### **2017** Optimization Workshop

Department of Mathematics National Taiwan Normal University

November 1, 2017

Sponsored by

National Center for Theoretical Sciences, Mathematics Division Department of Mathematics, National Taiwan Normal University

> Organized by Jein-Shan Chen

Table 1:	Program	Schedule.	Place:	M212,	Mathematics	Building
----------	---------	-----------	--------	-------	-------------	----------

	Speaker	Title
09:20	Muddappa Seetharama Gowda	Weakly homogeneous variational inequalities
10:00		
10:00	Bruno Figueira Lourenco	A bound on the Carathodory number and the
10:40		(non)homogeneity of <i>p</i> -cones for $p \neq 2$
	Tea Break	
11:00	Yu-Lin Chang	Some mean inequalities on SOC
11:40		
11:40	Yue Lu	No gap second-order optimality conditions
12:20		for circular conic programs
	Lunch Break	
13:40	Chun-Hsu Ko	User intention based maneuver for passive robot
14:20		walking helper
14:20	Yuh-Jye Lee	Reduced support vector machines: a compressed
15:00		learning viewpoint
	Tea Break	
15:20	Ngai-Ching Wong	Locating common fixed points of a nonlinear
16:00		representation of semigroup
16:00	Ruey-Lin Sheu	Solving a type of the Tikhonov regularization of the
16:40		total least squares problem by a new version of S-Lemma

### Some mean inequalities on SOC

Yu-Lin Chang Department of Mathematics National Taiwan Normal University Taipei 11677, Taiwan E-mail: ylchang@math.ntnu.edu.tw

**Abstract**. In this talk we will present mean inequalities on SOC by using some techniques we developed several years ago.

Keywords: SOC, mean inequalities

#### Weakly homogeneous variational inequalities

Muddappa Seetharama Gowda Department of Mathematics and Statistics University of Maryland, Baltimore County Baltimore, MD 21250, USA E-mail: gowda@umbc.edu

Abstract. Given a closed convex cone C in a finite dimensional real Hilbert space H, a weakly homogeneous map  $f: C \to H$  is a sum of two continuous maps h and g, where h is positively homogeneous of degree  $\gamma$  (> 0) on C and  $g(x) = o(||x||^{\gamma})$  as  $||x|| \to \infty$  in C. We denote h by  $f^{\infty}$  and call it the "leading term or the recession part" of f. Examples include polynomial maps over  $R^n_+$  (where the leading term is induced by a tensor) and Riccati maps  $f(X) := XAX + BX + XB^*$  over the cone of (real/complex) Hermitian positive semidefinite matrices.

Given a weakly homogeneous map f, a nonempty closed convex subset K of C, and a  $q \in H$ , we consider the variational inequality problem,  $\operatorname{VI}(f, K, q)$ , of finding an  $x^* \in K$  such that  $\langle f(x^*) + q, x - x^* \rangle \geq 0$  for all  $x \in K$ . When K is a cone, this becomes a complementarity problem. In this talk, we describe some results connecting the variational inequality problem  $\operatorname{VI}(f, K, q)$  and the cone complementarity problem  $\operatorname{VI}(f^{\infty}, K^{\infty}, 0)$ , where  $f^{\infty}$  is the recession part of f and  $K^{\infty}$  is the recession cone of K. Specializing, we extend a result of Karamardian formulated for homogeneous maps on proper cones to variational inequalities. As an application, we discuss the solvability of nonlinear equations corresponding to weakly homogeneous maps over closed convex cones.

**Keywords:** Weakly homogeneous map, variational inequality, complementarity problem, recession part/cone

#### User intention based maneuver for passive robot walking helper

Chun-Hsu Ko Department of Electrical Engineering I-Shou University Kaohsiung 84001, Taiwan E-mail: chko@isu.edu.tw

Abstract. With the growth of the elderly population, robot walking helpers play an important role in assisting their activities in daily lives. To be practical for use in daily lives for the elders, it should be able to follow user intention during maneuver and provide motion assistance when demanded. In this talk, an effective maneuver and control system is proposed for a passive robot walking helper. The proposed system detects users walking intention. A passive control law is then applied to derive the torques for steering the wheels of the walking helper. Experiments are conducted to demonstrate the effectiveness of the proposed system.

#### Reduced support vector machines: a compressed learning viewpoint

Yuh-Jye Lee Department of Applied Mathematics National Chiao Tung University Hsinchu 30010, Taiwan E-mail: yuhjye@math.nctu.edu.tw

Abstract. The reduced support vector machine was proposed for the practical objective to overcome the computational burden in generating a nonlinear SVM for the large scale classification problems. The reduced (rectangle) kernel trick has been applied to many machine learning algorithms in cooperating with kernel functions. In this talk, we try to link the random selection small reduced set to random projection. It provides a new interpretation of RSVM via a compressed learning viewpoint. We utilized sparse coding to represent the kernel vectors in a high dimensional feature space and random project these sparse coded vectors into a lower dimensional compressed domain. Our empirical results show that the nonlinear SVM with full kernel, RSVM, linear SVM in the high dimensional feature space and linear SVM in the compressed space all of them have a very similar performance. We also find that solving a machine learning in compressed space is much easier than recovering a sparse signal. That means you can be more aggressively using a small reduced set in generating a classifier. However, if you want to reconstruct the original sparse vector from the compressed signal than the dimension of compressed space has to bigger than the size of reduced set.

## A bound on the Carathodory number and the (non)homogeneity of *p*-cones for $p \neq 2$

Bruno Figueira Lourenco Department of Computer and Information Sciences Seikei University, Japan E-mail: lourenco@st.seikei.ac.jp

Abstract. This presentation is divided in two parts. In the first half, we discuss a geometric quantity of convex cones called "Carathodory number". We will explain why it is useful and we will present a new upper bound that is tight for several important families of cones. In the second half, we will discuss a recent proof that the *p*-cones are not homogeneous when the dimension is greater or equal than 3 and p is not 2. This proof makes use of the *T*-algebras, so we will also briefly discuss them. This is a joint work with Masaru Ito (Nihon University).

#### No gap second-order optimality conditions for circular conic programs

Yue Lu School of Mathematical Sciences Tianjin Normal University Tianjin 300387, China E-mail: jinjin403@sina.com

Abstract. In this paper, we study the second-order optimality conditions of a class of circular conic optimization problem. First, the explicit expressions of the tangent cone and the second-order tangent set for a given circular cone are derived. Then, we establish the closed-form formulation of critical cone and then calculate the "sigma" term of the aforementioned optimization problem. At last, in light of tools of variational analysis, we present the associated no gap second-order optimality conditions. Compared to analogous results in the literature, our approach is intuitive and straightforward, which can be manipulated and verified. An example is illustrated to this end.

#### Solving a type of the Tikhonov regularization of the total least squares problem by a new version of S-Lemma

Ruey-Lin Sheu Department of Mathematics National Cheng Kong University Tainan 70101, Taiwan E-mail: rsheu@mail.ncku.edu.tw

Abstract. Total Least Squares (TLS) is a method for fitting an overdetermined linear system Ax = b while assuming that both A and b are contaminated with noise. However, the TLS does not always have a solution; or even if it does, might frequently be ill-posed. The Tikhonov regularization which associated the TLS with a quadratic penalty is thus a technique for stabilizing the solution. Beck and Ben-Tal in 2006 (Siam J. Optim.) were the first to study the problem, but the formula (54) in their paper is unfortunately wrong. As such, all subsequent analysis and related computation after (54), including Algorithm TRTLSI itself, should be modified accordingly. In this talk, we provide another way to solve the problem via a new version of S-Lemma with two equalities. The result is neat, and can be generalized to optimize a special type of polynomial of degree 4. The result also covers our early papers on minimizing the double well potential problems.

# Locating common fixed points of a nonlinear representation of semigroup

Ngai-Ching Wong Department of Applied Mathematics National Sun Yat-sen University Kaohsiung 80424, Taiwan E-mail: wong@math.nsysu.edu.tw

Abstract. This talk is concerned with the problem of finding common fixed points for a family of Bregman relatively weak nonexpansive mappings. The motivation is due to our finding of some gaps in a recent paper of Kim, where the author was developing a hybrid iterative scheme for locating common fixed points of a nonlinear representation of a left reversible semigroup. After a brief discussion about the gaps and why they are fatal, we present a new approach by using Bergman type nonexpansive mappings. A correct version of Kim's convergence theorem is given as a consequence of our new results, which also improve and extend some recent results in the literature.