National Center for Theoretical Sciences

Taiwan Mathematics School

新學期學分課程上線預告

2025 Fall

Geometric Measure Theory I

Time 2025.9.5~12.9

Every Friday, 09:10-12:10

Venue M210, Department of Mathematics, NTNU

課號：NCTS5057　課程識別碼：V41 U3021

Speaker

Ulrich Menne
National Taiwan Normal University

Organizer

Nan-Kuo Ho
National Tsing Hua University

Course Background & Purposes

A very successful strategy for the study of geometric variational problems is to firstly prove existence in an enlarged class of competitors by means of compactness theorems and subsequently study the regularity of the solution therein. For instance, instead of considering only smooth submanifolds, one proves existence in the classes of boundaries of sets of finite perimeter, integral currents, or integral varifolds. The purpose of the course is to develop the necessary infrastructure on which any of these three theories rests.

Course Outline & Descriptions

The content of the present one-year course appertains to three topics: general measure theory, Grassmann algebra, and rectifiability. Regarding general measure theory, we treat covering theorems (adequate families, covering with enlargement, centred ball coverings, and Vitali relations), derivatives (existence of derivatives, indefinite integrals, density and approximate continuity, and curves of finite length), and Carathéodory's construction (the general construction, Hausdorff and spherical measure, relation to Riemann-Stieltjes integration, densities, Cantor sets, Steiner symmetrisation, equality of measures over Euclidean space, and Lipschitzian extensions of functions). Regarding Grassmann algebra, we cover the exterior algebra of a vector space, alternating forms and duality, interior multiplications, simple *m*-vectors, and inner products. Regarding rectifiability, we study differentials and tangents (differentiation, Rademacher's theorem, factorisation of maps near generic points, submanifolds of Euclidean space, tangent vectors, and relative differentiation) and area for Lipschitzian maps (Jacobians, area of maps of Euclidean space, rectifiable sets, approximate tangent vectors and differentials, area of maps of rectifiable sets, Cartesian products, equality of measures on rectifiable sets, and rectifiable sets and manifolds of class 1). General measure theory and rectifiability will be lectured by Ulrich Menne; Grassmann algebra will be treated as interlude to rectifiability by Myles Workman in the spring term. The main reference text will be typeset lecture notes, see [Men23]. They are based on and expand the relevant parts of Federer's treatise [Fed69].

Grading

Grading is solely determined by individual oral examinations conducted in English.

Prerequisites

We employ measures, measurable sets, Borel regular measures, measurable functions, Lebesgue integration, linear functionals, and product measures. Familiarity with the majority of these concepts is thus expedient.

Contact

Murphy Yu
murphyyu@ncts.tw