Invariance Principle and Dynamics

Time 10:30-11:45, 13:00-14:45
June 18, 2026

Venue R505 Cosmology Bldg., NTU

Speakers

Francois Ledrappier
University of Notre Dame

Organizer

Chih-Hung Chang
National University of Kaohsiung

Introduction & Purposes

The primary goal of the lecture is to explore the ergodic theory behind various examples of the "Invariance Principle" in dynamical systems. In the study of dynamical systems, analyzing the asymptotic properties of successive iterations is an active field of research, frequently utilizing Lyapunov exponents to understand a system's stability under perturbations. While systems that uniformly lack vanishing Lyapunov exponents exhibit established forms of stability, this lecture specifically addresses the question of how to understand systems where vanishing Lyapunov exponents do occur for invariant probability measures.

It turns out that the presence of vanishing Lyapunov exponents often implies the non-trivial invariance of a specific mathematical object, a phenomenon nicknamed the action of an "Invariance Principle." A common mechanism across these varying examples involves introducing a type of entropy—typically an average of Liebner-Küllback information—that quantifies how a given measure is distorted by the system's dynamics. When the Lyapunov exponent vanishes, it forces this entropy to also vanish. Consequently, the vanishing entropy necessitates the desired invariance, which is mathematically demonstrated by applying the equality case in Jensen's inequality.

Historically, the Invariance Principle unifies several classical findings into a single phenomenon. It connects earlier foundational work, such as Furstenberg's criterion for random walks, discoveries regarding Schrödinger operators, and findings on independent products of diffeomorphisms, revealing them all to be manifestations of this exact same underlying principle. The complete nonlinear version of the principle, along with its catchy name and numerous applications, was primarily formalized by A. Avila and M. Viana.

Outline & Descriptions

(1) Introduction of the "Invariance Principle" (IP), explain how vanishing Lyapunov exponents in dynamical systems force a non-trivial invariance of specific mathematical objects.

(2) Review of foundational, classical results that led to the formalization of the IP. This includes Furstenberg's criterion for random matrix products, one-dimensional Schrödinger operators, random homeomorphisms, and compact extensions of hyperbolic systems.

(3) Formalize the modern, non-linear IP established by Avila and Viana. It details the abstract mathematical setting, sketches the core theorem's proof utilizing entropy, and discusses related extensions like deformations and regularities.

(4) Explore direct, modern applications of the formal IP to hyperbolic systems utilizing invariant foliations and non-uniformly expanding random diffeomorphisms.

(5) Examine partially hyperbolic systems with invariant center laminations, specifically focusing on discretized Anosov flows and systems possessing quasi-isometric central leaves.

Prerequisites

Basic knowledge on measure theory and ergodic theory are strongly recommended.

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