# Newsletter Vol.16 Winter 2024

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## **Director's Message**

#### Happy New Year!

The NCTS was founded in 1997 and has been supported by National Science and Technology Councils (NSTC) which was named as National Science Council, and Ministry of Science and Technology before. The Center is currently in the last year of its Phase V-I (2021.1.1-2025.12.31). In 2024, we had an onsite International Advisory Committee Re-view meeting in June, submitted a full report for Phase V-I to NSTC in October, and had a NSTC review meeting in November. We are very happy to report that NTU has been granted to continue hosting the NCTS for another 5 years from 2026.1.1 to 2030.12.31 in its Phase V-II. Thank the support from all friends. Without the efforts and devoting of many local mathematicians, and the input of numerous international visitors, the Center is not able to thrive to today's stage.

Last December, the Center hosted three very successful international conferences on representative theories, harmonic analysis and geometry, respectively. We will continue to organize many exciting events in 2025. In particular, there will be Iwasawa 2025 and the Iwasawa Theory Taipei Summer School from 25th to 28th June before the conference. We will also host the 2025 NCTS Summer School on fluid dynamics and kinetic theory during 2nd to 13th June. Both summer schools welcome the participations of international students. Our goal is to coordinate with other institutions and train students together in Asia.

Look forward to your continuing support and participation in the NCTS activities. Wish you a healthy, peaceful and prosperous year of 2025 !

李慧英 Jug-Jy Zee

NCTS mathematics Division, Director Prof. Yng-Ing Lee



# Upcoming Opportunities

#### **2025 NCTS Research in Pairs**

The NCTS invites groups of 2–4 mathematicians from different institutions (including at least one from outside Taiwan) to apply for the Research in Pairs Program. Participants will spend 2–8 weeks at NCTS conducting impactful joint research, supported as visiting scholars (excluding airfare).

Calls are announced every April, August, and December. Priority is given to projects aligned with NCTS Topical Programs, particularly in:

- Number Theory and Representation Theory
- Algebraic Geometry
- Differential Geometry and Geometric Analysis
- Differential Equations and Stochastic Analysis
- Scientific Computing
- Interdisciplinary Studies

Application Requirements:

Submit the following to apply\_acad@ncts.tw (subject line: "NCTS Research in Pairs"): a. Cover letter



b. Project proposal (max. 10 pages):

- Overview, background, and known results
- Planned activities and timeline
- Expected outcomes and collaborations
- c. CVs of all participants

Apply at least 3 months before the proposed visit.

More information pleases refer to:



#### 2025 NCTS International Exchange Program

The National Center for Theoretical Sciences (NCTS) invites applications for its International Exchange Program, supporting collaboration between Taiwanese and international mathematicians. The program offers 1–3 month visits to partner institutes for academic exchange.

More information:





#### 2025 NCTS Postdoctoral Fellows

#### **Fellowship Details:**

Appointments last 1–2 years, extendable up to 3 years, with generous funding for travel and research. Support for Chinese language classes is also available for international fellows. Applications from all mathematical fields are welcome, but preference is given to those aligned with NCTS Topical Programs:

- Number Theory and Representation Theory
- Algebraic Geometry
- Differential Geometry and Geometric Analysis
- Differential Equations and Stochastic Analysis
- Scientific Computing
- Interdisciplinary Studies

Candidates specializing in mathematical foundations for AI are highly encouraged to apply. Applicants may suggest mentors from the NCTS Affiliate List.

#### How to Apply:

Submit your application via MathJobs. Required documents:

- Cover Letter (indicate start date and relevant Topical Program)
- CV
- 3 Reference Letters
- Research Statement
- Doctoral Dissertation and Reprints/Preprints

# Deadlines: November 15, 2024 (spring) and February 28, 2025 (fall).

More information:



#### 2025 Summer Undergraduate Research Program

The NCTS will host a 6-week Summer Undergraduate Research Program from July 7 to August 16, 2025, to inspire undergraduates to explore advanced mathematics through research. We plan to recruit six topics this year. Details of past programs are available here: https://sites.google.com/ncts.ntu.edu.tw/ nctsusrp

#### **Program Overview:**

- The program begins with *Boya Lectures* to introduce research topics and promote student-faculty interaction.
- Professors or postdoctoral researchers will guide students in weekly sessions.
- Collaboration with international scholars or coadvisors is encouraged. Refer to SLMath MSRI-UP.
- Teams should prioritize diversity and avoid students from a single department.

#### Submission Details:

Faculty interested in leading a group should submit a proposal (max. 3 pages) including the research topic, description, objectives, required knowledge, and recommended readings soon to apply2@ncts.tw (Contact: Ms. Tseng, Tel: 02-3366-8831).

#### Support Provided:

- NT\$30,000 advisor fee per group and up to NT\$20,000 for travel/accommodation.
- Teaching assistant support (NT\$20,000 and accommodation).

More information:





# Highlight of Events

#### NCTS-PIMS workshop in PDE

2024.06.11-2024.06.15

Organizers:

Chiun-Chuan Chen (National Taiwan University) Slim Ibrahim (University of Victoria) Chun-Hsiung Hsia (National Taiwan University) Jin-Cheng Jiang (National Tsing Hua University) Tai-Peng Tsai (University of British Columbia) Kung-Chien Wu (National Cheng Kung University)

NCTS and PIMS signed a collaboration MOU a few years ago and recently renewed it. As an effort to enhance the collaboration between these two institutes, Prof. Yng-Ing Lee, the director of the NCTS Mathematics Division (together with some members of NCTS), and Prof. Tai-Peng Tsai (University of British Columbia in Canada) held discussions on:

- 1. Mutual workshops with rotating locations and fields.
- 2. Mobility plans for students and scholars, including co-supervision.

We decided on PDE and geometric analysis as our focus fields. The 2024 workshop was held in Taipei, focusing on PDE, while the 2026 workshop will take place in Vancouver, focusing on geometric analysis. As mentioned, the subject of the 2024 workshop focused on PDE. More specifically, the following topics were emphasized:

- 1. Geometry and analysis of dispersive equations.
- 2. Regularity for solutions of certain fundamental equations.
- 3. Hyperbolic problems in traffic flow, kinetic theory, and the material sciences.
- 4. Modern approaches to asymptotic analysis in the calculus of variations.
- 5. Universal inequalities in relevant function spaces.
- 6. Elliptic and parabolic equations.





# **News**letter

## The List of NCTS Events

#### **Conferences & Workshops**

2024	
6/11-6/15	NCTS-PIMS workshop in PDE
6/19-6/21	2024 NCTS Workshop on Dynami- cal Systems
5/10	NCTS Interdisciplinary Distinguished Lectures on System Biology
6/3-6/7	NCTS/NTNU Conference on Frac- tional Integrals and Related Phe- nomena in Analysis
8/8-8/10	The 8th Asian Quantitative Finance Conference
8/12	Young Geometric Analysts Forum
8/12-8/16	The first student summer camp in algebraic geometry and number theory
8/29-8/30	2024 NCTS Workshop on Computa- tional Mathematics and Scientific Computing for Young Researchers
10/14-10/18	NCTS Workshop in Symplectic Ge- ometry
12/7	The 26th Taiwan Geometry Sym- posium
12/16-12/19	NCTS workshop on Harmonic Anal- ysis

This time, we had 13 invited speakers (8 from Taiwan and 5 from Canada). Moreover, there were 14 HQP (students and postdocs, 9 from Taiwan and 5 from Canada). The talks by invited speakers were scheduled in the morning, while the HQP talks took place in the afternoon. In both sessions, participants asked many interesting questions, and the speakers received valuable some speakers from Canada, working on fluid dynamics, brought new ideas and exchanged insights with local researchers.

This was particularly beneficial, as fluid dynamics researchers in Taiwan are very active; it provided a great opportunity to explore their research further.

Additionally, the speakers working on kinetic theory (from both Taiwan and Canada) shared their latest research. They had overlapping topics and were already familiar with each other's work. I believe this was the first time we had an HQP session at an international conference in Taiwan, and all HQP participants were well-prepared to present their research. It was an excellent opportunity for training students in Taiwan.

Thanks to the support of NCTS, the participants engaged in fruitful interactions in a very joyful atmosphere.



# **News**letter

# The List of NCTS Events

#### **Conferences & Workshops**

#### 2024

- 12/11-12/14 NCTS Workshop on Representation Theory and Lie Groups
- 12/18 NCTS One-day Workshop on Number Theory
- 12/27 NCTS Interdisciplinary Distinguished Lectures on Medical Innovations and AI

#### 2025

1/16	A Dialogue with the SIAM Preside- nt, Prof. Carol Woodward
1/18-1/20	NCTS Workshop on Algebra and Representation Theory
1/21-1/23	NCTS Workshop on Groupoids and Quantization
2/13-2/14	NCTS Workshop on Lie Theory and Representation Theory
2/14	2025 NCTS South Taiwan Workshop on Scientific Computing, Differen- tial Equations and Applications
2/20	NCTS One-Day Workshop on Arith- metic and Algebraic Geometry
3/3-3/5	The 15th Taiwan-Japan Joint Work- shop for Young Scholars in Applied Mathematics

#### NCTS International Scientific Advisory Committee Meeting

#### 2024/6/27-6/28

The NCTS Math International Advisory Committee (IAC) Meeting was held on June 27-28, 2024, to review the Center's operations and performance and to provide strategic recommendations for its future development. The Center invites world-renowned mathematicians who are familiar with the operation of leading international research centers to serve on the NCTS International Advisory Committee. The main role of the IAC is to oversee the scientific development of the NCTS and offer suggestions and recommendations regarding its operations to the National Science and Technology Council (NSTC) and the NCTS. Additionally, the IAC is responsible for nominating a new director in the event of a vacancy. The IAC typically holds an on-site meeting every two or three years; however, due to the pandemic, the meeting had been delayed. The current members of the IAC in Phase V-I (from January 1, 2021, to December 31, 2025) are:

Prof. Robert Bryant	(Duke University)
Prof. Russel Caflisch	(New York University)
Prof. Ching-Li Chai	(University of Pennsylvania, since August 1, 2023)
Prof. Hélène Esnault	(Freie Universität Berlin)
Prof. Horng-Tzer Yau	(Harvard University)
Prof. Shing-Tung Yau	(Tsinghua University)

We are also deeply grateful to Prof. Shigefumi Mori (Kyoto University), who served on the committee from January 1, 2016, to July 31, 2023.





## The List of NCTS Events

#### **Conferences & Workshops**

2025

3/19-3/20	AS-NCTS-NTU Distinguished Lect- ures in Geometry
3/20	NCTS One-Day Workshop on Non- linear Evolutionary Dynamics
3/20-3/21	2025 Conference on Advanced Topics and Auto Tuning in High-Pe- rformance Scientific Computing
4/9-4/11	NCTS Workshop on PDEs, Dynami- cal Systems and Applications
4/16-4/17	NCTS Workshop on Kinetic and Wave Equations
4/28-5/2	2025 Spring NSTC-NCTS-MRPC Joint Conference on Interdisciplinary Research
6/9-6/13	2025 NCTS International Confer- ence on PDEs
6/29-7/5	Iwasawa 2025
8/11-8/13	NCTS Tropical Geometry in Taiwan 2025
8/25-8/29	The 11th East Asia Number Theory Conference

More events are still being planned, so please stay tuned to NCTS Events (ncts.ntu.tw/ events.php) for more detailed information. Although Prof. Shing-Tung Yau was unable to attend, we were honored to have all other IAC members participate in the June 27-28 meeting. Over the two-day meeting, the committee evaluated the Center's operations and development, conducted an overview of all topic programs, analyzed strengths, weaknesses, and opportunities, and held interviews with NCTS members and postdoctoral fellows.

We would like to express our sincere gratitude to the IAC for their tremendous efforts, invaluable insights, and detailed report. In particular, they highlighted the issue of insufficient funding for the Center, which will be important for us to address moving forward. The IAC's praise of the Center's performance was a significant source of encouragement for our continued efforts to pursue excellence in mathematics and support the theoretical sciences community in Taiwan.



We also deeply appreciated the opening speech delivered by Dr. Chung-Chin Wu, Vice President for Research and Development at National Taiwan University, as well as the participation of all core members of the Center, including the executive, academic, and program committee members, Center Scientists, Young Theoretical Scientists, and postdoctoral fellows.

Finally, we would like to extend our heartfelt thanks to the entire NCTS staff for their great support, ensuring that the meeting was both smooth and successful.



NCTS Workshop on Representation Theory and Lie Groups

#### 2024/12/11-12/14

The workshop was originally gimed at honorina Professor Dan Barbasch of Cornell University, a leading expert in the representation theory of reductive groups. Although it evolved into a broader workshop, we successfully invited Professor Barbasch's former students, collaborators, and other researchers to foster connections among experts in related areas. The workshop covered key topics, including theta correspondence, unipotent representations, the unitary dual problem, the local Langlands correspondence, and the orbit method, with a focus on reductive groups over both Archimedean and non-Archimedean fields. We extend our heartfelt thanks to NCTS for their invaluable support in making this event a reality.

Reflecting on the event, we achieved:

#### -Focused and Engaging Content

Participants appreciated the well-structured program, which balanced depth and breadth, making discussions both accessible and enriching. Many talks were highly engaging, leaving attendees with new insights and ideas.

Interactive Online and In-Person Talks Professor Ciubotaru's interactive online lecture stood out, with postdocs from Academia Sinica actively asking questions. Follow-up communication ensured detailed answers, demonstrating meaningful academic exchange.

#### -Outstanding Presentations

A highlight was Professor Alexandre Afgoustidis's talk, stepping in for Professor Jeffrey Adams. The presentation was praised for its depth and interactivity, with positive feedback from participants, including Professor Shih-Yu Chen of NTHU.

#### -Diverse and Active Participation

The workshop attracted a wide range of participants, including local and international faculty, postdocs, and students. Notable attendees included Taiwanese Ph.D. students



from abroad and international postdocs, such as Dr. Rahul Dalal from the University of Vienna. Graduate and undergraduate students from several Taiwanese universities also benefited, gaining valuable exposure to international research.

#### -Hybrid Format Success

The inclusion of online participation allowed broader access, benefiting researchers unable to attend in person. Participants from Japan and Taiwan praised the convenience and quality of the hybrid format.

-Personal and Collaborative Gains

For both organizers and participants, the workshop offered inspiration and opportunities for collaboration. It facilitated progress on ongoing projects and sparked new research ideas.

Looking ahead to future plans of representation theory in Taiwan, it is expanding beyond its traditional focus on Lie algebras to include reductive groups, which have significant applications in number theory. To sustain this growth, we aim to organize this workshop regularly—potentially every one to two years —or to host shorter events with researchers from neighboring regions such as Japan, China, and Korea. These initiatives will provide a platform for showcasing new work and fostering collaborations.

This workshop has been an enriching experience, and we are deeply grateful to NCTS for their continued support in advancing representation theory research in Taiwan.

#### NCTS Workshop on Harmonic Analysis

#### 2024/12/16-12/19

Over the past decade, new approaches and developments in harmonic analysis have led to significant progress on a wide range of problems in areas such as combinatorics, number theory, geometric measure theory, and partial differential equations (PDEs). The aim of this workshop was to highlight these recent advancements in harmonic analysis and its applications.

Additionally, the workshop served as a platform to connect researchers across the Asia-Pacific region, fostering discussions and collaborations among various subfields of harmonic analysis.

Among the highlights of the workshop:

Prof. Alex losevich delivered a talk on the efficient use of Fourier analysis in addressing signal recovery problems.

Prof. Polona Dursik presented cutting-edge research on the triangular Hilbert transform. Prof. Joonil Kim discussed the challenging topic of double discrete Hilbert sum estimates.

These talks sparked vibrant discussions and inspired further exploration in the field.

We hope to continue organizing such workshops in the future and express our heartfelt gratitude to NCTS for their support, which has significantly contributed to the development of harmonic analysis in Taiwan.



#### 2025 Events Preview

2025 NCTS Summer School on Fluid Dynamics and Kinetic Theory

#### 2025/6/2-6/13

NCTS had the pleasure of co-organizing the 2022 MSRI-NCTS Joint Summer School: Recent Topics in Well-Posedness at the University of Hawaii's Hilo campus from July 18–29, 2022. During the program, international graduate students and lecturers shared meals, fostering discussions on both mathematics and life experiences. Participants also organized sightseeing activities, creating a vibrant and memorable experience.

Building on this successful model, NCTS is pleased to announce the 2025 NCTS Summer School on Fluid Dynamics and Kinetic Theory, to be held at National Taiwan University, Taipei, from June 2–13, 2025. We warmly invite domestic and international graduate students to join us for this enriching academic experience.

The summer school will feature three distinguished lecturers:

• Kazuo Aoki (Professor Emeritus, Kyoto University; Professor, National Cheng Kung University)

• Jerry L. Bona (Professor Emeritus, University of Illinois at Chicago; Visiting Professor, National Taiwan University)

• **Zhouping Xin** (William M.W. Mong Professor of Mathematics, The Chinese University of Hong Kong)

The program aims to introduce graduate students to fundamental results in kinetic theory and fluid dynamics, with a special focus on:

- 1. The relationship between kinetic theory and fluid dynamics
- 2. Dispersive water waves
- 3. Transonic flows

These topics have a rich research history. Through comprehensive lectures, the speakers will not only cover foundational results but also present intriguing open problems for exploration. We hope this summer school will help participants enhance their research skills, deepen their understanding of fluid dynamics and kinetic theory, and expand their academic networks.

For more information and regis-tration details, visit: https://sites.google.com/ncts.ntu.edu.tw/ fluid-dynamics-kinetic-theory/homepage

#### lwasawa 2025

2025/6/25-6/28 (Summer School) 2025/6/25-7/4 (Conference)

This is the 10th Iwasawa Conference, following previous conferences held in Besançon (2004), Limoges (2006), Irsee (2008), Toronto (2010), Heidelberg (2012), London (2015), Tokyo (2017), Bordeaux (2019), and Cambridge (2023).

This series of Iwasawa Conferences is the largest and one of the most important gatherings in Iwasawa theory. It aims to bring together different strands of research within and closely related to the field of Iwasawa theory.

In addition, we organize a summer school before the conference to introduce fundamental topics such as p-adic L-functions, Euler systems, and the modular construction of Selmer classes.

We believe that hosting this summer school and the conference will promote research in number theory in Taiwan and enhance the international visibility of our number theory community.

For more information and registration details, visit: https://sites.google.com/ncts.ntu.edu.tw/ iwasawa-summerschool/homepage

NCTS looks forward to welcoming you to Taipei!

# Cultivation of the Youth

### NCTS Research Abroad Program

The NCTS Research Abroad Program was launched in October 2023 to support outstanding students in conducting short-term research visits at international universities or institutes. The program funds up to 10 students annually, enabling them to collaborate with professors at international institutions for 2 to 4 months. It aims to enhance the research capabilities of Taiwanese students, broaden their perspectives, and strengthen their global competitiveness. A total of 6 students have been granted support for the period of June 2024 to May 2025.

Here's a reflection from 2024 participant: Bang-Shien Chen.

#### **Bang-Shien Chen**

(National Taiwan Normal University) Minneapolis, MN, USA June 14, 2024 - August 31, 2024

The Mathematics Division of NCTS (National Center for Theoretical Sciences) announced a research abroad program last year, and I was fortunate to be awarded a grand from NCTS. I visited d Prof. Shuzhong Zhang at the Department of Industrial and Systems Engineering, University of Minnesota (UMN), from June 14 to August 31. Prof. Zhang is an expert in optimization, with applications span various aspects of Operation Research, Machine Learning, and Statistical Learning. During the visit, we come up with new algorithms based on my previous work, and have roughly finished a manuscript that is ready to publish in a few months.

When I arrived at mid-June, Prof. Zhang intro-

duced a PhD student Ting-Tsen Lin, who is also from Taiwan, to help me settle down. I lived at a subleased apartment, and the subleasor's friends also helped me a lot. The public transport is very convenient in Minneapolis, and there is also free campus buses near my apartment. Overall, I settled down smoothly thanks to the people above.

During the first week, Prof. Zhang organized a seminar for all graduate students and let me introduce my previous work there. After that, we brainstormed together and immediately came up various potential ideas to improve my work. I have weekly meetings with Prof. Zhang for the remaining visit time, which he will give me constructive suggests and research directions. He has a good intuition about how optimization should look like in real-world applications, which I will discuss about that in the following section. I have my own desk at the graduate student's office, and a temporary access card to the department 24/7, which is convenient and comfort-able to study there.

Besides that, I also visited Prof. Volkan Isler at the Robotics Institute and he also gave me a chance to introduce my work to his team (the application of my previous work is in Robotics). I made some friends in the Robotics Institute that are working on Lidar-based SLAM problems, and I had a lot of fun discussing with them and tried implementing my work with theirs.

To sum up, I am happy to spend my scholarship at UMN and had an enjoyable and productive summer. I am sincerely thankful to NCTS for giving me this opportunity.

As mentioned above, our results are based on my previous work, a fractional programming technique for Geman-McClure (GM) robust estimator, where we demonstrate our solver on 3-D point cloud registration, a wellknown problem in Robotics. The main idea of my previous work is to reformulate GM estimation as sum-of-ratio problems, which we leverage fractional programming techniques to solve it.

After Introducing my previous work to Prof. Zhang, he point out immediately that the relationship between the numerators and denominators of GM are not utilized. We further simplified the sum-of-ratio formulation to sum of convex reciprocal functions, and constructed an iterative process by the first-order conditions which results in and quadratic program (QP). That is, we can now solve the problem by quadratic programming techniques instead of fractional programming techniques.

To demonstrate the new formulation on point cloud registration problems, the optimization problem becomes a QP with manifold constraints. There are various methods to deal with this kind of situation, we compare three commonly used method and propose our own relaxation method. The first approach is linear relaxation, where we simply omit the manifold constraint, solve the QP on Euclidean space, then project the solution back to the manifold at last. The second approach is



Prof. Shuzhong Zhang



**UMN TSA** 

to reformulate QP with manifold constraints toan equivalent nonconvex auadratically constrained quadratic program (QCQP), then rewrite it to a rank-constrained semidefinite program (SDP). We solve the rank-relaxed SDP, and such approach also comes along with sufficient conditions of the tiahtness of relaxation using the rank constraint. The third approach is by Riemannian gradient descent, and we further showed that the Riemannian aradient defined by the smooth function on the ambient space is equivalent the Lie derivative of the manifold. For the last approach, we first relax the rotation manifold to the convex hull of orthogonal manifold, which results in a conic program.

Experiments showed that our proposed relaxation method has the best performance overall (even among state-of-the-art robust point cloud registration solvers), and the linear relaxation QP has the same result as our previous fractional programming approach, but with less computational time. We also planned to generalize the relation between Riemannian gradient and Lie theory to some theorem. A manuscript of the results are roughly done.

The research visit to UMN is fruitful and meaningful to me. The meeting format there and here in Taiwan are quite different, and I think the experience will benefit my future studies a lot. I've also made some connections with Professors and graduate students there, which will also increase my chance to collaborate with others in the future. I believe this is the first year NCTS provide the research abroad program, and I highly recommend other students participate.

During the research visit, I had a lot of discussions with Prof. Zhang's team member, everyone is very knowledgeable and I benefit a lot from them. I also made some friends at the UMN Taiwan Student Association, everyone is very friendly and accommodating, which helped me a lot during my stay there. I hope I have a chance to visit there again !



Prof. Zhana's team member

#### **Program Details**

The list of students for the June 2025–May 2026 period was determined and announced in November 2024. The call for applications for the June 2026–May 2027 period will be announced around August 2025. The program offers funding of up to NT\$150,000 per student, and participants must finalize their host professors and research plans by February 2026.

#### Here are some previous participants:

Bang-Shien Chen (NTNU)	University of Minnesota
Yung-Chen Li (NTU)	Kyoto University
Ming Hsiao (NTU)	SLMath
<b>Jia-Lin Syu</b> (NTU)	SLMath
Chih-Huan Chang (NTU)	RIMS & IPMU University
Yi-Hsin Tsai (NTU)	of Cologne

While the application process has not yet opened, interested students can start preparing their documents and refer to the previous announcement for more details:



## SLMath Summer Graduate Schools

Every summer, the Simons Laufer Mathematical Sciences Institute (SLMath, formerly MSRI) organizes 12–13 topic-oriented Summer Graduate Schools, each typically lasting two weeks. Some are held at SLMath, while others take place at partner institutions worldwide. As a sponsoring member, the NCTS has the privilege of nominating participants for these prestigious programs each year. After the Center submits its nominations, SLMath selects the final participants, who receive full funding to attend their chosen summer schools.

Ping-Hong Lee attended one of the 2024 SLMath Summer Graduate Schools as a recent graduate of National Taiwan University. Here is his reflection on the experience:

**Ping-Hong Lee** (National Taiwan University) Mathematics of General Relativity and Fluids FORTH, Greece July 20, 2024 - August 5, 2024

The SLMath Institute held a two-week summer school on General Relativity and Fluids from July 20 to August 5 this year. The event took place at the Foundation for Research and Technology in Heraklion, Greece. The purpose of this program was to provide an accessible introduction to the mathematical knowledge of general relativity and fluid mechanics.

The participants of this course were quite diverse in terms of research interests, age and also race. Among the more than fifty participants, a small portion were postdoctoral fellows and recent university graduates, while the rest were currently pursuing doctoral studies. In addition to a portion of participants, while the rest were currently pursuing doctoral studies. In addition to a portion of participants whose research topics were general relativity or fluid mechanics, there were also many whose focus was on other areas of research from geometric measure theory and geometric flows on the geometric side, to control theory and stability analysis in the field of differential equations; some students had not yet selected a specific research topic. The diversity of this course was not only evident among the participants, but also in the broad research interests of the lecturers, among the ten lecturers, each had their own research field and had made outstanding research achievements in their respective research topics.

The two main academic components of the

entire event were the daily courses and the problem-solving sessions. I will elaborate on them separately in the following. Lastly, I will add some remarks on other activities.

In this summer school, the speakers gave a variety of courses to help participants gain a broad understanding of both the basics and the recent developments in general relativity and fluid mechanics; they also attempted to let students have a deeper understanding standing of these subjects. During the weekdays of the two-week event, three onehour courses were arranged each day, and a two-hour problem-solving session was reserved on Mondays to Thursdays in the afternoon. Throughout the event the two themes, general relativity and fluid mechanics, were interspersed, with some courses dependent on both of the themes.

There were about ten lecturers in the summer school, including professors in the field and several teaching assistants who were mostly their PhD students. on average, each lecturer taught for about two to three hours. In addition to giving several basic introductory courses, the professors mostly gave lectures related to there own research. Meanwhile, one of the main purposes of the teaching assistants, classes was to conduct more discussions or some pre-course detailed preparations on the relevant contents, ensuring that the participants could achieve better absorption, of course, the style of each lecturer was quite different. Some lecturers liked to write down tons of mathematical formulas, while others preferred to convey the insights they wanted to present using only simple mathematical formulas and images.



The follow topics are several materials that, needs to have a deep understanding of the among some other things, were presented in the summer school.

i) The fundamental contents in general relativity: Lorentzian mainfold, Einste's field equations, Schwarzs child's solution together with Lemaitre's extension, Penrose diagrams, Cauchy's surfaces, black holes, red-shift effects, etc. We got in touch with most of of these topics in the first two days, and they appeared once and again later in the summer school.

ii) Linear wave equations and their well-sedness in the Minkowski spaces and more general Lorentzian manifolds; the energy methods and the bootstrapping argument. this is an important topic. One reason is that wave equations appear Einstein's scalar field equations. Even though they do so in a highly nonlinear way, it is conceivable, that to understand the stability of black holes, one

stability of linear wave equations. Then, one tries to deal with the nonlinear issues. This is indeed what mathematicians, are doing, and the general results are still incomplete.

iii) A construction of solutions to Einstein's vacuum equations that contain naked singularities<sup>1</sup>. Naked singular are something we dislike. Roughly speaking, in a Lorentzian manifold with naked singularities, observers may be affected by the singular parts of the manifold however they are close to the infinity. However, this phenomenon would contradict determinism; singularities should be hidden behind an event horizon and not be perceived by observers outside. Anyway, understanding how naked singularities may appear is important.

Some other topics include gravitational collapses<sup>2</sup> and a restricted version of the shock development problem<sup>3</sup>.

In the daily problem-solving sessions, students were divided into thirteen aroups to have discussions. Each group consisted of about five members. Because people in our group had different backgrounds and were good at different areas of mathematics, we could learn from each other. For example, on the third day, the summer school was teaching the theory related to wave equations; therefore, knowledge of the Sobolev spaces was crucial throughout the day's classes and problem sets. However, one student lacked prior experience with Sobolev spaces. As a result, he did not participate in our discussion. Seeing this, a group member kindly volunteered to spend a long time sharing with him many key concepts and results of the Sobolev spaces, such as embedding theorems. He also shared his experiences in learning the subject.

Overall, the content of the exercises in the problem sessions was quite rich and supplemented many parts that could not be dealt with in detail or were not mentioned in the course. Because the teaching assistants who set the questions were not the same, the style and difficulty of the questions varied from one day to the next; sometimes many questions were start. Of course from not easy to time to time, the teaching assistants would provide appropriate hints to help everyone solve the problems. For example, in the fluid mechanics part, there was once a problem that required us to obtain some estimates of the decay rate, but many people did not know how to start. Later teaching assistant suggested that we consider some auantities and calculate their evolution equations, from which we would realize the monotonicity or other key properties of these quantities. With these results, the problem could be solved easily. Indeed, our group did solve the problem by the hints she gave us.

However, although we successfully solved th problem, we were still puzzled about how to

think of considering these quantities, which was a pity.

Here are some examples of the problem sets:

- Uniformization Theorem for Lorentzian
  surfaces Topology of Lorentzian manifolds
- Geometric properties of null hypersurfaces
- Properties of Einstein's equations in spherically symmetric cases
- Uses of ordinary differential equation theory, such as the stable manifold theorem and the Poincaré linearization theorem, to study the construction of naked singularities
- Construction of an example of a trapped surface
- Decay rate estimates of energy flux for several solutions to Einstein's equations
- Asymptotic behavior of the is entropic Euler
  equations

Besides the lectures and the problem-solving sessions, lunchtime and the time spent on the shuttle bus between the hotel and the classroom were also convenient opportunities to interact with others. During this period one could meet many people and learn about their expertise or experiences. Indeed, conversations and discussions with others, I heard many different mathematical the theories. In general, people there were good at expressing their thoughts and ideas smoothly. Thus, talking to them was a good experience.

In there spare time, students also engaged in various leisure activities. In addition to enjoying the sunlight on the beach, going hiking in the gorge, or visiting archeological museums to learn about the local culture and history some of the students even took a short trip to other places, e.g. Athens, by airplane. Evidently, these extracurricular activities also helped foster more communication and interaction among each other.

In the summer school, participants saw various topics, including:

- Important concepts in general relativity
- Solvability and stability properties of wave equations and of the compressible Euler flow

equations and of the compressible Euler flow

- The existence of naked singularities and their implication
- Gravitational collapse
- The shock development problem

Overall, this Summer school offers valuable insights into the work of modern mathematicians in general relativity and fluid mechanics. Additionally, participants have many opportunities to learn about new mathematical ideas. It is definitely worth attending.

Lastly, I would like to thank: NCTS and SLMath for providing me with the opportunity to participate in this summer school and for their support in all aspects !

<sup>1</sup> Igor R., & Shlapentokh-Rothman, Y. (2023). Naked singularities for the Einstein vacuum equations: The exterior solution. *Annals of Mathematics*, 198(1).

<sup>2</sup> Guo, Y., Hadžić M., & Jang, J. (2021). Larson-Penston Self-similar Gravitational Collapse. Communications in Mathematical Physics, 386(3), 1551-1601.

Guo, Y., Hadžić M., Jang, J., & Schrecker M. (2022). Gravitational Collapse for Polytropic Gaseous Stars: Self-Similar Solutions. Archive for Rational Mechanics and Analysis, 246(2-3) 957-1066.

<sup>3</sup> Christodoulou, D. (2019). The Shock Development Problem. In EMS monographs in mathematics.

#### **Program Details**

The NCTS Mathematics Division invites full-time students from universities and research institutions in Taiwan to apply for the NCTS Nomination for the SLMath Summer Graduate Schools 2026 (SGS). For students nominated by the Center and admitted by SLMath, the NCTS will cover the remaining costs not fully funded by SLMath and will also provide financial support for the other fee students who are approved by SLMath.

Previous Admitted Nominees in 2024:

Song-Yun Chen	(NTU)
Meng-Hao Liang	(NTHU)
Ming Hsiao	(NTU)
Ping-Hung Lee	(NTU)
Yi-Hsin Tsai	(NTU)
Jia-Lin Syu	(NTU)
Juei-Yin Lin	(NTNU)
Yung-Chen Li	(NTU)
Jian-An Wang	(NTU)

The call for the NCTS Nomination for SLMath Summer Graduate Schools 2026 (SGS) will announce in August. In the meantime, please refer to previous announcements for more information.



## 2025 Course

#### Exciting Courses in the Spring Semester

The upcoming spring semester will feature a series of remarkable courses, covering cutting-edge topics in supercomputing, algebraic geometry, and differential geometry.

#### Integration of Simulation, Data, and Machine Learning on a Heterogeneous Supercomputer System

In an era where AI is a dominant trend, Professor Weichung Wang (National Taiwan University) has invited four distinguished scholars—Takashi Arakawa, Kengo Nakajima, Shinji Sumimoto, and Hisashi Yashiro—to deliver the course in February 17 and 18. This course will focus on supercomputing technology and heterogeneous system applications, particularly the integration of simulation, machine learning (ML), and multiphysics coupling in advanced technological fields.

# Geometry, Algebra, and Topology with a view towards application and computation

In the field of algebraic geometry, we are honored to host Professor Chris Peterson, an expert in liaison theory and computational algebraic geometry. His recent work on geometric data analysis has also gained significant attention. His research interests span a wide range of topics, from computing Hilbert series—rooted in pure algebraic geometry to analyzing mappings between datasets using algebraic geometry concepts. Additionally, Prof. Peterson collaborates with scientists and physicians on topics such as geometric data analysis and beyond.

During his stay, we will offer a concise 8-hour minicourse (0.5 credits) for students to engage with his expertise.

#### From Differential Geometry to #\* Geometry (2)

The second half of "From Differential Geometry to tt\* Geometry" will continue next semester, with an increased emphasis on student training to deepen their understanding and skills.

# The List of NCTS Courses

#### Lectures & Courses

#### 2024

9/2-12/13	Taiwan Mathematics School: Nor- mal Approximation and Related Topics
9/5-12/19	Taiwan Mathematics School: From Differential Geometry to tt* Geom- etry-(1)
11/5-11/6	Lecture Series: Computer-Assisted Proofs in Nonlinear Analysis
12/3-12/6	Lecture Series: Introduction to Adic Spaces

#### 2025

1/3-6/30	Lectures on WKB methods of 2 <sup>nd</sup> order complex ODE
2/17-2/8	Integration of Simulation, Data, and Machine Learning on a Heter- ogeneous Supercomputer System
2/27-6/12	Taiwan Mathematics School: From Differential Geometry to tt* Geom- etry-(2)
2/24-3/17	Taiwan Mathematics School: Ge- ometry, Algebra, and Topology with a view towards application and computation
6/2-6/13	2025 NCTS Summer School on Fluid Dynamics and Kinetic Theory
6/25-6/28	lwasawa 2025: Taipei Summer School

Please refer to NCTS News (ncts.ntu.tw/news.php) for more detailed information

# Spotlight

Research

Prof. Chieh-Yu Chang has been awarded the 2023 Academic Award by the Ministry of Education. A professor at National Tsing Hua University, he is renowned for his contributions to transcendence theory and multiple zeta values in positive characteristic. He has received the 2013 Morningside Silver Medal and the NSTC Outstanding Research Award twice. Prof. Chang has long been connected with the NCTS, having started his postdoctoral career here. We are honored to invite him to share a brief discussion of his award-winning research.

# Results and Problems on Multiple Zeta Values in Characteristic Zero

Prof. Cheih Yu Chang | National Tsing Hua University



Riemann zeta function is one of the most significant functions in mathematics, appearing in many research areas. Its values at positive integers  $\geq 2$  are called special zeta values. Real multiple zeta values, abbreviated as MZV's, are a generalization of special zeta values initiated by Euler, and later developed by Hoffman and Zagier in the 1990s. They are multiple series defined at admissible indices, which are tuples of positive integers

 $\mathbf{s} = (s_1, ..., s_r)$  with :  $s_1 \ge 2 : \zeta(s) = \sum_{n_1 > \dots > n_r \ge 1} \frac{1}{n_1^{s_1} \dots n_r^{s_r}} \in \mathbb{R}^{\times}$ 

The weight and the depth of the presentation  $\zeta(\mathbf{s})$  are defined by  $wt(\mathbf{s}) = \sum_{i=1}^{r} s_i$  and  $dep(\mathbf{s}) = r$  respectively.

Over the past 30 years, real MZV's have been extensively studied due to their connections to various topics such as arithmetic geometry and number theory. One central problem is to understand and determine all the algebraic relations among MZV's over Q. Regularized double shuffle relations, developed by Racinet and Ihara-Kaneko-Zagier, generate many-Q linear relations among real MZV's of the same weight. Goncharov conjectured that the real MZV's forms a graded algebra (graded by weight), meaning that there are no Q-linear relations among real MZV's of different weights.

Let  $Z_n$  be the Q-vector space spanned by all real MZV's of weight  $n \ge 2$ . Zagier's dimension conjecture asserts that  $\dim_Q Z_n = d_n$ , where  $\{d_n\}$  are defined by the generating function  $\frac{1}{1-X^2-X^3} = \sum_{n=0}^{\infty} d_n X^n$ . It was shown by Terasoma and Goncharov that  $d_n$  is the upper bound of for  $\dim_Q Z_n$  each n. For a fixed weight  $n \ge 2$ , Hoffman's  $\{2,3\}$ -basis conjecture asserts that the set

$$\beta_{Hn} := \left\{ \zeta(s_1, \dots, s_r) \, | \, r \in \mathbb{N}, \quad s_i \in \{2,3\} \, \forall 1 \le i \le n, \text{ and } \sum_{i=1}^r s_i = n \right\}$$

is a  $\mathbb{Q}$ -basis of  $Z_n$ . In 2012, Brown showed that  $\beta_{H,n}$  is a generating set for  $Z_n$ , and hence derived the upper bound result for  $\dim_{\mathbb{Q}} Z_n$ .

In the *p*-adic world, Furusho defined *p*-adic MZV's using Coleman's p-adic integration theory. Furusho and Jafari demonstrated that these *p*-adic MZV's satisfy the regularized double shuffle relations. Furusho proposed a conjecture asserting that p-adic MZV's satisfy the same Q -algebraic relations that the corresponding real MZV's satisfy. In other words, conjecturally there is a well-defined Q -algebra homomorphism from the real MZV's to *p*-adic MZV's, and this conjectural phenomenon can be also illustrated via Grothendieck's period conjecture for motivic multiple zeta values. My recent works with my collaborators are devoted to prove that the analogues of the above conjectures are valid in the world of function fields in positive characteristic.

#### **Recent Advances in Positive Characteristic**

Let  $A := \mathbb{F}_{a}[\theta]$  be the polynomial ring in the variable  $\theta$  over the finite field  $\mathbb{F}_q$  of q elements, where q is a power of a prime number p. Let k be the field of fractions of and be the completion of k at the  $\infty$  place. Thakur's *p*-adic MZV's generalize Carlitz zeta values and are defined at all tuples of positive integers  $\mathbf{s} = (s_1, \dots, s_r)$  :  $\zeta_A(\mathbf{s}) \coloneqq \sum \frac{1}{a_1^{s_1} \cdots a_r^{s_r}} \in k_\infty$  , where  $a_1, \ldots, a_r$  run over all monic polynomials in A with deg  $a_1 > \cdots > \deg a_r$ . The weight and depth of the presentation of  $\zeta_A(\mathbf{s})$  are defined in the same way such as the classical case. Let  $\mathcal{Z}_n$  be the k-vector space spanned by Thakur's  $\infty$  -adic MZV's of weight n. Thakur's work on q-shuffle product implies that  $\mathcal{Z}_{n_1} \cdot \mathcal{Z}_{n_2} \subset \mathcal{Z}_{n_1+n_2}$  , and so Thakur's  $\infty\text{-adic}$ MZV's form a k-algebra. In 2014, I proved the analogue of Goncharov's direct sum coniecture in this setting. Namely, Thakur's  $\infty$ -adic MZV's form a graded algebra (graded by weight).

In his PhD thesis, Todd studied k-linear relations among Thakur's  $\infty$ -adic MZV's of the same weight. He proposed a dimension conjecture for dim<sub>k</sub>  $\mathcal{Z}_n$ , and later, Thakur formulated a conjectural basis for  $\mathcal{Z}_n$  for each weight  $n \geq 1$ . My recent collaborative works are summarized as follows:

(I) Analogue of Furusho's conjecture: In [CM21], for each finite place v of k, we defined v -adic MZV's as function field analogues of Furusho's *P*-adic MZV's. We demonstrated that v-adic MZV's satisfy the same k-linear relations as their corresponding  $\infty$ -adic MZV's satisfy. In [CCM22], we completely established the analogue of Furusho's conjecture by proving that there is a well-defined k-algebra homomorphism  $\phi_v$  from  $\infty$ -adic MZV's to v-adic MZV's.

(II) Todd-Thakur's conjectures: In [CCM23], we proved Thakur's basis conjecture and, as a consequence, derived Todd's dimension conjecture. Note that around the same time, Ngo Dac's group independently proved Thakur's basis conjecture, and their paper was published in the same journal. For project (I), our primary transcendence tool is Yu's sub-t-module theorem, which plays the analogue of Wuestholz's analytic subgroup theorem. For project (II), our proof is based on Anderson-Brownawell-Papanikolas criterion, which can be viewed as an analogue of Siegel-Shidlovskii theorem. Finally, we mention that in project (I), we used a computer program to compute some examples, leading to a conjecture about the kernel of . This kernel seems to be a specific ideal but is independent of . If this conjecture can be proved to be true, it would be fascinating, as it would establish a concrete connection between positive characteristic MZV's of different worlds.

#### References

[CM21] C.-Y. Chang and Y. Mishiba, On a conjecture of Furusho over function fields, Inventiones mathematicae 223, 49-102 (2021).

[CCM22] C.-Y. Chang, Y.-T. Chen and Y. Mishiba, Algebra structure of multiple zeta values in positive characteristic, Cambridge Journal of Mathematics Vol. 10, No. 4,743-783, 2022.

[CCM23] Y.-T. Chen and Y. Mishiba, On Thakur's basis conjecture for multiple zeta values in positive characteristic, Forum of Mathematics, Pi (2023), Vol. 11:e26 1–32.

# Honors in 2024

Congratulations !

The NCTS Mathematics Division is excited to share some great news! In 2024, several core members working closely with NCTS received major recognition in academia and education. We're proud of their success and look forward to continuing our collaborations, pushing the boundaries of mathematical research and education, and inspiring future generations.

- Professor Chieh-Yu Chang, a core member of the center, received the 2023 Academic Award of the Ministry of Education (MOE).
- Professor Fu-Tsun Wei, a Program chair, received the 2023 Outstanding Research Award of the National Science and Technology Council (NSTC).
- Professor Hsueh-Yung Lin, a center scientist and academic committee member, received the 2023 Outstanding Research Award of the NSTC.
- Professor Tai-Chia Lin, a Program chair, received the NSTC outstanding specially appointed research fellow award.
- Dr. Jun-Wen Peng, NCTS postdoc fellow, awarded the 2023 Academic Award of the NSTC for Postdoctoral Fellows.
- Professor Hao-Chung Cheng, recipient of the Young Theoretical Scholar Award, was honored with the 2023 K. T. Li Young Researcher Award and the 2024 Renmin Outstanding Young Scholar Lectureship.
- Professor Cheng-Chiang Tsai, recipient of the Young Theoretical Scholar Award and an academic committee member, received the 2024 Wu Ta-You Memorial Award of the NSTC and the Academia Sinica Early-Career Investigator Research Achievement Award.

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