Data.	Computational Statistics and Data Analysis	2014
	Discrete Particle Swarm Optimization for Constructing Uniform Design on Irregular Regions.	Computational Statistics and Data Analysis	2014
	Contour Estimation via Two Fidelity Computer Simulators under Limited Resources.	Computational Statistics	2014
Cheong, Wan-Keng	From GW invariants of symmetric product stacks to relative invariants of threefolds	Algebraic Geometry in East Asia	2015
Chang, Chieh-Yu	Frobenius difference equations and difference Galois groups	Proceedings of the Banff workshop ont-motives	to appear
	On characteristic p multizeta values	RIMS Kokyuroku Bessatsu	to appear
	Linear independence of monomials of multizeta values in positive characteristic	COMPOS MATH.	2014
Chen, Lung-Chi	Asymptotic Behavior for a Version of Directed Percolation on the honeycomb Lattice	PHYSICA A	2015
	Critical two-point functions for long-range statistical-mechanical models in high dimensions	ANN. PROBAB	2015
	A monotonicity result for the range of a perturbed random walk''	J THEOR PROBAB	2014
Chen, hao-Nien	Traveling pulse solutions to FitzHugh-Nagumo equations	CALC VAR PARTIAL DIF	2015
	Spectral comparison and gradient-like property in the FitzHugh-Nagumo type equations	Nonlinearity	2015
	Stability analysis for standing pulse solutions to FitzHugh-Nagumo equations	CALC VAR PARTIAL DIF	2014
	Planar standing wavefronts in the FitzHugh-Nagumo equations	SIAM J MATH ANAL	2014
	A variational approach for standing waves of FitzHugh-Nagumo type systems	J DIFFER EQUATIONS	2014
	Connecting orbits for subharmonic solutions in time reversible Hamiltonian systems	Special issue in honor of Neil Trudinger	2014
Chen, Jungkai	Explicit birational geometry of 3-folds and 4-folds of general type, III	COMPOS MATH	2015
	The Noether inequality for Gorenstein minimal 3-folds	COMMUN ANAL GEOM	2015
	Varieties fibered over abelian varieties with fibers of log general type	ADV MATH	2015
	Varieties with vanishing holomorphic Euler characteristic	J REINE ANGEW MATH	2014

	Factoring threefold divisorial contractions to points	ANN SCUOLA NORM-SCI	2014
	Positivity in varieties of maximal Albanese dimension	J REINE ANGEW MATH	to appear
Chuah, Meng-Kiat	Dirac cohomology and geometric quantization	J REINE ANGEW MATH	to appear
	Journal of Functional Analysis	J ALGEBRA	2015
	Partially harmonic forms and models of H-series	J FUNCT ANAL	2014
Dang, Tuan-Hiep	Rational curves on Calabi-Yau threefolds: Verifying mirror symmetry predictions	J SYMB COMPUT	to appear
	On the degree of Fano schemes of linear subspaces on hypersurfaces	KODAI MATH J	to appear
Fang, Xiang	Two weight inequalities for the Bergman projection with doubling measures.	TAIWAN J MATH	2015
	Fiber dimension for invariant subspaces	J FUNCT ANAL	2015
Fei, Jia-Rui	Constructing coherently G-invariant modules, to appear	J ALGEBRA	to appear
	General presentations of algebras	ADV MATH	2015
	On some quiver determinantal varieties	J ALGEBRA	2015
	Counting using Hall algebra II. Extensions from quivers,	ALGEBR REPRESENT TH	2015
Fok, Chi-Kwong	The Real K-Theory of Compact Lie Groups	SIGMA	2014
Furutani, Kenro	Spectral zeta function on pseudo h-type nilmanifolds	INDIAN J PURE AP MAT	2015
Gau, Hwa-Long	Numerical radii for tensor products of matrices.	LINEAR MULTILINEAR A	2015
	Power partial isometry index and ascent of a finite matrix.	LINEAR ALGEBRA APPL	2014
	Excursions in numerical ranges.	BULL INST MATH ACAD SIN	2014
	Numerical radii for tensor products of operators.	INTEGR EQUAT OPER TH	2014
	Equality of higher-rank numerical ranges of matrices.	LINEAR MULTILINEAR A	2014
	Refinements of numerical ranges of weighted shift matrices.	LINEAR MULTILINEAR A	2014
	Structures and numerical ranges of power partial isometries.	LINEAR ALGEBRA APPL	2014
	Zero-dilation index of a finite matrix.	LINEAR ALGEBRA APPL	2014
	Zero-dilation indices of KMS matrices.	ANN FUNCT ANAL	2014
Guo, Jong-Shenq	The existence of traveling wave solutions for a bistable three-component lattice dynamical system	J DIFFER EQUATIONS	to appear
	On a free boundary problem for the curvature flow with driving force	ARCH RATIONAL MECH ANAL	to appear
	The minimal speed of traveling wave solutions for a diffusive three species competition system	TAIWAN J MATH	to appear
	On the dead-core problem for the p-Laplace equation with a strong absorption	TOHOKU MATH J	to appear
	No touchdown at zero points of the permittivity profile for the MEMS problem	SIAM J MATH ANAL	2015

	Dynamics for a two-species competition-diffusion model with two free boundaries	Nonlinearity	2015
	Recent developments on a nonlocal problem arising in the micro-electro mechanical system	TAMKANG J MATH	2014
	Hyperbolic quenching problem with damping in the micro-electro mechanical system device	DISCRETE CONTIN DYN SYST	2014
Guo, Mei-Hui	Optimal Restricted Quadratic Estimator of Integrated Volatility.	ANN INST STAT MATH	2015
	COPICA - Independent Component Analysis Via Copula Techniques.	STAT COMPUT	2015
	Estimation of Inverse Autocovariance Matrices for Long Memory Processes.	Bernoulli Journal.	2014
	Assessing Influential Trade Effects via High Frequency Market Reactions.	J APPL STAT	2014
	Model Risk of the Implied GARCH-normal Model.	Quantitative Finance	2014
	The Bickel-Rosenblatt Test for Continuous Time Stochastic Volatility Models.	TEST	2014
	Monitoring Change Point for Diffusion Parameter Based on Discretely Observed Sample from SDE Models.	APPL STOCH MODELS BUS IND	2014
Hsieh, Ming-Lun	Special values of anticyclotomic L-functions for modular forms.	J REINE ANGEW MATH	to appear
	On the anticyclotomic Iwasawa main conjecture for modular forms.	COMPOS MATH	2015
	Eisenstein congruence and Iwasawa main conjectures for CM fields.	J AM MATH SOC	2014
	On the vanishing of mu-invariant of anticyclotomic p-adic L-functions for CM fields.	J REINE ANGEW MATH	2014
Ho, Nan-Kuo	Connected Components of Surface Group Representations for Complex reductive Lie groups	NEW YORK J MATH	2015
	Intersection Cohomology of the Universal Imploded Cross-section of SU(3)	PURE APPL MATH Q	2014
Hsu, Sze-Bi	Multiplesteady-state in phytoplankton population induced by photo-inhibition	J DIFFER EQUATIONS	2015
	A pivotal eigenvalue problem in river ecology	J DIFFER EQUATIONS	2015
	On a nonlocal reaction-diffusion-advection system modeling the growth of phytoplankton with cell quota structure	J DIFFER EQUATIONS	2015
	A reaction-diffusion-advection model of harmful algae growth with toxin degradation	J DIFFER EQUATIONS	2015
	A nonlocal problem from conservation biology	SIADS	2015
	Dynamics of two phytoplankton Species Competing for light and nutrient with internal storage.	DISCRET CONTIN DYN S -S	2014
	Further studies of a reaction-diffusion system for an unstirred chemostat with internal storage	DISCRET CONTIN DYN S -B	2014

Hu, Zheng-Yu	Valuations and log canonical thresholds	PURE APPL MATH Q	2015
	Log canonical pairs and good augmented base loci	COMPOS MATH	2014
	Polarized pairs, log minimal models, and Zariski decomposition	NAGOYA MATH J	2014
	Valuative multiplier ideal.	PAC J MATH	2014
Huang, Hsin-Yuan	Uniqueness of topological multi-vortex solutions for a skew-symmetric Chern-Simons system	J MATH PHYS.	2015
	Classification of the entire radial self-dual solutions to non-Abelian Chern-Simons systems	J FUNCT ANAL	2014
	Uniqueness of non-topological solutions for the Chern-Simons system with two Higgs particles	KODAI MATH J	2014
Hwang, Gyeongha	On the finite time blowup for Hartree equations	P ROY SOC EDINB A	2015
	Well-posedness and ill-posedness for the cubic fractional Schrödinger equations	DISCRET CONTIN DYN S -A	2015
	Profile decompositions of fractional Schrödinger equations with angularly regular data	J DIFFER EQUATIONS	2014
	On the orbital stability of fractional Schrödinger equations	COMMUN PUR APPL ANAL	2014
Kaji, Hajime	Higher gauss maps of veronese varieties—a generalization of boole's formula	Preprint	2015
Lai, Ming-Chih	An unconditionally energy stable penalty immersed boundary method for simulating the dynamics of an inextensible interface interacting with a solid particle	J SCI COMPUT	2015
	A hybrid immersed boundary and immersed interface method for electrohydrodynamic simulations	J COMPUT PHYS	2015
	An immersed interface method for axisymmetric electrohydrodynamic simulations in Stokes flow	COMMUN COMPUT PHYS	2015
	A conservative scheme for solving coupled surface-bulk convection-diffusion equation with an application to interfacial flows with soluble surfactant	J COMPUT PHYS	2014
	A coupled immersed interface and level set method for three-dimensional interfacial flows with insoluble	COMMUN COMPUT PHYS	2014
	An immersed boundary method for simulating the dynamics of three-dimensional axisymmetric vesicles in Navier-Stokes flows	J COMPUT PHYS	2014
	Numerical simulations of three-dimensional foam by the immersed boundary method	J COMPUT PHYS	2014
Lam, Ching-Hung	Orbifold construction of holomorphic vertex operator algebras associated to inner automorphisms	COMMUN COMPUT PHYS	to appear
	Groups of Lie type, vertex algebras, and modular moonshine	INT MATH RES NOTICES	To appear
	Griess algebras generated by the Griess algebras of two	J MATH SOC JPN	2015

	3A-algebras with a common axis		
	Classification of holomorphic framed vertex operator algebras of central charge 24	AM J MATH	2015
	A level rank duality for parafermion vertex operator algebras of type	P AM MATH SOC	2014
	An explicit Majorana representations of the group 32: 2 of 3C pure type	PACIFIC J MATH	2014
	Zhu's algebras, C2-algebras and C2-cofiniteness of parafermion vertex operator algebras	ADV MATH	2014
	Weyl groups and vertex operator algebras generated by Ising vectors satisfying (2B; 3C)-condition	REV MATH PHYS	2014
	On 3-transposition groups generated by δ-involutions associated to c = 4/5 Virasoro vectors	J ALGEBRA	2014
	Coset construction of Z/3Z orbifold vertex operator algebra	J PURE APPL ALGEBRA	2014
Lee, Ming-Yi	Characterization of Campanato spaces associated with parabolic sections	ASIAN J MATH	to appear
	VMO Space associated with parabolic sections and its application	CAN MATH BULL	2015
	Boundedness of Calderon-Zygmund operators on weighted product Hardy spaces	J OPERAT THEOR	2014
	Ap,E weights, maximal operators, and Hardy spaces associated with a family of general sets	J FOURIER ANAL APPL	2014
Lin, Tai-Chia	Exponential decay estimates for the stability of boundary layer solutions to Poisson-Nernst-Planck systems: one spatial dimension case.	SIAM J MATH ANAL	to appear
	Asymptotic Analysis of Poisson-Boltzmann equations with Constrained Ionic Densities for Multi-species Ions.	COMM MATH SCI	to appear
	Multiple solutions of steady-state Poisson-Nernst-Planck equations with steric effects.	NONLINEARITY	2015
	Diffusion limit of kinetic equations for multiple species charged particles.	ARCH RATIONAL MECH ANAL.	2015
	Transport of Charged Particles: Entropy Production and Maximum Dissipation Principle.	J MATH ANAL APPL	2015
	Ground states of nonlinear Schr¨odinger systems with saturable nonlinearity in R2 for two counterpropagating beams.	J MATH PHYS	2014
	Development of traveling waves to an interacting two-species chemotaxis model.	DISCRETE CONT DYN S-A	2014
	A new approach to the Lennard-Jones potential and a new model: PNP-steric equations.	COMM MATH SCI	2014
Lin, Chin-Cheng	Criterion of the L2 boundedness and sharp endpoint estimates for singular integral operators on product spaces of homogeneous type.	ANN SC NORM SUPER PISA CL SCI.	to appear
	Characterization of Campanato spaces associated with parabolic	ASIAN J MATH	to appear

	sections.		
	Boundedness of Monge-Ampere singular integral operators acting on Hardy spaces and their duals.	T AMER MATH SOC	to appear
	Atomic Hp spaces and their duals on open subsets of R^d.	FORUM MATH	2015
	A p,E weights, maximal operators, and Hardy spaces associated with a family of general sets.	J FOURIER ANAL APPL	2014
	Boundary values of harmonic functions in spaces of Triebel-Lizorkin type.	INTEGR EQUAT OPER TH	2014
	Weighted norm inequalities for multilinear singular integral operators and applications.	ANAL APPL	2014
Lin, Wen-Wei	A Robust Numerical Algorithm for Computing Maxwell's Transmission Eigenvalue Problems.	SIAM SCI COMP	2015
	Backward Perturbation Analysis and Residual-Based Error Bounds for the Linear Response Eigenvalue Problem.	BIT NUMER MATH	2015
	A Positivity Inexact Noda Iteration for Computing the Smallest Eigenpair of a Large Irreducible M-Matrix.	NUMER MATH	2015
	On Spectral Analysis and a Novel Algorithm for Transmission Eigenvalue Problems.	J SCI COMPUT	2015
	Singular Value Decompositions for Single-Curl Operators in Three-Dimensional Maxwell's Equations for Complex Media.	SIAM MATRIX ANAL APPL	2015
	A Null Space Free Jacobi-Davidson Iteration for Maxwell's Operator.	SIAM SCI COMP	2015
	An Efficient Algorithm of Yau-Yau Method for Solving Nonlinear Filtering Problems.	COMMUN INF SYSTEM	2014
	A Novel Efficient Homotopy Continuation Method in Tracking.	COMMUN INF SYSTEM	2014
	A Novel Symmetric Skew-Hamiltonian Isotropic Lanczos Algorithm for Spectral Conformal Parameterizations.	J SCI COMPUT	2014
	Eigenvalue Solvers for Three Dimensional Photonic Crystals with Face-Centered Cubic Lattices.	J COMP APPL MATH	2014
	High Performance Computing for Spherical Conformal and Riemann Mappings.	IMAG COMPUT	2014
	New Solvers for Higher Dimensional Poisson Equations by Reduced B-Splines.	NUM METHOD FOR PDE	2014
Lee, Yng-Ing	Spacelike Spherically Symmetric CMC Foliation in the Extended Schwarzschild Space	ANN HENRI POINCARE	to appear
	The Stability of Self-shrinkers of Mean Curvature Flow in Higher Co-dimension	T AM MATH SOC	2015
	Stability of the Minimal Surface System and Convexity of Area Functional	T AM MATH SOC	2014
Lin, Min-Hsiung	An algorithm for constructing nonnegative matrices with prescribed real eigenvalues	APPL MATH COMPUT	2015

	A Note on Sylvester-type Equations	J FRANKLIN INST	2015
	On the finite rank and finite-dimensional representation of bounded semi-infinite Hankel operators	IMA J NUMER ANAL	2015
	A study of singular spectrum analysis with global optimization techniques	J GLOB OPTIM	2014
	Numerical methods for solving nonnegative inverse singular value problems with prescribed structure	INVERSE PROBL	2014
	Nonnegative rank factorization|a heuristic approach via rank reduction	NUMER ALGORITHMS	2014
Lin, Chun-Chi	The second-order L^2-flow of inextensible elastic curves with hinged ends in the plane, J. Elasticity	J ELASTICITY	2015
	Interior continuity of two-dimensional weakly stationary-harmonic multiple-valued functions	J GEOM ANAL	2014
	Evolution of open elastic curves in R^n subject to fixed length and natural boundary conditions	Analysis (Berlin)	2014
Peng, Yung-Ning	Parabolic presentations of the super yangian y (glm|n) associated with arbitrary 01-sequences	COMMUN MATH PHYS	2015
Seol, Yun-Chang	Numerical simulations of three-dimensional foam by the immersed boundary method	J COMPUT PHYS	2014
Shen, Chun-Yen	Extension estimates on a system of homogeneous equations for finite fields	J GEOM ANAL	to appear
	A note on failure of energy reversal for classical fractional singular integrals	INT MATH RES NOTICES	to appear
	A two weight theorem for fractional singular integrals with an energy condition	REV MAT IBEROAM	to appear
	Two weight inequality for the Hilbert transform: A real variable characterization	DUKE MATH J	2014
Sheu, Yuan-Chung	Disorder chaos in the spherical mean-field model.	J STAT PHYS	2015
Tsai, Yen-Hsi	Fast sweeping methods for hyperbolic systems of conservation laws at steady state II	J COMPUT PHYS	2015
Tsai, Je-Chiang	Stationary waves on the sphere	SIAM J APPL MATH	2015
	A Mathematical model of bimodal epigenetic control of miR-193a in ovarian cancer stem cell	PLOS ONE	2014
	Curvature dependence of propagating velocity for a simplified calcium model	SIAM J APPL MATH	2014
	The evolution of traveling waves in a simple isothermal chemical system modelling quadratic autocatalysis with strong decay	J DIFFER EQUATIONS	2014

Tsui, Mao-Pei	Generalized lagrangian Mean Curvature Flows	preprint	2015
	Curvature Decay Estimates of Graphical Mean Curvature Flow in Higher Codimensions	T AM MATH SOC	2015
	Stability of the minimal surface system and convexity of area functional	T AM MATH SOC	2014
Wang, Wei-Cheng	A Nullspace-Free Jacobi-Davidson Iteration for Maxwell's Operator.	SIAM J SCI COMP	2015
	A Kernel-free Boundary Integral Method for Variable Coefficient Elliptic PDEs	COMMUN COMPUT PHY	2014
Wang, Yong-Jie	The Steinbery Lie algebra st2(S),	ALGEBR COLLOQ	to appear
	Simple Harish-Chandra supermodules over the super Schrodinger algerba	SCI CHINA MATH	to appear
	A family of representations of the Lie superalgebra[glm|n(Cq)	J ALGEBRA	2015
	Color cyclic homology and Steinberg Lie color algebra	FRONT MATH CHINA	2015
	Simple Harish-Chandra modules over the super Schrodinger algerba in (1 + 1) dimensional spacetime	J MATH PHYS	2014
Wang, Wei-Chung	Effective Anatomical Priors for Emission Tomographic Reconstruction	J Medical and Biological Engineering	2015
	Minimax Optimal Designs via Particle Swarm Optimization Methods	STAT COMPUT	2015
	A Complete Study of the Ground State Phase Diagrams of Spin-1 Bose-Einstein Condensates in a Magnetic Field via Continuation Methods	J SCI COMPUT	2015
	Optimizing Two-level Supersaturated Designs by Particle Swarm	Technometrics	2015
	Singular Value Decompositions for Single-Curl Operators in Three-Dimensional Maxwell's Equations for Complex Media	SIAM J MATRIX ANAL APPL	2015
	A Modified Particle Swarm Optimization Technique for Finding Optimal Designs for Mixture Models	PLOS ONE	2015
	Discrete Particle Swarm Optimization for Constructing Uniform Design on Irregular Regions	Computational Statistics and Data Analysis	2014
	Performance Models and Workload Distributions for Optimizing a Hybrid CPU-GPU Multifrontal Solver	COMPUT MATH APPL	2014
	Eigenvalue Solvers for Three Dimensional Photonic Crystals with Face-Centered Cubic Lattice	J COMPUT APPL MATH	2014
	Adaptive Block Size for Dense QR Factorization in Hybrid CPU-GPU Systems via Statistical Modeling	Parallel Computing	2014
	Using Animal Instincts to Design Efficient Biomedical Studies	SWARM EVOLUT COMPUT	2014
Wang, Jenn-Nan	Inverse boundary value problem for the Stokes and the Navier-Stokes equations in the plane.	ARCH RATIONAL MECH ANAL	to appear

	Estimate of an inclusion in a body with discontinuous conductivity.	BULL INST MATH ACAD SIN	to appear
	Quantitative uniqueness estimates for second order elliptic equations with unbounded drift.	MATH RES LETT	2015
	On Landis' conjecture in the plane.	COMMUM PDE	2015
	Increasing stability for determining the potential in the Schrődinger equation with attenuation from the Dirichlet-to-Neumann map.	INVERSE PROBL IMAG	2014
	Increasing stability of the inverse boundary value problem for the Schrődinger equation.	CONTEMP MATH	2014
	Quantitative uniqueness estimates for the general second order elliptic equations.	J FUNCT ANAL	2014
	Bounds on the volume fraction of the two-phase shallow shell using one measurement.	J ELASTICITY	2014
Wu, Kung-Chien	Nonlinear stability of the Boltzmann Equation in a periodic box	J MATH PHYS	2015
	Pointwise discription for the linearized Fokker-Planck-Boltzmann Model	J STATIS PHYS	2015
	Low Froude number limit of the rotating shallow water and Euler equations	P AM MATH SOC	2014
	Global in time estimates for the spatially homogeneous Landau equation with soft potentials	J FUNCT ANAL	2014
	Pointwise Behavior of the Linearized Boltzmann Equation on a Torus	SIAM J MATH ANAL	2014
	An incompressible limit for a Navier-Stokes system with capillary effects	COMM MATH PHYS	2014
Yang, Suh-Yuh	An unconditionally energy stable penalty immersed boundary method for simulating the dynamics of an inextensible interface interacting with a solid particle	J SCI COMPUT	2015
	Effects of network structure on the synchronizability of nonlinearly coupled Hindmarsh-Rose neurons	PHYS LETT A	2015
	Eventual dissipativeness and synchronization of nonlinearly coupled dynamical network of Hindmarsh-Rose neurons	APPL MATH MODEL	2015
	A novel technique for constructing diference schemes for systems of singularly perturbed equations	COMMUN COMPUT PHYS	2015
	Effect of permeability-porosity functions on simulated morphological evolution of a chemical dissolution front in a fluid-saturated porous medium	Hydrological Processes	2014
	Analysis of the small viscosity and large reaction coefficient in the computation of the generalized Stokes problem by a novel stabilized finite element method	COMPUT. METHODS APPL. MECH. ENG	2014
	Analysis of epidemic spreading of an SIRS model in complex heterogeneous networks	COMMUN NONLINEAR SCI NUM SIMUL	2014

	A high-accuracy finite difference scheme for solving reaction-convection-diffusion problems with a small diffusivity	ADV APP MATH MECH	2014
	Error and attack tolerance of synchronization in Hindmarsh-Rose neural networks with community structure	PHYS LETT SECT A	2014
	Computation of Maxwell singular solution by nodal-continuous elements	J COMPUT PHYS	2014
	An SPD stabilized finite element method for Stokes equations	IMA J NUMER ANAL	2014
Yu, Jeng-Daw	E1-degeneration of the irregular Hodge ltration	J REINE ANGEW MATH	to appear
	On the irregular Hodge ltration of exponentially twisted mixed Hodge modules.	FORUM MATH	2015
	Irregular Hodge ltration on twisted de Rham cohomology.	MANUSCRIPTA MATH	2014
Yu, Chia-Fu	On existence and density of the ordinary locus of certain Shimura varieties	PROC 6th ICCM	to appear
	A Note on the Mumford-Tate conjecture for CM abelian varieties	TAIWAN J MATH	to appear
	Abelian varieties over finite fields as basic abelian varieties	MPIM Preprint	to appear
	Abelian varieties without a prescribed Newton polygon reduction	P AM MATH SOC	2015
	Monomial, Gorenstein and Bass orders	J PURE APPL ALGEBRA	2015
	Variants of mass formulas for definite division algebras	J ALGEBRA	2015
	Endomorphism algebras of factors of certain hypergeometric Jacobians	T AM MATH SOC	2015
	Class numbers of central simple algebras over global function fields	INT MATH RES NOTICES	2014
	Fixed points and homology of superelliptic Jacobians	MATH Z	2014
	Notes on density of the ordinary locus	BULL INST MATH ACAD SIN	2014
	Embeddings of fields into simple algebras over global fields	ASIAN J MATH	2014

Table 9. List of Publications of NCTS Key Members