National Center for Theoretical Sciences

NCTS Distinguished Lecture Series

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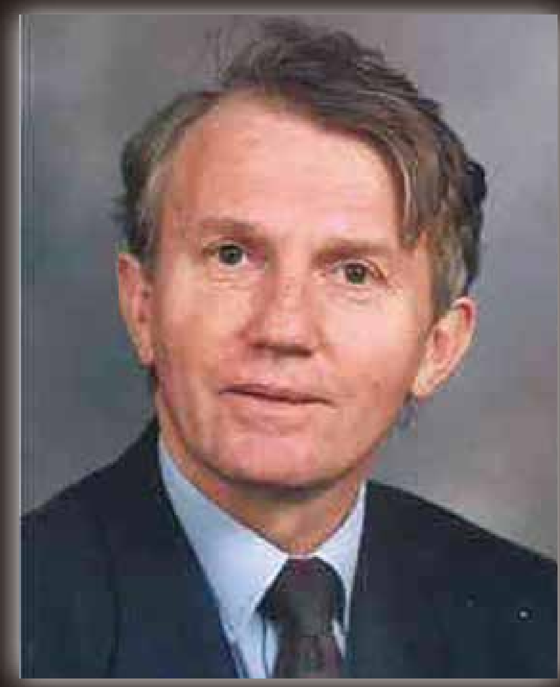

Prof.
John Coates
Cambridge University

Title: On the conjecture of Birch and Swinnerton-Dyer for quadratic twists of X_0 (49).
Time: 10:45–12:00, 14:15–15:30 on 7/5 and 7/7.
Venue: Rm 202, NCTS (Astro-Math Bldg., NTU)
Organizer: Ming-Lun Hsieh (NTU)

On the conjecture of Birch and Swinnerton-Dyer for quadratic twists of $X_0(49)$

Let $X_0(49)$ be the modular elliptic curve with equation $y^2 + x\,y = x^3 - x^2 - 2x - 1$, and let E be any elliptic curve defined over the rational number field Q which is a quadratic twist of $X_0(49)$. Let E(Q) be the group of rational points of E, Sha(E/Q) its Tate-Shafarevich group, and L(E/Q, S) its complex L-series. The aim of my four lectures will be to discuss in some detail the proof of the following theorem.

Theorem. We have L(E/Q, 1) is non-zero if and only if both E(Q) and the 2- primary subgroup of Sha(E/Q) are finite. When these equivalent conditions hold, the full Birch-Swinnerton-Dyer conjecture is valid for E.

To my knowledge, a result like this is not known for the family of all quadratic twists of any other elliptic curve defined over Q. The proof involves earlier work of K. Rubin and C. Gonzalez-Aviles, as well as some joint recent work by Y. Kezuka, Y. Li, Y. Tian and myself. If time allows, I will also explain a partial generalization to a large family of quadratic twists of the Gross curves A(q), with complex multiplication by the ring of integers of an imaginary quadratic field $K = Q(\sqrt{-q})$, where q is any prime which is congruent to 7 modulo 8.

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