The Regularity Theory for the Double Obstacle Problem

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Abstract In this talk, I will introduce the regularity of the free boundary for the double obstcle problems. Preciesly, I will discuss the interior C^1 regularity of the free boundary for the elliptic and parabolic double obstacle for linear and Fully nonlinear operator. Furthermore, I will introduce the C^1 regularity of the free boundary near fixed boundary for Laplacian problem.

First, I am going to introduce the proof of local C^1 regularity of free boundaries for the double obstacle problem with an upper obstacle ψ,

$$\Delta u = f\chi_{\Omega(u)\cap\{u<\psi\}} + \Delta\psi\chi_{\Omega(u)\cap\{u=\psi\}}, \qquad u \le \psi \quad \text{in } B_1,$$

where $\Omega(u) = B_1\backslash(\{u=0\}\cap\{\nabla u = 0\})$ under a thickness assumption for u and ψ. The function ψ satisfies

$$\psi \in C^{1,1}(B_1)\cap C^{2,1}(\overline{\Omega(\psi)}), \quad \Omega(\psi) = B_1 \setminus (\{\psi=0\}\cap\{\nabla\psi = 0\})\,.$$

Our result is very close to the well-known regularity theory of L. Caffarelli for the obstacle problem [Caf77], and also the no-sign obstacle problem due to Caffarelli-Karp-Shahgholian [CKS00]. This is a joint work with Ki-ahm Lee and Henrik Shahgholian.

Next, I will talk about the problem for elliptic fully nonlinear opeator. Because of the nonlinearity of the operator, the main difficulty in our work is the lack of monotonicity formulas, which is the important tools for the classification of the global solutions for the double obstacle problem for the Laplace operator. Thus, we exploit the method that use the value $\partial_e u/x_n$, where u is the global solution with the upper obstacle $\psi = c(x_n^+)^2$. The method using the first derivative first introduced in [IM16], in the study of the regularity of the free boundary near fixed boundary for sigle fully nonlinear obstacle problem.

Lastly, I will breifly introduce the result on parabolic double obstacle problem and the C^1 regularity of the free boundary near fixed boundary for Laplacian problem.

Bibliography

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