# Notions of Convexity on Carnot Groups

饒維明 (中央大學)

**Abstract:** Convex functions and convex analysis play a substantial role in analysis, in particular in some non-linear problems and variational problems. We introduce the notion of convexity on Carnot groups and highlight some of its applications to sub-elliptic PDEs.

# Tb theorem on product spaces

## 李明憶 (中央大學)

**Abstract:** In this talk, we show a Tb theorem on product spaces  $\mathbb{R}^n \times \mathbb{R}^m$ , where  $b(x_1, x_2) = b_1(x_1)b_2(x_2)$ ,  $b_1$  and  $b_2$  are para-accretive functions on  $\mathbb{R}^n$  and  $\mathbb{R}^m$ , respectively.

### Extremality of numerical radii of matrix products

#### 高華隆 (中央大學)

Abstract: For two *n*-by-*n* matrices *A* and *B*, it was known before that their numerical radii satisfy the inequality  $w(AB) \leq 4w(A)w(B)$ , and the equality is attained by the 2-by-2 matrices  $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ . Moreover, the constant "4" here can be reduced to "2" if *A* and *B* commute, and the corresponding equality is attained by  $A = I_2 \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_2$ . In this talk, we give a complete characterization of *A* and *B* for which the equality holds in each case. Analogous characterizations for the extremal equalities for tensor products and Hadamard products are also proven. For doubly commuting matrices, we obtain a unitary similarity model, namely, *A* and *B* satisfy AB = BA and  $AB^* = B^*A$  if and only if they are simultaneously unitarily similar to matrices of the form  $\sum_{j=1}^{k} \oplus (A_j \otimes I_{n_j})$  and  $\sum_{j=1}^{k} \oplus (I_{m_j} \otimes B_j)$ . For commuting 2-by-2 matrices *A* and *B*, we show that w(AB) = w(A)w(B) if and only if either *A* or *B* is a scalar matrix, or *A* and *B* are simultaneously unitarily similar to  $\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$  and  $\begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}$  with  $|a_1| \ge |a_2|$  and  $|b_1| \ge |b_2|$ .

## Monge-Ampère singular integral operators acting on Hardy spaces and their duals

林欽誠 (中央大學)

**Abstract:** We study the Hardy spaces  $H^p_{\mathcal{F}}$  associated with a family  $\mathcal{F}$  of sections which are closely related to the Monge-Ampère equation. We characterize the dual spaces of  $H^p_{\mathcal{F}}$ , which can be realized as Carleson measure spaces, Campanato spaces, and Lipschitz spaces. Then we prove that Monge-Ampère singular operators are bounded from  $H^p_{\mathcal{F}}$  into  $L^p_{\mu}$ , and bounded on both  $H^p_{\mathcal{F}}$  and their dual spaces.

# Composition operators on the Bergman space via quasiconformal mappings

# 方向 (中央大學)

**Abstract:** We extend the theory of composition operators on the Bergman space from analytic symbols to non-analytic symbols. The symbols we choose are quasiconformal mappings, which form a beautiful area in analysis but previously disjoint from operator theory.